Solow Growth Model, Part 1

Agenda

• The Saving Function.
• The Balanced Investment Function.
• The Solow Growth Model.

The Saving Function

• Saving, \( S \), is how Investment, \( I \), is financed.
  \[ S = I_a \]
  \( I_a \) = actual investment
  Which, over time, determines how much \( K \).

• Transform \( S = \nu \ast Y \) by dividing by \( N \).
  \[ S/N = \nu \ast Y/N \]
  \[ = \nu \ast A \ast f(K/N) \]

  Remember \( S = I_a \), so
  \( S/N = I_a/N \)  (This is an identity.)
  \[ \Rightarrow \text{where } I_a = \text{actual } I. \]

The Production and Saving Functions

The Saving Function

• What happens if:
  \[ \nu \text{ changes?} \]
  \[ A \text{ changes?} \]
**The Investment Function**

- $I_g = k \cdot \text{dot} + \delta \cdot K$

- $I_g = k \cdot \text{dot} + \delta \cdot K$

- $I_g = (k \cdot \text{dot} + \delta) \cdot K$

- $\frac{I}{N} = (k \cdot \text{dot} + \delta) \cdot \frac{K}{N}$

**Balanced Investment Function**

- How much $I/N$ is needed to keep $K/N$ constant?

  - For $K/N$ to be constant, $k \cdot \text{dot} = n \cdot \text{dot}$
    - This is called “The Balanced Growth Path.”

  - If $\frac{I}{N} = (k \cdot \text{dot} + \delta) \cdot \frac{K}{N}$ and $k \cdot \text{dot} = n \cdot \text{dot}$

  - Then $Ib/N = (n \cdot \text{dot} + \delta) \cdot \frac{K}{N}$
    - This is called “balanced investment”, $Ib$. 

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**The Investment Function**

- $I_g = I_n + \text{depreciation}$
  - $I_g$ = gross investment.
  - $I_n$ = net investment.

- $I_n = \Delta K = k \cdot \text{dot} \cdot K(t-1)$
  - Because $\Delta K$ is small, $K(t)$ is approximately $\approx K(t-1)$.
  - Therefore, $\Delta K = k \cdot \text{dot} \cdot K(t)$.

- Depreciation = $\delta \cdot K(t-1)$
  - $\delta$ is the rate of economic depreciation of $K$, $0 < \delta < 1$.
  - Because $K(t)$ is approximately $\approx K(t-1)$, depreciation is approximately $\approx \delta \cdot K(t)$.
Balanced Investment Function

\[ \frac{I_b}{N} = (n - \text{dot} + \delta) \times \frac{K}{N} \]

The Solow Growth Model

- The Solow Growth Model combines:
  - The production function,
  - The saving function, and
  - The balanced investment function.

- All in per capita terms.
  - Measured by \( N \) as a proxy for population.

- Assumes \( A = \alpha \), a constant. Then, \( \text{a-dot} = 0 \).

Balanced Investment Function

- What happens if:
  - \( n - \text{dot} \) changes?
  - \( \delta \) changes?

Solow Growth Model

- \( A \) is called “steady-state equilibrium.”
  - \( S/N = I/N = I_b/N \)
    - \( S = I_a = I_b \).
  - Once the economy is at its steady state there are no economic pressures to move away from it.
    - This is a stable equilibrium.

The Solow Growth Model

- \( Y/N = A \times f\left(\frac{K}{N}\right) \)
- \( S/N = v \times Y/N = I_a/N \)
- \( I_b/N = (n - \text{dot} + \delta) \times K/N \)
- \( S/N = I_a/N = I_b/N \)
**Solow Growth Model**

- How fast is the economy growing at A?
  - At the steady-state, \( \frac{Y}{N} \) is constant.
  - Therefore, \( y-\text{dot} = n-\text{dot} \).
    - The economy is growing at the same rate as labor input is growing.

**Solow Growth Model**

- How fast is the capital stock growing at A?
  - At the steady-state, \( \frac{K}{N} \) is constant.
  - Therefore, \( k-\text{dot} = n-\text{dot} \).
    - The capital stock is growing at the same rate as labor input is growing.

**The Solow Growth Model**

- Therefore, in a steady state:
  - \( y-\text{dot} = n-\text{dot} = k-\text{dot} \)

**Solow Growth Model**

- Most advanced economies are generally near their steady states.
  - What is potential GDP, \( Y^* \)?
    - The \( Y \) in \( Y/N \) at the steady state.
  - How fast is \( Y^* \) growing?
    - \( y^*-\text{dot} = n-\text{dot} \) if \( a-\text{dot} = 0 \).
Solow Growth Model

• Disequilibria dynamics
  ➢ What if the economy is not at its steady-state?
    • $K/N < (K/N)_A$
      ➢ German reunification, increase $N$
      ➢ Natural disasters, decrease $K$
    • $I_a > I_b$ and $K$ increases

Solow Growth Model

• Disequilibria dynamics
  ➢ What if the economy is not at its steady-state?
    • $K/N > (K/N)_A$
      ➢ Plagues, decrease $N$
      ➢ Foreign wars, decrease $N$
      ➢ Neutron bomb, decrease $N$
    • $I_a < I_b$ and $K$ decreases
Solow Growth Model

- Disequilibria dynamics:
  - Growth process is stable.
  - Economy will always converge over time to the SAME steady state.
  - However, growth rates during the transition period will be different.
    - When $K/N < (K/N)_A$, $y$-dot $> n$-dot.
    - When $K/N > (K/N)_A$, $y$-dot $< n$-dot.