The Multiplier

Agenda

• Keynesian Cross (or Multiplier) Model
  - The Multiplier
    • The (Simple) Spending Multiplier
    • Multipliers with Income Tax Rates
    • Multipliers with Endogenous Imports

The Multiplier

• What happens to Ye when G increases?
  - Graphically, Ep line shifts up.
  - Same for increases in C0, Ip, X or decreases in M.
    • Or for any exogenous change in spending.

The Multiplier

• By how much does Ye change?
  - $\delta Y_e > \delta G$
  - This concept is the simple income multiplier
    • Multiplier for short

The Multiplier

• Ye, an endogenous variable, depends on:
  - Ap, the sum of several exogenous variables, and
  - mpc, a parameter

The Multiplier

• Algebraically,
  - $Y = C + I + G + (X - M)$
  - And $C = C_0 + mpc( Y - T )$
  - $Y = C_0 + mpcY - mpcT + I + G + (X - M)$
  - Let $Ap = C_0 - mpcT + I + G + (X - M)$
  - $Y = Ap + mpcY$
  - $Y = Ap / (1 - mpc)$
The Multiplier

• If
  \[ Y = \frac{Ap}{1 - mpc} \]
• Then
  \[ \Delta Y = \frac{\Delta Ap}{1 - mpc} \]
• or
  \[ \frac{\Delta Y}{\Delta Ap} = \frac{1}{1 - mpc} \]
• So
  \[ k(Ap) = \frac{\Delta Y}{\Delta Ap} = \frac{1}{1 - mpc} \]

The Multiplier

• If
  \[ G = 400, Ap = 1,000 \text{ and } mpc = 0.9, \]
• then
  \[ Y = \frac{Ap}{1 - mpc} \]
  \[ = \frac{1,000}{0.1} \]
  \[ = 10,000 \]

The Multiplier

• Now if
  \[ G = 500, Ap = 1,100 \text{ and } mpc = 0.9, \]
• then
  \[ Y = \frac{Ap}{1 - mpc} \]
  \[ = \frac{1,100}{0.9} \]
  \[ = 1,100 / 0.1 \]
  \[ = 11,000 \]
• And \( \Delta Y = 1,000 > \Delta G = 100 \)

The Multiplier

• Continuing further
  \[ \frac{1}{1 - mpc} \] is called the simple multiplier
  • Because \( 0 < mpc < 1, \)
  • \( (1 - mpc) < 1, \) and
  • \( 1/(1 - mpc) > 1 \)
  \[ \text{The larger is } mpc, \text{ the greater is the multiplier} \]
  • The steeper the spending line (the slope of Ep), the
greater is the multiplier and the change in Ye.

The Multiplier

• Why Does This Work?
  \[ \text{When } G \text{ increases, } Y \text{ increase as well} \]
  • \( G \) is a part of \( Y \)
  \[ \text{When } Y \text{ increases, } YD \text{ increase, so } C \text{ increases} \]
  \[ \text{When } C \text{ increases, } Y \text{ increases further} \]
  • \( C \) is a part of \( Y \)
  \[ \text{This process continues but gets smaller and}
smaller with each round of spending} \]
### The Multiplier

**mpc = 0.9**

<table>
<thead>
<tr>
<th>Spending Round</th>
<th>Delta G</th>
<th>Delta C</th>
<th>Delta Y</th>
<th>Total Delta Y</th>
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<tr>
<td>1</td>
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<td>---</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>2</td>
<td>---</td>
<td>90.0</td>
<td>90.0</td>
<td>190.0</td>
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<tr>
<td>3</td>
<td>---</td>
<td>81.0</td>
<td>81.0</td>
<td>271.0</td>
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<td>4</td>
<td>---</td>
<td>72.9</td>
<td>72.9</td>
<td>343.9</td>
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<tr>
<td>5</td>
<td>---</td>
<td>65.6</td>
<td>65.6</td>
<td>409.5</td>
</tr>
<tr>
<td>6</td>
<td>---</td>
<td>59.0</td>
<td>59.0</td>
<td>468.6</td>
</tr>
</tbody>
</table>

**Equilibrium**

100.0  900.0  1,000.0  1,000.0

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### The Multiplier

- **Why Does This Work?**
  - The increase in Y is > the increase in G because of the increase in C.
  - The multiplier measures the amount of Y stimulated by an increase in G or any other categories of autonomous spending (that is not itself sensitive to Y).
    - This spending cycle takes place very quickly
    - Less than a year

- **Observations**
  - Ye depends on the exogenous variables
  - Ye is a multiple of these exogenous variables
  - Relatively small changes in these exogenous variables can lead to large changes in Ye
  - The size of the multiplier depends on the magnitude of the “leakages”.
    - The greater the leakages, the smaller the multiplier.

### Multipliers w/ Income Tax Rates

- **Multipliers with Income Tax Rates:**
  - Assume \( T = tY \)
    - \( t \) is the marginal tax rate
    - \( 0 < t < 1 \)
    - \( tY \) are induced taxes

- **Reformulated Consumption Function**
  - \( C = C_0 + mpc ( Y - T ) \)
  - \( C = C_0 + mpc ( Y - tY ) \)
  - \( C = C_0 + mpc ( 1 - t ) Y \)

- **Equilibrium**

\[
Y = \frac{Ap}{1 - mpc(1 - t)}
\]

\[
\Delta Y = \frac{\Delta Ap}{1 - mpc(1 - t)}
\]

\[
k(Ap) = \frac{1}{1 - mpc(1 - t)}
\]

- If \( mpc = 0.9 \) and \( t = 0.1 \), then
  \[
k(Ap) = \frac{1}{1 - 0.9(1 - 0.1)} = 1 / 0.19 = 5.26\]
Multipliers w/ Income Tax Rates

- Implications of adding income tax rate, \( t \)
  - The size of the multiplier is reduced.
  - There are more leakages into taxes.
  - A smaller multiplier \( \Rightarrow \) shallower business cycles.
  - \( t \) serves as an “automatic” stabilizer.

- \( Y_e \) changes when \( t \) changes.
  - Higher \( Y_e \) when \( t \) declines; lower \( Y_e \) when \( t \) increases.
  - \( E_p \) line rotates.
    - Fixed on the vertical axis.

Multipliers w/ Income Tax Rates

- If
  - \( A_p = 1,000, \) \( mpc = 0.9, \) and \( t = 0.2 \)
- then
  - \( Y = A_p / [1 - mpc \ (1 - t)] \)
  - \( = 1,000 / [1 - 0.9 \ (1 - 0.2)] \)
  - \( = 1,000 / 0.28 \)
  - \( = 3,571 \)
- And \( k(A_p) = 3.57 \)

Multipliers w/ Income Tax Rates

- If
  - \( A_p = 1,000, \) \( mpc = 0.9, \) and \( t = 0.15 \)
- then
  - \( Y = A_p / [1 - mpc \ (1 - t)] \)
  - \( = 1,000 / [1 - 0.9 \ (1 - 0.15)] \)
  - \( = 1,000 / 0.235 \)
  - \( = 4,255 \)
- And \( k(A_p) = 4.255 \)

Multipliers w/ Endogenous M

- Multipliers with Endogenous Imports.
  - Assume \( M = M_0 + mY \)
    - \( M_0 \) are autonomous imports.
    - \( m \) is the marginal propensity to import.
    - \( 0 < m < 1 \)
    - And \( mY \) are induced imports.

Multipliers w/ Endogenous M

- Algebraically,
  - \( Y = C + I + G + (X - M) \)
    where \( C = C_0 + mpc \ (1 - t)Y \) and \( M = M_0 + mY \)
  - \( Y = C_0 + mpc \ (1 - t)Y + I + G + X - M_0 - mY \)
  - Let \( A_p = C_0 + I + G + X - M_0 \)
  - Then \( Y = A_p + mpc \ (1 - t)Y - mY \)
  - or \( Y = A_p + [mpc \ (1 - t) - m]Y \)
  - or \( Y = A_p / [1 - mpc \ (1 - t) + m] \)
Multipliers w/ Endogenous M

- Multipliers with Endogenous Imports:
  - $Y = \frac{Ap}{1 - mpc (1 - t) + m}$
  - $\Delta Y = \frac{\Delta Ap}{1 - mpc (1 - t) + m}$
  - $k(Ap) = \frac{1}{1 - mpc (1 - t) + m}$
  - If $mpc = 0.9$, $t = 0.1$, and $m = 0.2$, then
    \[ k(Ap) = \frac{1}{1 - 0.9 \times (1 - 0.1) + 0.2} = \frac{1}{1 - 0.81 + 0.2} = 2.56 \]

Implications of Endogenizing Imports:

- The size of the multiplier is reduced.
- There are more leakages into imports.
- A smaller multiplier $\Rightarrow$ shallower business cycles.

The is called the open economy multiplier.
- Always smaller than the closed economy multiplier.

- Ye changes when $m$ changes.
  - Higher $Ye$ when $m$ declines; lower $Ye$ when $m$ increases.
  - Ep line rotates.

If $Ap = 1,000$, $mpc = 0.9$, $t = 0.2$, and $m = 0.2$

- then
  - $Y = \frac{1,000}{1 - 0.9 \times (1 - 0.2) + 0.2} = \frac{1,000}{0.48} = 2083$
  - And $k(Ap) = 2.083$

If $Ap = 1,000$, $mpc = 0.9$, $t = 0.2$, and $m = 0.15$

- then
  - $Y = \frac{1,000}{1 - 0.9 \times (1 - 0.2) + 0.15} = \frac{1,000}{0.43} = 2326$
  - And $k(Ap) = 2.326$
The Multiplier

- Summarizing the Multiplier:
  - \( k(A_p) = \frac{1}{\text{marginal leakage rate}} \)
    - Consumption Only: \( \frac{1}{1 - mpc} \)
    - With Income Tax Rates: \( \frac{1}{1 - mpc(1 - t)} \)
    - With Endogenous Imports: \( \frac{1}{1 - mpc(1 - t) + m} \)
  - If \( mpc = 0.9 \), \( t = 0.1 \), and \( m = 0.2 \), then
    - Consumption: \( \frac{1}{1 - mpc} = 10 \)
    - w/ Income Tax Rates: \( \frac{1}{1 - mpc(1 - t)} = 5.26 \)
    - w/ Endogenous Imports: \( \frac{1}{1 - mpc(1 - t) + m} = 2.56 \)

The Multiplier

- Implications for Business Cycles
- Implications for Stabilization Policy

The Multiplier

- Major Points:
  - \( Ye \) is determined where \( E = E_p \).
  - The multiplier expresses the simple relationship between changes in \( A_p \) and \( Ye \).
  - Introduction of income tax rates and endogenous imports reduces the size of the multiplier.