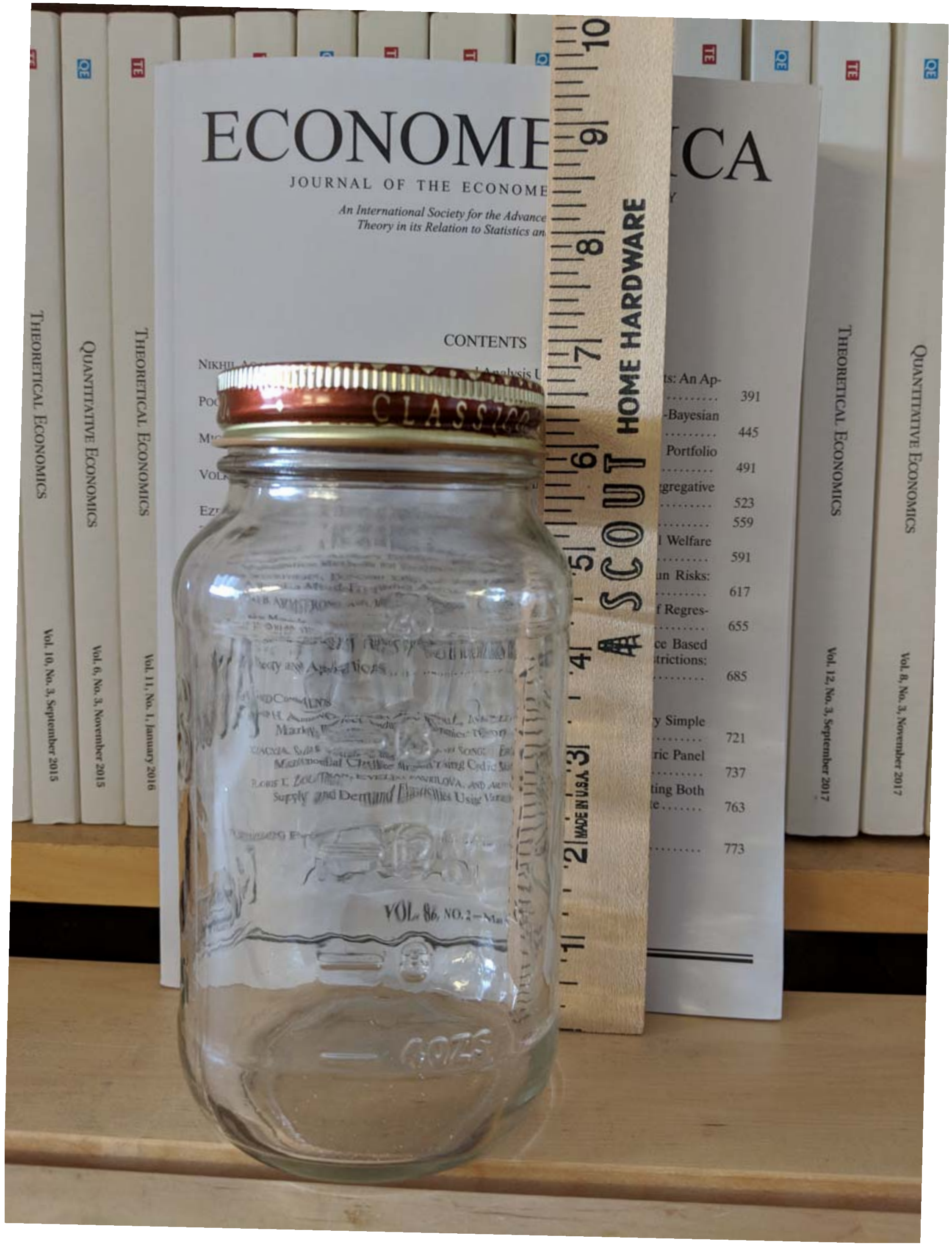
University of California – Berkeley
Department of Economics
Game Theory in the Social Sciences
(ECON C110 | POLSCI C135)
Fall 2023

Lecture II
Static interactions and strategic-form games

Aug 31, 2023



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Types of auctions

Sequential / simultaneous

Bids may be called out sequentially or may be submitted simultaneously in sealed envelopes:

- English (or oral) – the seller actively solicits progressively higher bids and the item is soled to the highest bidder.
- Dutch – the seller begins by offering units at a "high" price and reduces it until all units are soled.
- Sealed-bid – all bids are made simultaneously, and the item is sold to the highest bidder.

First-price / second-price

The price paid may be the highest bid or some other price:

- First-price – the bidder who submits the highest bid wins and pay a price equal to her bid.

- Second-prices – the bidder who submits the highest bid wins and pay a price equal to the second highest bid.

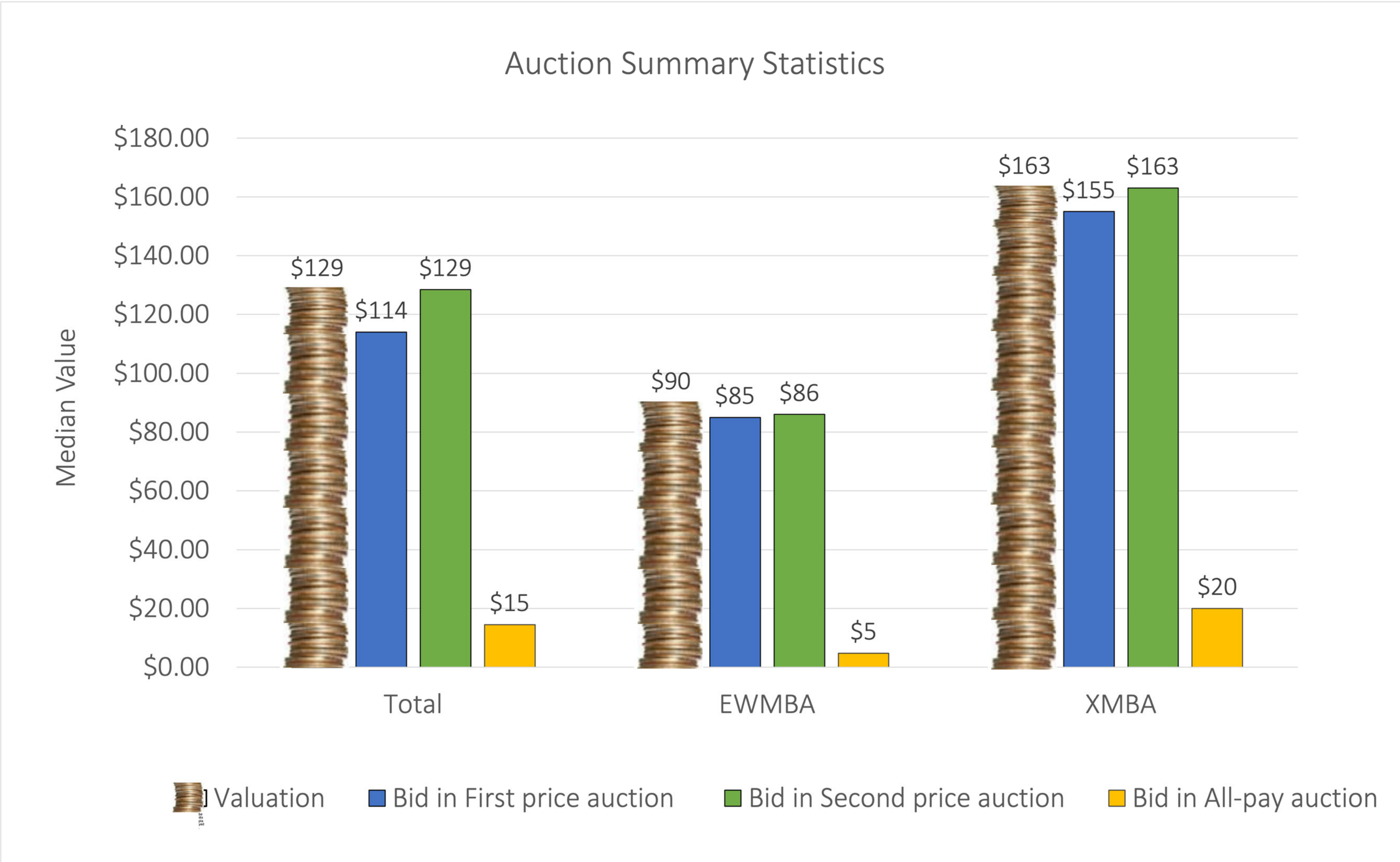
Variants: all-pay (lobbying), discriminatory, uniform, Vickrey (William Vickrey, Nobel Laureate 1996), and more.

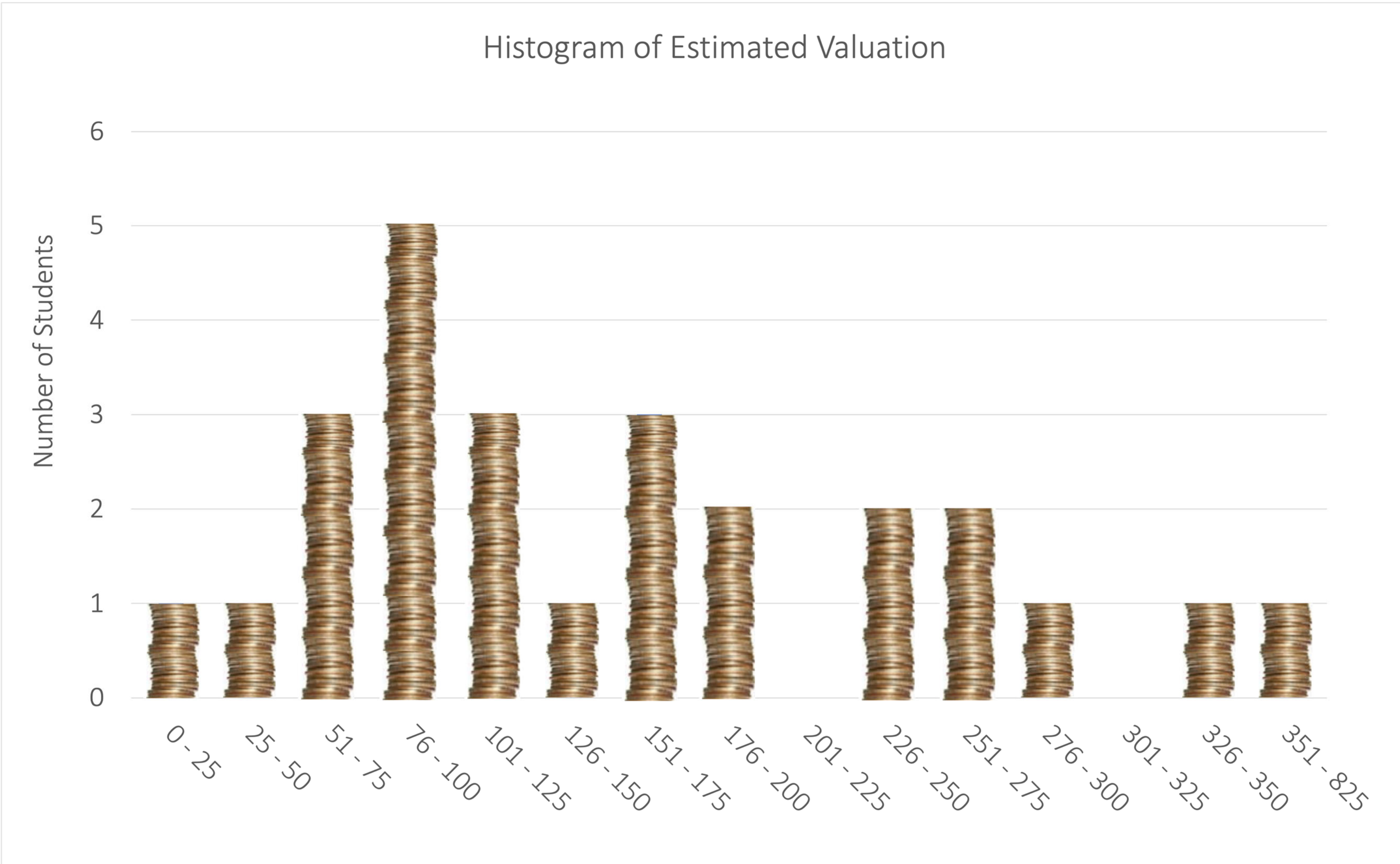
Private-value / common-value

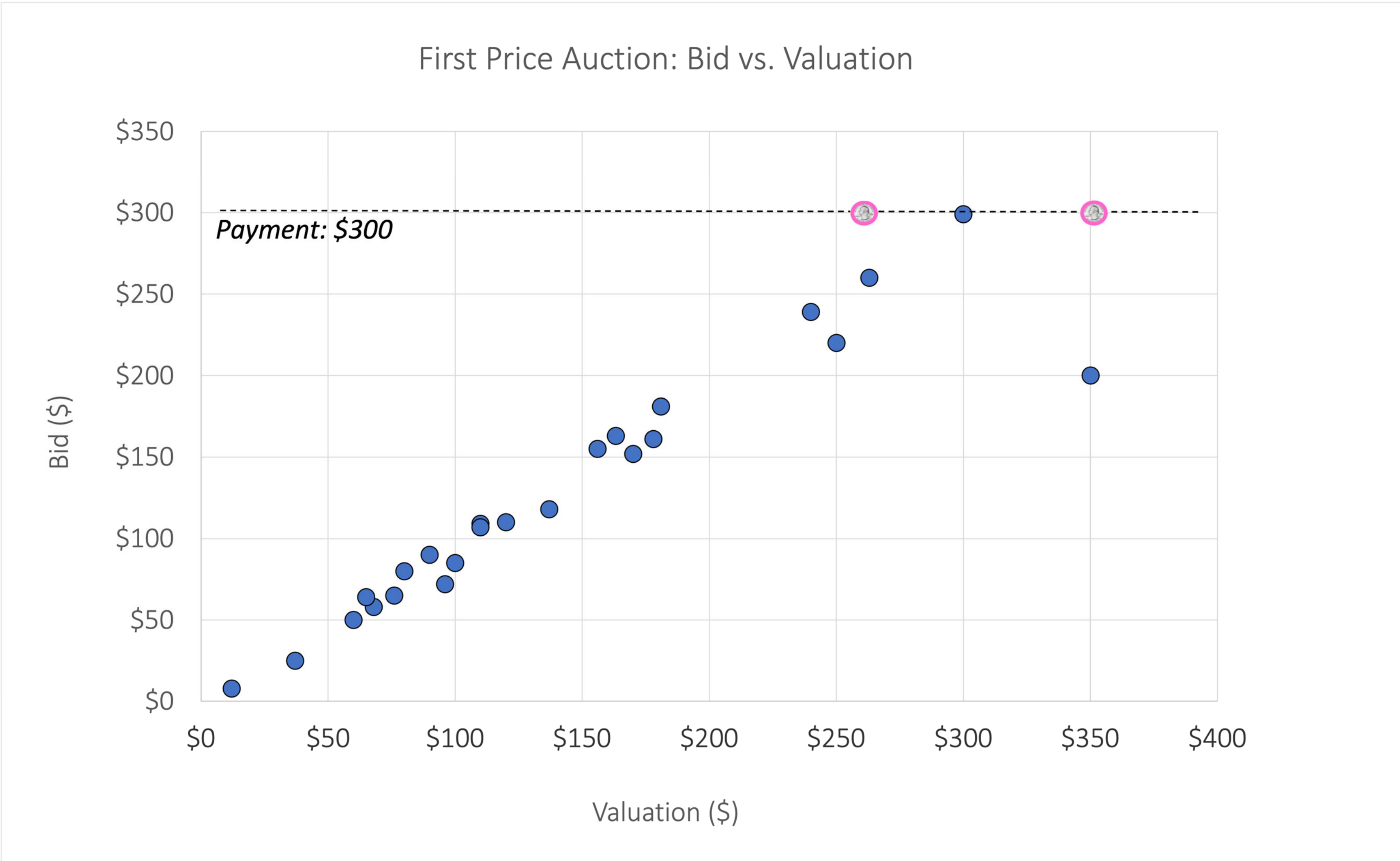
Bidders can be certain or uncertain about each other's valuation:

- In private-value auctions, valuations differ among bidders, and each bidder is certain of her own valuation and can be certain or uncertain of every other bidder's valuation.

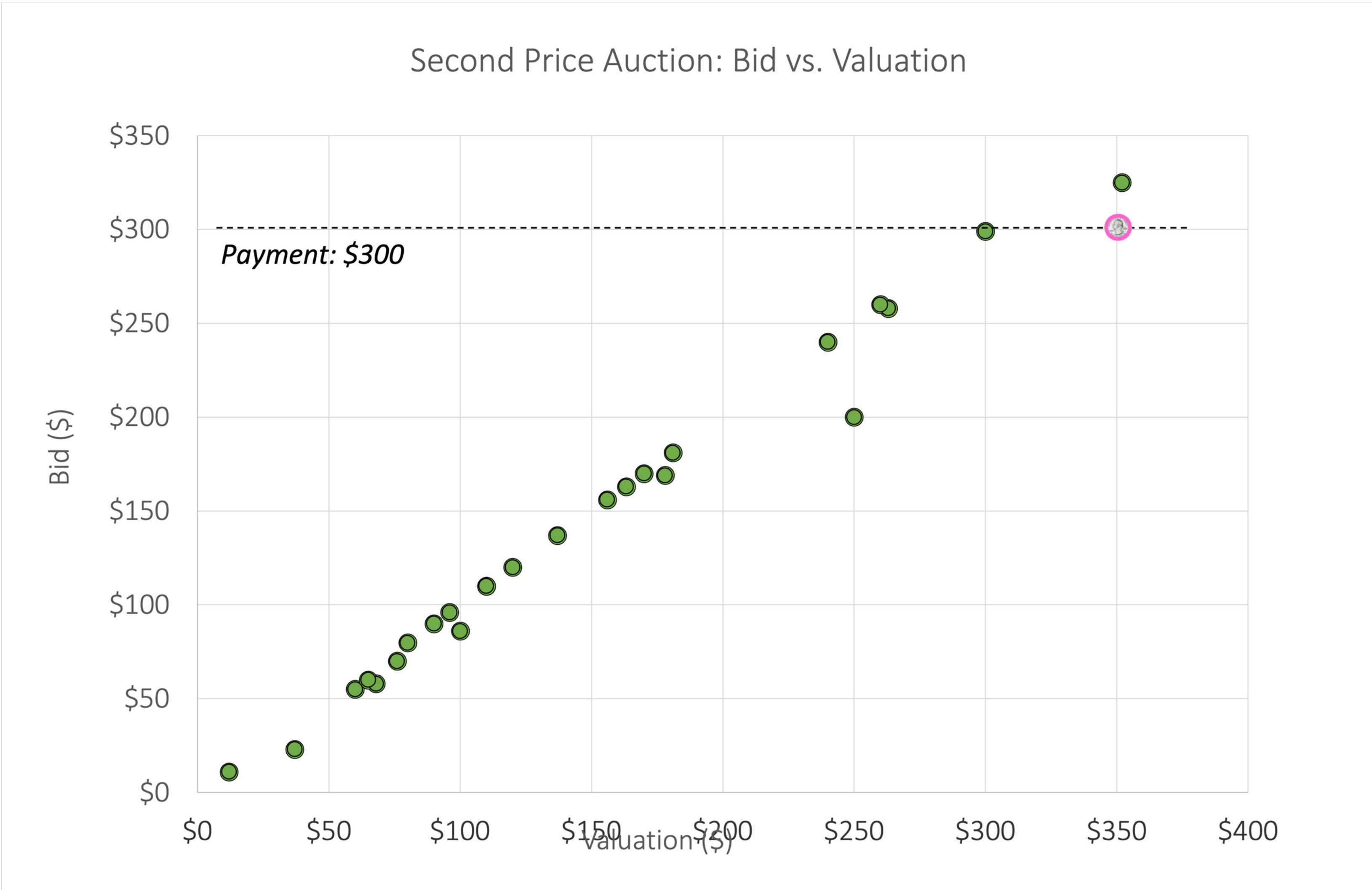
- In common-value auctions, all bidders have the same valuation, but bidders do not know this value precisely and their estimates of it vary.

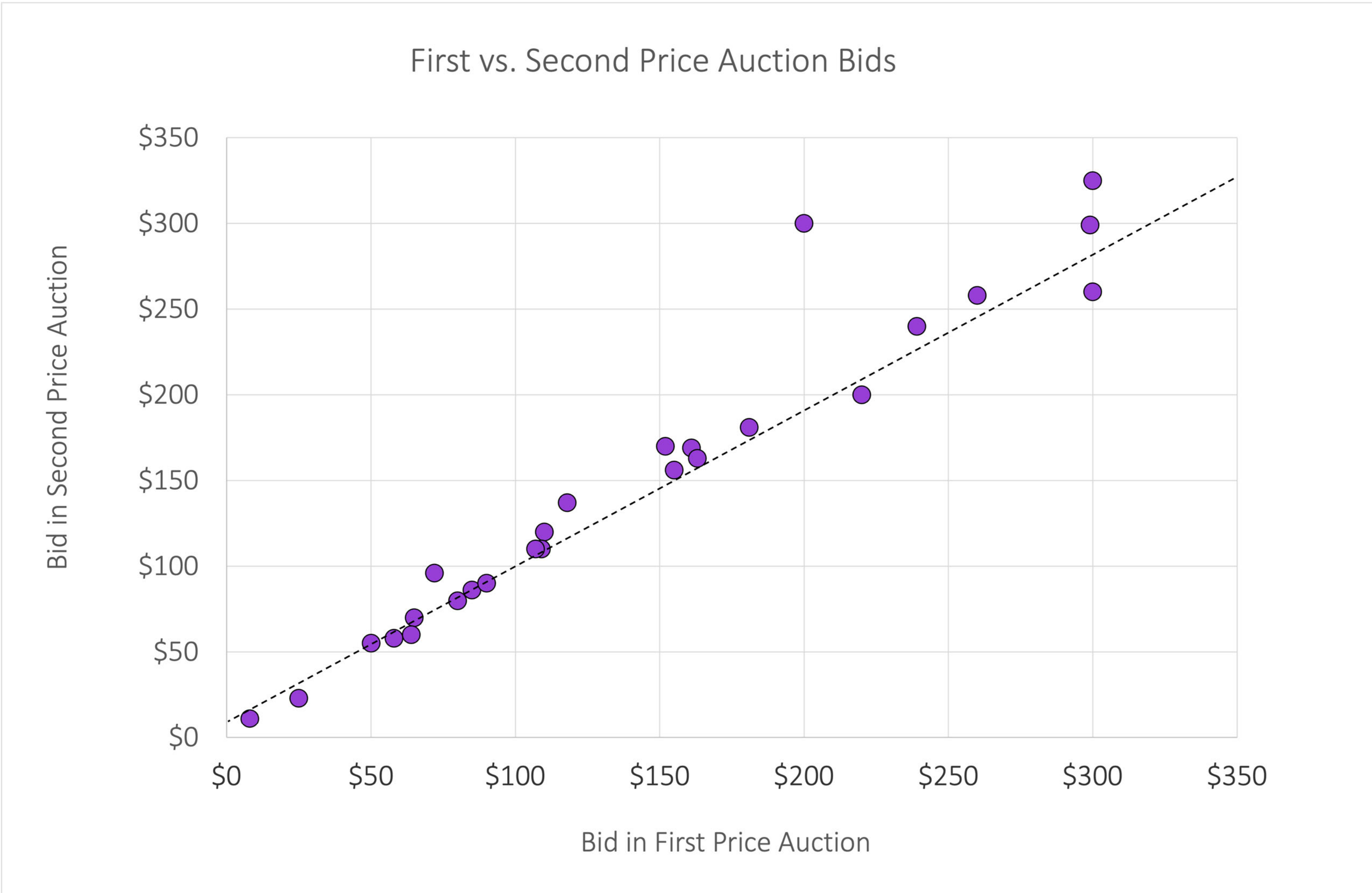

Auction Summary Statistics
Median Value
$180.00
$160.00
$140.00
$120.00
$100.00
$80.00
$60.00
$40.00
$20.00
$0.00
$129
$114
$129
$15
$90
$85
$86
$5
$163
$155
$163
$20
Total
EWMBA
XMBA
Valuation
Bid in First price auction
Bid in Second price auction
Bid in All-pay auction


Histogram of Estimated Valuation
Number of Students
0
1
2
3
4
5
6
0 - 25
25 - 50
51 - 75
76 - 100
101 - 125
126 - 150
151 - 175
176 - 200
201 - 225
226 - 250
251 - 275
276 - 300
301 - 325
326 - 350
351 - 825



First Price Auction: Bid vs. Valuation
Payment: $300
Bid ($)
Valuation ($)
$0
$50
$100
$150
$200
$250
$300
$350
$400


Second Price Auction: Bid vs. Valuation
Payment: $300
Bid ($)
Valuation ($)
$0
$50
$100
$150
$200
$250
$300
$350
$400


First vs. Second Price Auction Bids
Bid in Second Price Auction
$350
$300
$250
$200
$150
$100
$50
$0
$0
$50
$100
$150
$200
$250
$300
$350
Bid in First Price Auction

Common-value auctions and the winner's curse

Suppose we all participate in a sealed-bid auction for a jar of coins. Once you have estimated the amount of money in the jar, what are your bidding strategies in first- and second-price auctions?

The winning bidder is likely to be the bidder with the largest positive error (the largest overestimate).

In this case, the winner has fallen prey to the so-called the winner's curse. Auctions where the winner's curse is significant are oil fields, spectrum auctions, pay per click, and more.

Types of games

We study four groups of game theoretic models:

I strategic games

II extensive games (with perfect and imperfect information)

III repeated games

IV coalitional games

Strategic games

A strategic game consists of

- a set of players (decision makers)
- for each player, a set of possible actions
- for each player, preferences over the set of action profiles (outcomes).

In strategic games, players move simultaneously. A wide range of situations may be modeled as strategic games.

A two-player (finite) strategic game can be described conveniently in a so-called bi-matrix.

For example, a generic 2×2 (two players and two possible actions for each player) game

	L	R
T	A_1, A_2	B_1, B_2
B	C_1, C_2	D_1, D_2

where the two rows (resp. columns) correspond to the possible actions of player 1 (resp. 2).

Applying the definition of a strategic game to the 2×2 game above yields:

- Players: $\{1, 2\}$
- Action sets: $A_1 = \{T, B\}$ and $A_2 = \{L, R\}$
- Action profiles (outcomes):

$$A = A_1 \times A_2 = \{(T, L), (T, R), (B, L), (B, R)\}$$

- Preferences: $\succsim_1$ and $\succsim_2$ are given by the bi-matrix.

Rock-Paper-Scissors
(over a dollar)

	R	P	S
R	$0, 0$	$-1, 1$	$1, -1$
P	$1, -1$	$0, 0$	$-1, 1$
S	$-1, 1$	$1, -1$	$0, 0$

Each player's set of actions is $\{Rock, Papar, Scissors\}$ and the set of action profiles is

$$\{RR, RP, RS, PR, PP, PS, SR.SP, SS\}.$$

In rock-paper-scissors

$$PR \sim_1 SP \sim_1 RS \succ_1 PP \sim_1 RR \sim_1 SS \succ_1 PS \sim_1 SR \sim_1 PS$$

and

$$PR \sim_2 SP \sim_2 RS \prec_2 PP \sim_2 RR \sim_2 SS \prec_2 PS \sim_2 SR \sim_2 PS$$

This is a zero-sum or a strictly competitive game.

Dominance Solvability and Rationalizability

- In a Nash equilibrium, each player knows (correctly conjectures) the other players' equilibrium strategies.

- Dominance solvability and rationalizability are solution concepts that do not entail this assumption.

- Players' beliefs about each other's action are not assumed to be correct, but are constrained by (some) considerations of rationality.

Dominance I

	b_1	b_2	b_3
a_1	1, _	1, _	1, _
a_2	1, _	0, _	1, _
a_3	0, _	0, _	0, _

For player 1, action a_2 is weakly dominated by a_1, and action a_3 is weakly dominated by a_2 and strictly dominated by a_1.

Dominance II

	b_1	b_2	b_3
a_1	4, 3	5, 1	6, 2
a_2	2, 1	8, 4	3, 6
a_3	3, 0	9, 6	2, 8

	b_1	b_3
a_1	4, 3	6, 2
a_2	2, 1	3, 6
a_3	3, 0	2, 8

	b_1	b_3
a_1	4, 3	6, 2

$\Longrightarrow$ by iterated elimination of strictly dominated strategies.

Rationalizability

	b_1	b_2	b_3	b_4
a_1	$0, 7$	$2, 5$	$7, 0$	$0, 1$
a_2	$5, 2$	$3, 3$	$5, 2$	$0, 1$
a_3	$7, 0$	$2, 5$	$0, 7$	$0, 1$
a_4	$0, 0$	$0, -2$	$0, 0$	$10, -1$

The rationalizable actions are a_1, a_2, a_3 for player 1 and b_1, b_2, b_3 for player 2.

Classical 2×2 **games**

- The following simple 2×2 games represent a variety of strategic situations.

- Despite their simplicity, each game captures the essence of a type of strategic interaction that is present in more complex situations.

- These classical games "span" the set of almost *all* games (strategic equivalence).

Game I: Prisoner's Dilemma

	$Work$	$Goof$
$Work$	$3, 3$	$0, 4$
$Goof$	$4, 0$	$1, 1$

A situation where there are gains from cooperation but each player has an incentive to "free ride."

Examples: team work, duopoly, arm/advertisement/R&D race, public goods, and more.

Game II: Battle of the Sexes (BoS)

	$Ball$	$Show$
$Ball$	$2, 1$	$0, 0$
$Show$	$0, 0$	$1, 2$

Like the Prisoner's Dilemma, Battle of the Sexes models a wide variety of situations.

Examples: political stands, mergers, among others.

Game III-V: Coordination, Hawk-Dove, and Matching Pennies

	Ball	*Show*
Ball	2, 2	0, 0
Show	0, 0	1, 1

	Dove	*Hawk*
Dove	3, 3	1, 4
Hawk	4, 1	0, 0

	Head	*Tail*
Head	1, −1	−1, 1
Tail	−1, 1	1, −1

Best response and dominated actions

Action T is player 1's *best response* to action L player 2 if T is the optimal choice when 1 *conjectures* that 2 will play L.

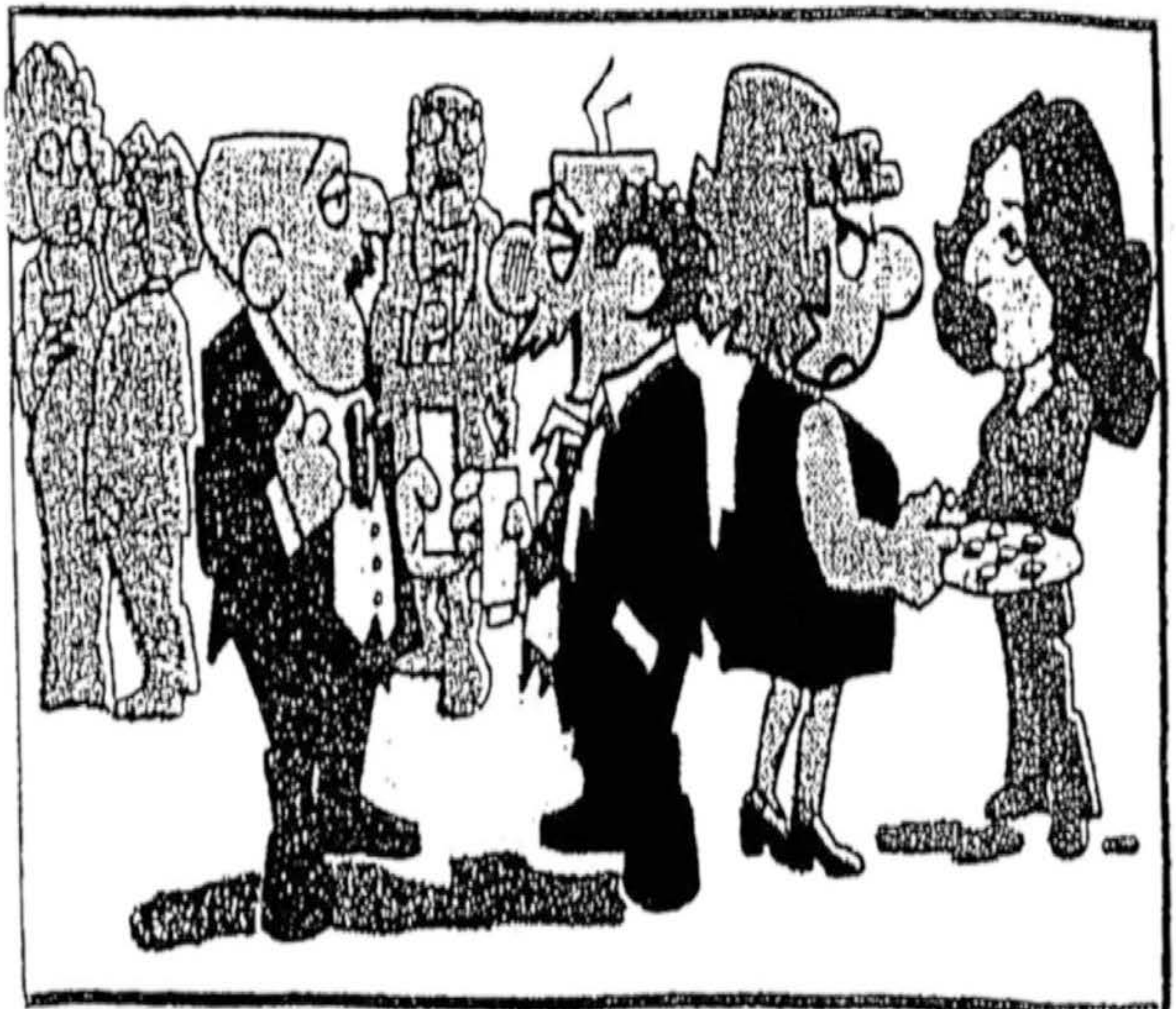
Player 1's action T is *strictly* dominated if it is never a best response (inferior to B no matter what the other players do).

In the Prisoner's Dilemma, for example, action $Work$ is strictly dominated by action $Goof$. As we will see, a strictly dominated action is not used in any Nash equilibrium.

Nash equilibrium

Nash equilibrium (NE) is a steady state of the play of a strategic game – no player has a profitable deviation given the actions of the other players.

Put differently, a NE is a set of actions such that all players are doing their best given the actions of the other players.



"LORETTA'S DRIVING BECAUSE I'M DRINKING, AND I'M DRINKING BECAUSE SHE'S DRIVING."

Food for thought

LUPI

Many players simultaneously chose an integer between 1 and 99,999. Whoever chooses the lowest unique positive integer (LUPI) wins.

Question What does an equilibrium model of behavior predict in this game?

The field version of LUPI, called Limbo, was introduced by the government-owned Swedish gambling monopoly Svenska Spel. Despite its complexity, there is a surprising degree of convergence toward equilibrium.

Morra

A two-player game in which each player simultaneously hold either one or two fingers and each guesses the total number of fingers held up.

If exactly one player guesses correctly, then the other player pays her the amount of her guess.

Question Model the situation as a strategic game and describe the equilibrium model of behavior predict in this game.

The game was played in ancient Rome, where it was known as "micatio."

Maximal game
(sealed-bid second-price auction)

Two bidders, each of whom privately observes a signal X_i that is independent and identically distributed (i.i.d.) from a uniform distribution on $[0, 10]$.

Let $X^{\mathsf{max}} = \mathsf{max}\{X_1, X_2\}$ and assume the ex-post common value to the bidders is X^{max}.

Bidders bid in a sealed-bid second-price auction where the highest bidder wins, earns the common value X^{max} and pays the second highest bid.