Economics 280C Problem
Optimal Currency Regime in a Stochastic Keynesian Model

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The model is a stochastic variant on the basic Dornbusch setup, with different assumptions about price behavior and output determination.

The familiar elements are the LM relationship and uncovered interest parity (UIP):

\[ m_t - p_t = y_t - \lambda i_t + \nu_t, \]

\[ i_t = i^* + E_t \epsilon_{t+1} - \epsilon_t + \epsilon_i, \]

where \( \nu_t \) and \( \epsilon_t \) are, respectively, velocity and risk-premium "shocks" assumed to be i.i.d. such that \( E_t \nu_{t+1} = E_t \epsilon_{t+1} = 0. \)

Let \( w_t \) be the (log) nominal wage; it is predetermined (and thus sticky on date \( t \)), set at date \( t - 1 \) to equal the price level expected for the following period:

\[ w_t = E_{t-1} p_t. \]

Finally, how is output determined? As the intersection of aggregate demand,

\[ y_t = \delta (e_t + p^* - p_t) + g_t, \]

where \( g_t \) is an i.i.d. demand shock such that \( E_t g_{t+1} = 0, \) and aggregate supply,

\[ y_t^* = \theta (p_t - w_t). \]

(a) Under the assumption that \( m \) is constant and that the exchange rate therefore floats, calculate the equilibrium values of \( e_t, p_t, \) and \( y_t. \) [Hint: Because all shocks are i.i.d. and mean-zero, the expected value of \( e \) and of \( p \) is \( m \) while that of \( y \) is zero.] For simplicity take \( i^* = p^* = 0. \)

(b) Calculate the variance of output \( y \) (where you may assume that the covariance matrix of \( g, \nu, \) and \( \epsilon \) is diagonal). (For ease of notation define the composite "financial" shock \( \varphi \equiv \nu - \lambda \epsilon. \) )
(c) Solve the model under a fixed exchange rate, such that $e$ is constant at $\bar{e}$. [Hint: Solve for $p$ and $y$, noting that $E_{t-1}p_t = \bar{e}$.]

(d) Compute the variance of output under a fixed exchange rate.

(e) Prove that when the variance of financial shocks $\sigma^2_\varphi$ is zero, output variance is lower under a floating exchange rate, and that when the variance of demand shocks $\sigma^2_\delta$ is zero, a fixed exchange rate delivers lower output variance.

(f) Now assume that the monetary authority sets interest rates according to a rule

$$i_t = \psi p_t + u_t$$

where $u_t$ is i.i.d. with mean zero. Calculate the equilibrium, including equilibrium output, and the variance of output, $\sigma^2_y$, under this type of floating-rate regime. Now the composite financial shock $\varphi$, which depends on policy “errors” as well as random investor-preference shifts, is defined as $\varepsilon - u$, and you may assume that it is uncorrelated with $g$. [Hint: $E\varepsilon = i^* = 0$, so $E\varepsilon = 0 = E\varphi$.]

(g) When $\sigma^2_\varphi = 0$, which regime, floating or fixed, delivers lower output variability? What about when financial shocks $\varphi$ dominate? Why don’t the $\varphi$-shocks from the LM curve above matter anymore?