University of California – Berkeley
Department of Economics
Game Theory in the Social Sciences
(ECON C110 | POLSCI C135)
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Lecture III
Nash equilibriaum

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Morra

A two-player game in which each player simultaneously hold either one or two fingers and each guesses the total number of fingers held up.

If exactly one player guesses correctly, then the other player pays her the amount of her guess.

Question Model the situation as a strategic game and describe the equilibrium model of behavior predict in this game.

The game was played in ancient Rome, where it was known as "micatio."

In Morra there are two players, each of whom has four (relevant) actions, S_1G_2, S_1G_3, S_2G_3, and S_2G_4, where S_iG_j denotes the strategy (Show i, Guess j).

The payoffs in the game are as follows

	S_1G_2	S_1G_3	S_2G_3	S_2G_4
S_1G_2	$0, 0$	$2, -2$	$-3, 3$	$0, 0$
S_1G_3	$-2, 2$	$0, 0$	$0, 0$	$3, -3$
S_2G_3	$3, -3$	$0, 0$	$0, 0$	$-4, 4$
S_2G_4	$0, 0$	$-3, 3$	$4, -4$	$0, 0$

Strategic games
(review)

A two-player (finite) strategic game

The game can be described conveniently in a so-called bi-matrix. For example, a generic 2×2 (two players and two possible actions for each player) game

	L	R
T	a_1, a_2	b_1, b_2
B	c_1, c_2	d_1, d_2

where the two rows (resp. columns) correspond to the possible actions of player 1 (resp. 2). The two numbers in a box formed by a specific row and column are the players' payoffs given that these actions were chosen.

In this game above a_1 and a_2 are the payoffs of player 1 and player 2 respectively when player 1 is choosing strategy T and player 2 strategy L.

Applying the definition of a strategic game to the 2×2 game above yields:

- Players: $\{1, 2\}$
- Action sets: $A_1 = \{T, B\}$ and $A_2 = \{L, R\}$
- Action profiles (outcomes):

$$A = A_1 \times A_2 = \{(T, L), (T, R), (B, L), (B, R)\}$$

- Preferences: $\succsim_1$ and $\succsim_2$ are given by the bi-matrix.

Best response and dominated actions

Action T is player 1's *best response* to action L player 2 if T is the optimal choice when 1 *conjectures* that 2 will play L.

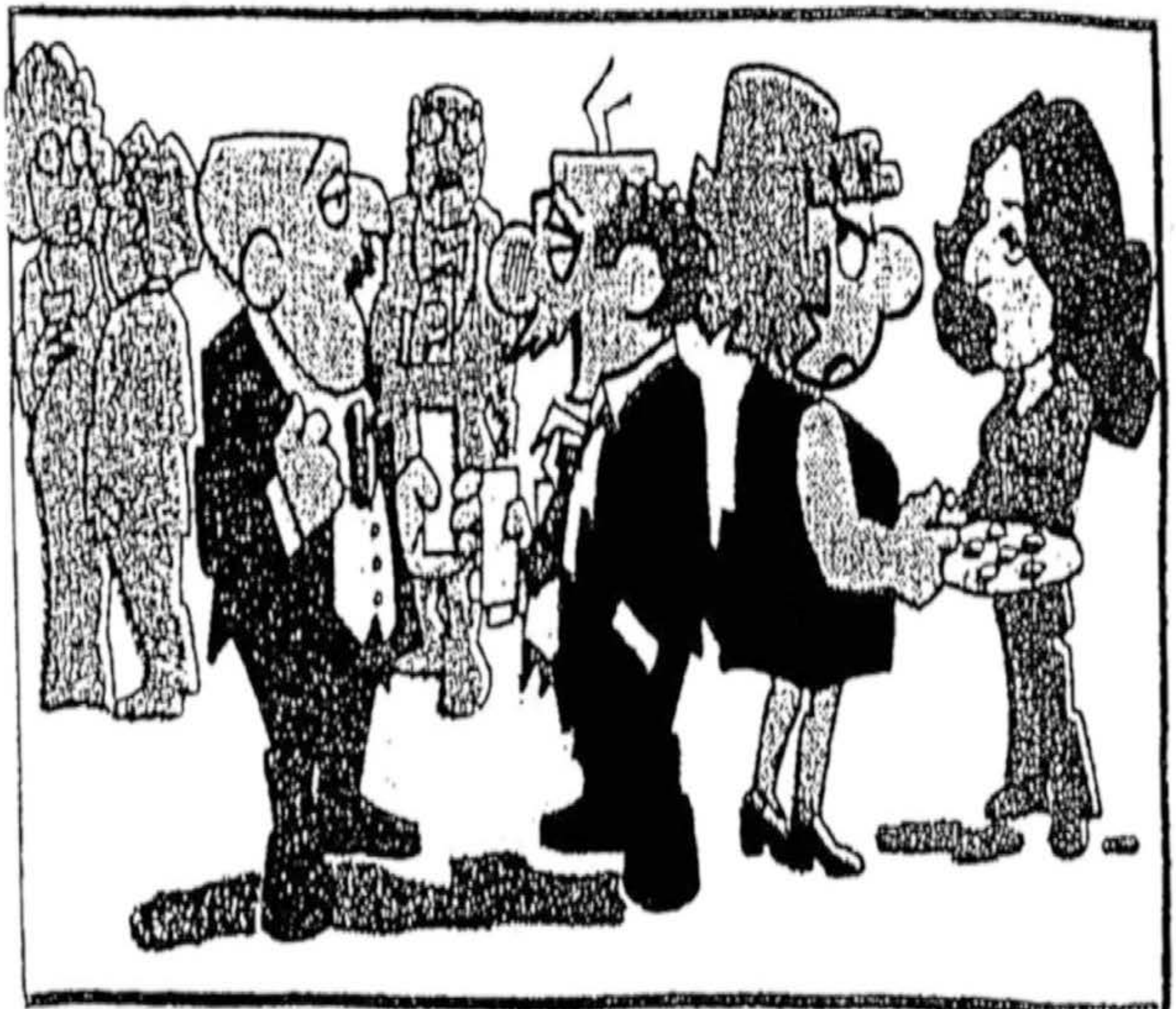
Player 1's action T is *strictly* dominated if it is never a best response (inferior to B no matter what the other players do).

In the Prisoner's Dilemma, for example, action $Work$ is strictly dominated by action $Goof$. As we will see, a strictly dominated action is not used in any Nash equilibrium.

Nash equilibrium

Nash equilibrium (NE) is a steady state of the play of a strategic game – no player has a profitable deviation given the actions of the other players.

Put differently, a NE is a set of actions such that all players are doing their best given the actions of the other players.



"LORETTA'S DRIVING BECAUSE I'M DRINKING,
AND I'M DRINKING BECAUSE SHE'S DRIVING."

Classical 2×2 games

- The following simple 2×2 games represent a variety of strategic situations.

- Despite their simplicity, each game captures the essence of a type of strategic interaction that is present in more complex situations.

- These classical games "span" the set of almost *all* games (strategic equivalence).

Game I: Prisoner's Dilemma

	$Work$	$Goof$
$Work$	3, 3	0, 4
$Goof$	4, 0	1, 1

A situation where there are gains from cooperation but each player has an incentive to "free ride."

Examples: team work, duopoly, arm/advertisement/R&D race, public goods, and more.

Game II: Battle of the Sexes (BoS)

	$Ball$	$Show$
$Ball$	$2,1$	$0,0$
$Show$	$0,0$	$1,2$

Like the Prisoner's Dilemma, Battle of the Sexes models a wide variety of situations.

Examples: political stands, mergers, among others.

Game III-V: Coordination, Hawk-Dove, and Matching Pennies

	Ball	*Show*
Ball	2, 2	0, 0
Show	0, 0	1, 1

	Dove	*Hawk*
Dove	3, 3	1, 4
Hawk	4, 1	0, 0

	Head	*Tail*
Head	1, −1	−1, 1
Tail	−1, 1	1, −1

Mixed strategy Nash equilibrium in the BoS

Suppose that, each player can randomize among all her strategies so choices are not deterministic:

		q L	$1-q$ R
p	T	pq	$p(1-q)$
$1-p$	B	$(1-p)q$	$(1-p)(1-q)$

Let p and q be the probabilities that player 1 and 2 respectively assign to the strategy *Ball*.

Player 2 will be indifferent between using her strategy B and S when player 1 assigns a probability p such that her expected payoffs from playing B and S are the same. That is,

$$1p + 0(1-p) = 0p + 2(1-p)$$
$$p = 2 - 2p$$
$$p^* = 2/3$$

Hence, when player 1 assigns probability $p^* = 2/3$ to her strategy B and probability $1 - p^* = 1/3$ to her strategy S, player 2 is indifferent between playing B or S any mixture of them.

Similarly, player 1 will be indifferent between using her strategy B and S when player 2 assigns a probability q such that her expected payoffs from playing B and S are the same. That is,

$$2q + 0(1-q) = 0q + 1(1-q)$$
$$2q = 1 - q$$
$$q^* = 1/3$$

Hence, when player 2 assigns probability $q^* = 1/3$ to her strategy B and probability $1 - q^* = 2/3$ to her strategy S, player 2 is indifferent between playing B or S any mixture of them.

In terms of best responses:

$$B_1(q) = \begin{cases} p = 1 & if \quad p > 1/3 \\ p \in [0,1] & if \quad p = 1/3 \\ p = 0 & if \quad p < 1/3 \end{cases}$$

$$B_2(p) = \begin{cases} q = 1 & if \quad p > 2/3 \\ q \in [0,1] & if \quad p = 2/3 \\ q = 0 & if \quad p < 2/3 \end{cases}$$

The BoS has two Nash equilibria in pure strategies $\{(B, B), (S, S)\}$ and one in mixed strategies $\{(2/3, 1/3)\}$. In fact, any game with a finite number of players and a finite number of strategies for each player has Nash equilibrium (Nash, 1950).

Three Matching Pennies games in the laboratory

		.48	.52
		a_2	b_2
.48	a_1	80, 40	40, 80
.52	b_1	40, 80	80, 40

		.16	.84
		a_2	b_2
.96	a_1	320, 40	40, 80
.04	b_1	40, 80	80, 40

		.80	.20
		a_2	b_2
.08	a_1	44, 40	40, 80
.92	b_1	40, 80	80, 40