Econ 250a
Problem Set 4

1. Consider a simple search model with no on-the-job search, no job destruction, and similar utility while working or searching. As shown in Lecture, the reservation wage \( w^* \) satisfies the equation:

\[
w^* = b + \frac{\lambda}{r} \int_{w^*}^{\infty} (w - w^*) f(w) dw
\]

Assume that the offered wage distribution is discrete, \( w \in \{w_1, ..., w_n\} \) with d.f.:

\[
P(w = w_1) = \Phi\left(\frac{w_1 - \mu}{\sigma}\right),
\]
\[
P(w = w_2) = \Phi\left(\frac{w_2 - \mu}{\sigma}\right) - \Phi\left(\frac{w_1 - \mu}{\sigma}\right)
\]
\[
...\]
\[
P(w = w_{n-1}) = \Phi\left(\frac{w_{n-1} - \mu}{\sigma}\right) - \Phi\left(\frac{w_{n-2} - \mu}{\sigma}\right)
\]
\[
P(w = w_n) = 1 - \Phi\left(\frac{w_{n-1} - \mu}{\sigma}\right),
\]

where \( \Phi \) is the standard normal d.f., and \((\mu, \sigma)\) are parameters. Assume that time in measured in months, and that \( b = 1000 \), \( r = 0.02 \). Set up a numerical procedure to find the optimal reservation wage, using a grid with \( n = 200 \), and setting \( w_1 = 0 \) and \( w_n = 8000 \). Assuming \( \lambda = 0.1 \) and \( \mu = 1800 \), find \( w^* \) for \( \sigma = 100, 500, 1000, 1500, 2000 \). Find the expected duration of job search \( d \) for each \( \sigma \). Graph the relationships between \( \sigma \), \( w^* \) and \( d \).

2. Consider the simplified model with on the job search described in Lecture 9, where wage offers are distributed on the interval \([0, \bar{w}]\) according to a given d.f. \( F(w) \), and the two value functions are:

\[
U(w) = \frac{w - c}{r + \delta} + \frac{\delta}{r + \delta} V + \frac{\lambda(1 - \delta)}{r + \delta} \int_{w}^{\bar{w}} (U(\tilde{w}) - U(w)) f(\tilde{w}) d\tilde{w} \tag{1}
\]

and:

\[
V = \frac{b}{r} + \frac{\lambda (1 - \delta)}{r} \int_{w^*}^{\bar{w}} (U(\tilde{w}) - V) f(\tilde{w}) d\tilde{w} \tag{2}
\]

where \( w^* = b + c \) is the reservation wage.

a) Find the derivative \( U'(w) \). What is \( U'(\bar{w}) \)? What is \( U'(w^*) \)? Draw a picture of \( U(w) \) and \( V \).

b) Suppose \( b = 800 \) (per month), \( c = 0 \), \( \lambda = 0.4 \), \( r = 0.02 \), and \( \delta = 0.25 \), and \( F(w) = N(1200, 400) \). Assume that with this distribution \( \bar{w} = 2500 \) (which is almost true). Develop a numerical procedure to solve for \( U(w) \) and \( V \).
3. Read Card-Chetty-Weber, QJE 2008. The model is discrete time, with variable search intensity and endogenous asset accumulation. Each worker has a fixed wage $w$; all jobs last indefinitely (no job destruction); and utility is additively separable in consumption and search effort. Some details:

- Discount rate $\delta$; interest rate $r$ (non-random)
- Flow utility $u(c_t) - \psi(s_t)$; $c_t =$ consumption; $s_t =$ search effort
- Beginning-of-period: start with assets $A_t$, choose $s_t$
  - If successful (prob $= s_t$) start working, receive $w$,
  - If not successful (prob $= 1 - s_t$), receive benefit $b$,

End-of-period: choose $c_t$ or $c^*_t$ depending on search outcome.

Value function at the end of period $t$ for an individual who finds a job:

$$V_t(A_t) = \max_{A_{t+1} \geq L} u(A_t - A_{t+1}/(1+r) + w_t) + \frac{1}{1+\delta} V_{t+1}(A_{t+1}).$$

Value function at end of period $t$ for an individual who does not find a job:

$$U_t(A_t) = \max_{A_{t+1} \geq L} u(A_t - A_{t+1}/(1+r) + b_t) + \frac{1}{1+\delta} J_{t+1}(A_{t+1}).$$

Beginning-of-period value function: $J_t(A_t) = \max_{s_t} s_t V_t(A_t) + (1-s_t)U_t(A_t) - \psi(s_t)$.

a) Find the first order condition for optimal search intensity, $s^*_t$. Show that the marginal utility of effort depends on the gap $V_t(A_t) - U_t(A_t)$.

b) Using your answer in (a) and the derivatives of $V_t$ and $U_t$, find $\partial s^*_t / \partial A_t$, $\partial s^*_t / \partial w_t$, and $\partial s^*_t / \partial b_t$ and discuss the conditions under which $\partial s^*_t / \partial A_t = 0$.

c) Suppose that benefits $b$ are available indefinitely, that $u(c_t) = \log c_t$, that $\psi(s_t) = a s^2_t$, and that $\delta = r$. Suppose in addition that $L = 0$ (i.e., assets have to be positive). Can you derive an algorithm to compute $U_t(A_t)$ and the optimal search intensity choice $s^*_t(A_t)$?

HINTS: find $V_t(A_t) = V(A_t)$. Now think about how to compute $U_t(A_t) = U(A_t)$. 