1. Peter lives for three periods. He is currently considering three alternative education-work options. He can start working immediately, earning $100,000 in period 1, $110,000 in period 2 (as his work experience leads to higher productivity), and $90,000 in period 3 (as his skills become obsolete and physical abilities deteriorate). Alternatively, he can spend $50,000 to attend college in period 1 and then earn $180,000 in periods 2 and 3. Finally, he can receive a doctorate degree in period 2 after completing his college education in period 1. This last option will cost him nothing when he is attending graduate school in the second period as his expenses on tuition and books will be covered by a research assistantship. After receiving his doctorate, he will become a professor in a business school and earn $400,000 in period 3. Peter’s discount rate is 20 percent per period.
   a. What are Peter’s opportunity costs versus his direct costs of schooling in both options where he invests in further education?
   b. Assume Peter’s discount rate is 20 percent per period. What education path maximizes Peter’s net present value of his lifetime earnings?
   c. What is the maximum value of Peter’s discount rate so that he chooses to get more education over no further schooling?

(a) If Peter goes to college in period 1, the direct cost of schooling is $50,000. The opportunity cost is the lost wage income in period 1 $100,000.
(b) The present discounted values of Peter’s earnings associated with each of the alternatives are

\[ PV_{HS} = 100,000 + \frac{110,000}{1.2} + \frac{90,000}{1.2^2} = $254,167 \, , \]

\[ PV_{COL} = -50,000 + \frac{180,000}{1.2} + \frac{180,000}{1.2^2} = $225,000 \, , \]

and

\[ PV_{PhD} = -50,000 + \frac{0}{1.2} + \frac{400,000}{1.2^2} = $227,778 \, . \]

Thus, the best option for Peter is to start working upon completing high school.
(c) r=11%

2. Suppose Carl’s wage-schooling locus is given by

<table>
<thead>
<tr>
<th>Years of Schooling</th>
<th>Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>$18,500</td>
</tr>
<tr>
<td>10</td>
<td>$20,350</td>
</tr>
<tr>
<td>11</td>
<td>$22,000</td>
</tr>
<tr>
<td>12</td>
<td>$23,100</td>
</tr>
<tr>
<td>13</td>
<td>$23,900</td>
</tr>
<tr>
<td>14</td>
<td>$24,000</td>
</tr>
</tbody>
</table>
(a) Derive the marginal rate of return schedule. When will Carl quit school if his discount rate is 4 percent? What if the discount rate is 12 percent?
(b) Why might one assume that different workers have different wage-schooling loci? How would differences in the wage-schooling locus influence the empirical estimates of the returns to schooling in conventional cross-section studies? Do you know of any empirical evidence that support the hypothesis that wage schooling loci differ strongly across workers?

(a) The marginal rate of return is given by the percentage increase in earnings if the worker goes to school one additional year.

<table>
<thead>
<tr>
<th>Schooling</th>
<th>Earnings</th>
<th>MRR</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>$18,500</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>$20,350</td>
<td>10.0</td>
</tr>
<tr>
<td>11</td>
<td>$22,000</td>
<td>8.1</td>
</tr>
<tr>
<td>12</td>
<td>$23,100</td>
<td>5.0</td>
</tr>
<tr>
<td>13</td>
<td>$23,900</td>
<td>3.5</td>
</tr>
<tr>
<td>14</td>
<td>$24,000</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Carl will quit school when the marginal rate of return to schooling falls below his discount rate. If his discount rate is 4 percent, therefore, he will quit after 12 years of schooling; if his discount rate is 12 percent, he will quit after 9 years of schooling.

(b) If workers differ in their ability to succeed in school and in their ability to be productive on the job, this would result in different wage-schooling loci for each type of worker. Cross sectional wage schooling regressions assume that all workers face the same wage schooling locus. If there are difference in wage schooling locus, workers with low ability, and a low wage-schooling locus will decide to invest little into schooling, while workers with high ability and a high wage-schooling locus will invest in more schooling. Comparing these worker’s earnings and schooling levels will obviously overestimate the return to schooling.

All the studies we discussed in class, which tried to control for an ability bias in the estimates for return to schooling, did not find any evidence of a bias.

3. Jill is planning the timing of her on-the-job training investments over the life cycle. What happens to Jill’s OJT investments at every age if

(a) the market-determined rental rate to an efficiency unit falls?

The marginal revenue of investing in OJT declines so that Jill will invest less at each age.

(b) Jill’s discount rate increases?

If Jill’s discount rate increases she becomes more “present oriented”, reducing the future benefits associated with OJT. Thus her OJT investments fall.

(c) the government passes legislation delaying the retirement age until age 70.
The marginal revenue of investing in OJT increases because the payoff period to the investment is longer. Thus, she undertakes more OJT in this case.

(d) technological progress is such that much of the OJT acquired at any given age becomes obsolete within the next 10 years.

The marginal revenue to investing in OJT declines and the amount of OJT acquired falls.

(e) Jill expects to change her job frequently while she is young and take a more stable job once she is older. Jill will decide to invest mostly in general training while she is young and changes jobs. Once she has a stable job, she will start also investing in specific training.

4. Consider the following (highly) simplified description of the U.S. wage distribution and income and payroll tax schedule. Suppose 50 percent of households earn $40,000, 30 percent earn $70,000, 15 percent earn $120,000, and 5 percent earn $500,000. Marginal income tax rates are 0 percent up to $30,000, 15 percent on income earned from $30,000 to $60,000, 25 percent on income earned from $60,000 to $150,000, and 35 percent on income earned in excess of $150,000. There is also a 7.5 percent payroll tax on all income up to $80,000.

(a) What is the marginal tax rate and average tax rate for each of the four types of households? What is the average household income, payroll, and total tax bill? What percent of the total income tax is paid by each of the four types of households? What percent of the total payroll tax bill is paid by each of the four types of household?

Marginal tax rates:
- Households earning $40,000 is 15 percent + 7.5 percent = 22.5 percent.
- Households earning $70,000 is 25 percent + 7.5 percent = 32.5 percent.
- Households earning $120,000 is 25 percent + 0 percent = 25 percent.
- Households earning $500,000 is 35 percent + 0 percent = 35 percent.

Income tax bill:
- Households earning $40,000 is 15 percent of $40,000 = $6,000.
- Households earning $70,000 is 25 percent of $70,000 = $17,500.
- Households earning $120,000 is 25 percent of $120,000 = $30,000.
- Households earning $500,000 is 35 percent of $500,000 = $175,000.

Payroll tax bill:
- Households earning $40,000 is 7.5 percent of $40,000 = $3,000.
- Households earning $70,000 is 7.5 percent of $70,000 = $5,250.
- Households earning $120,000 is 7.5 percent of $120,000 = $9,000.
- Households earning $500,000 is 7.5 percent of $500,000 = $37,500.

Total tax bill and average tax rates:
- Households earning $40,000 is $6,000 + $3,000 = $9,000 => ATR = 11.25 percent.
- Households earning $70,000 is $17,500 + $5,250 = $22,750 => ATR = 17.5 percent.
- Households earning $120,000 is $30,000 + $9,000 = $39,000 => ATR = 14.00 percent.
- Households earning $500,000 is $175,000 + $37,500 = $212,500 => ATR = 35.00 percent.
Average tax bills over all households are:

Income: \(0.5(1,500) + 0.3(7,000) + 0.15(19,500) + 0.05(149,500) = 13,250\)

Payroll: \(0.5(3,000) + 0.3(5,250) + 0.15(6,000) + 0.05(6,000) = 4,275\).

Total: \(17,525\).

Percent of the total income tax collected by the government that is paid by each household group:

- \(\$40,000\) households pay \(\frac{0.5(1,500)}{13,250} = 5.67\) percent.
- \(\$70,000\) households pay \(\frac{0.3(7,000)}{13,250} = 15.85\) percent.
- \(\$120,000\) households pay \(\frac{0.15(19,500)}{13,250} = 22.08\) percent.
- \(\$500,000\) pay \(\frac{0.05(149,500)}{13,250} = 56.42\) percent.

Percent of the total payroll tax collected by government that is paid by each household group:

- \(\$40,000\) households pay \(\frac{0.5(3,000)}{4,275} = 35.09\) percent.
- \(\$70,000\) households pay \(\frac{0.3(5,250)}{4,275} = 36.84\) percent.
- \(\$120,000\) households pay \(\frac{0.15(6,000)}{4,275} = 21.05\) percent.
- \(\$500,000\) pay \(\frac{0.05(6,000)}{4,275} = 7.02\) percent.

(b) What are the 50-10 wage gap and the 90-10 wage gap for gross earnings? What do these measures tell us about the income distribution?

The 50-10 wage gap = \(\frac{\$40,000 - \$40,000}{\$40,000} = 0\). The 90-10 wage gap = \(\frac{\$120,000 - \$40,000}{\$40,000} = 2\). There is no inequality at the bottom of the income distribution. But the gap in wages between the 90th and the 10th percentile is 200% of the wages in the 10th percentile.

(c) What is the Gini coefficient for the economy when comparing after-tax incomes across households? (Hint: assume there are 1,000 households in the economy.) What happens to the Gini coefficient if all taxes were replaced by a single 20 percent flat tax on all incomes?

Suppose there were 1,000 households. Total after-tax income is \(500 \times \$35,500 + 300 \times \$57,750 + 150 \times \$94,500 + 50 \times \$344,500 = \$66,475,000\).

The cumulative gross income shares of the four income groups, therefore, are:

- \(500 \times \$35,500 / \$66,475m = 26.7\) percent.
- \(26.7 + 300 \times \$57,750 / \$66,475m = 52.8\) percent.
- \(52.8 + 150 \times \$94,500 / \$66,475m = 74.1\) percent.
- \(74.1 + 50 \times \$344,500 / \$66,475m = 100.0\) percent.

The area under the Lorenz curve, therefore, is \((0.5)(\frac{1}{2})(0.267) + (0.3)[0.267 + \frac{1}{2}(0.528 - 0.267)] + (0.15)[0.528 + \frac{1}{2}(0.741 - 0.528)] + (0.05)[0.741 + \frac{1}{2}(1 - 0.741)] = 0.6675 + 0.11925 + 0.95175 + 0.043525 = 0.3247\). The Gini coefficient, therefore, is \(\frac{0.5 - 0.3247}{0.5} = 0.3506\).

Suppose there was a flat tax of 20 percent. Total after-tax income is \(500(0.8)(\$40,000 + 300(0.8)(\$70,000) + 150(0.8)(\$120,000) + 50(0.8)(\$500,000) = \$67,200,000\).

The cumulative gross income shares of the four income groups, therefore, are:

- \(500 \times \$32,000 / \$67.2m = 23.8\) percent.
- \(23.8 + 300 \times \$56,000 / \$67.2m = 48.8\) percent.
- \(48.8 + 150 \times \$96,000 / \$67.2m = 70.2\) percent.
- \(70.2 + 50 \times \$400,000 / \$67.2m = 100.0\) percent.
The area under the Lorenz curve, therefore, is \((.5)(.238) + (.3)[.238+(.15)(.488-.238)] + (.15)[.488+(.15)(.702-.488)] + (.05)[.702+(.05)(1-.702)] = .0595 + .1089 + .08925 + .04255 = .3002\). The Gini coefficient if there was a 20 percent flat tax, therefore, would be \([.5 - .3002]/.5 = .3996\). This means inequality is higher if a flat tax is of 20 percent is imposed and the tax system above reduces inequality in incomes.

(d) A presidential candidate wants to remove the cap on payroll taxes so that every household would pay payroll taxes on all of its income. To what level could the payroll tax rate be reduced under the proposal while keeping the total amount of payroll tax collected the same?

Suppose there are 1,000 households: 500 pay $3,000, 300 pay $5,250, and 200 pay $6,000. The total payroll tax receipts, therefore, are $4,275,000, while total income is $84 million. Thus, generating $4.275 million of tax on $84 million of income requires a tax rate of 5.089 percent.

5. The two points for the international income distributions reported in Table 8-1 can be used to make a rough calculation of the Gini coefficient. Use a spreadsheet to estimate the Gini coefficient for each country. Which three countries have the most equal income distribution? Which three countries have the most unequal income distribution?

If one considers the percent of income received by the poorest and richest 10 percent of households called \(P\) and \(R\) respectively, the Gini coefficient is

\[.5 - \left(\frac{1}{2}\right)(.1)P - .8P - .8(\frac{1}{2})(R-P) - .1R - .1(\frac{1}{2})(1-R) \].

Using excel, the results are:

<table>
<thead>
<tr>
<th>Country</th>
<th>Gini Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guatemala</td>
<td>0.4716</td>
</tr>
<tr>
<td>Chile</td>
<td>0.4815</td>
</tr>
<tr>
<td>Mexico</td>
<td>0.5148</td>
</tr>
<tr>
<td>Dominican Republic</td>
<td>0.5400</td>
</tr>
<tr>
<td>India</td>
<td>0.5670</td>
</tr>
<tr>
<td>United States</td>
<td>0.6093</td>
</tr>
<tr>
<td>Germany</td>
<td>0.6147</td>
</tr>
<tr>
<td>Israel</td>
<td>0.6246</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.6336</td>
</tr>
<tr>
<td>Italy</td>
<td>0.6363</td>
</tr>
<tr>
<td>Australia</td>
<td>0.6534</td>
</tr>
<tr>
<td>France</td>
<td>0.6561</td>
</tr>
<tr>
<td>Canada</td>
<td>0.6606</td>
</tr>
<tr>
<td>Norway</td>
<td>0.6669</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.6705</td>
</tr>
<tr>
<td>Austria</td>
<td>0.6777</td>
</tr>
<tr>
<td>Hungary</td>
<td>0.6786</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.6885</td>
</tr>
</tbody>
</table>

Thus, the three countries with the most inequality are Guatemala, Chile, and Mexico. The three countries with the most equality are Austria, Hungary, and Sweden. It should be emphasized that these are very crude measures as they rely on only two points in the income distribution.