

University of California – Berkeley  
Department of Economics  
Game Theory in the Social Sciences  
(ECON C110 | POLSCI C135)  
Fall 2025

**Lecture V**  
**Applications of strategic-form games 1 (out of 2):**  
**Oligopolistic competition**

Sep 25, 2025

## **The Cournot model of oligopoly**

A French philosopher and mathematician (1801-1877)

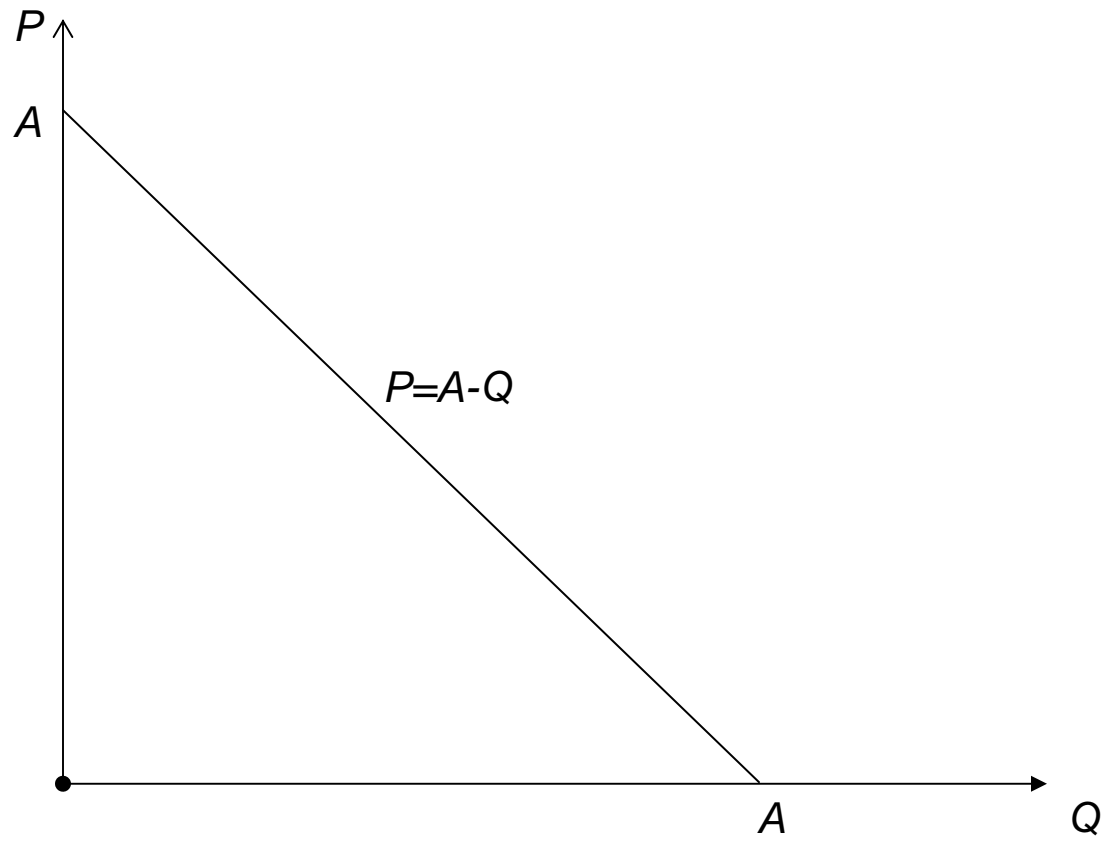
## Cournot's oligopoly model (1838)

- A single good is produced by two firms (the industry is a “duopoly”).
- The cost for firm  $i = 1, 2$  for producing  $q_i$  units of the good is given by  $c_i q_i$  (“unit cost” is constant equal to  $c_i > 0$ ).
- If the firms' total output is  $Q = q_1 + q_2$  then the market price is

$$P = A - Q$$

if  $A \geq Q$  and zero otherwise (linear inverse demand function). We also assume that  $A > c$ .

## The inverse demand function



To find the Nash equilibria of the Cournot's game, we can use the procedures based on the firms' best response functions.

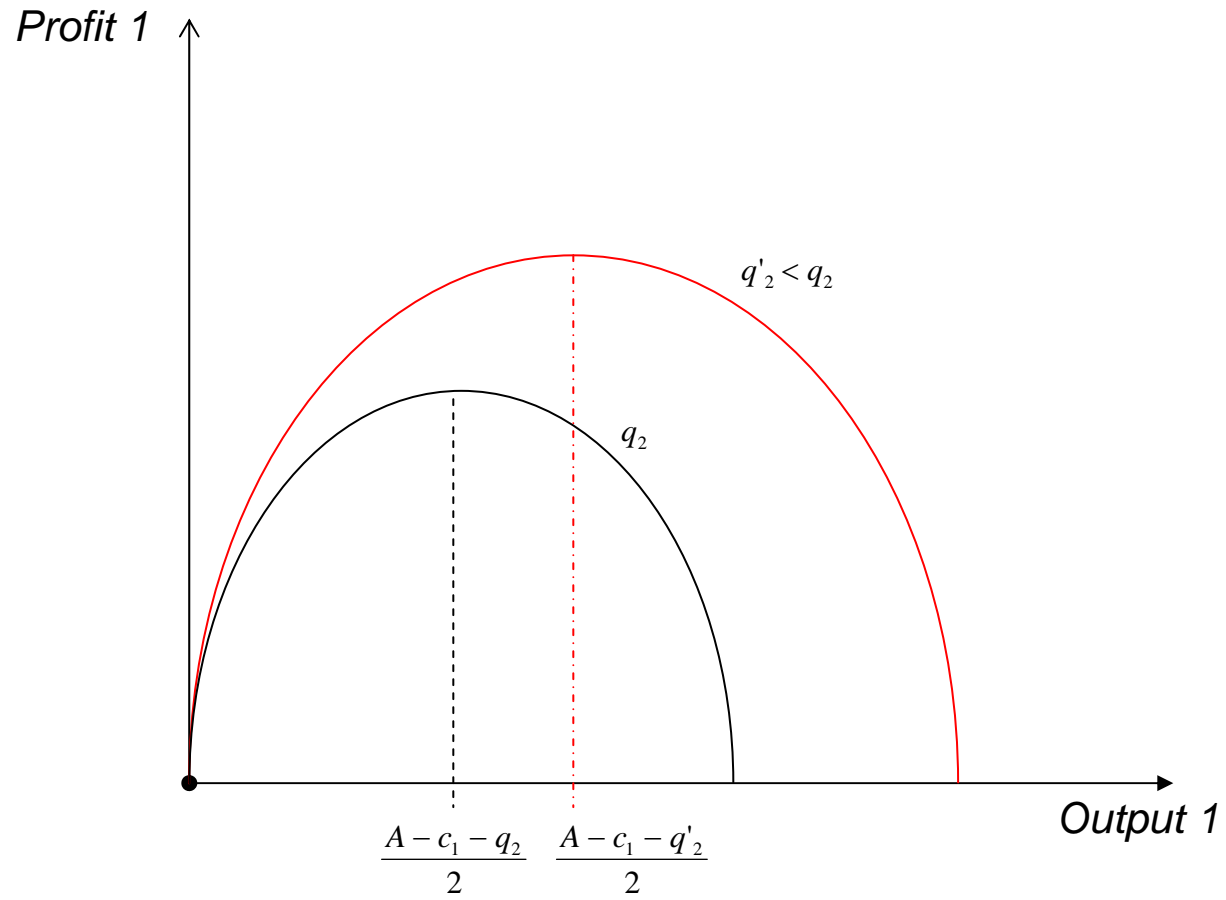
But first we need the firms payoffs (profits):

$$\begin{aligned}\pi_1 &= Pq_1 - c_1q_1 \\ &= (A - Q)q_1 - c_1q_1 \\ &= (A - q_1 - q_2)q_1 - c_1q_1 \\ &= (A - q_1 - q_2 - c_1)q_1\end{aligned}$$

and similarly,

$$\pi_2 = (A - q_1 - q_2 - c_2)q_2$$

**Firm 1's profit as a function of its output  
(given firm 2's output)**



To find firm 1's best response to any given output  $q_2$  of firm 2, we need to study firm 1's profit as a function of its output  $q_1$  for given values of  $q_2$ .

Using calculus, we set the derivative of firm 1's profit with respect to  $q_1$  equal to zero and solve for  $q_1$ :

$$q_1 = \frac{1}{2}(A - q_2 - c_1).$$

We conclude that the best response of firm 1 to the output  $q_2$  of firm 2 depends on the values of  $q_2$  and  $c_1$ .

Because firm 2's cost function is  $c_2 \neq c_1$ , its best response function is given by

$$q_2 = \frac{1}{2}(A - q_1 - c_2).$$

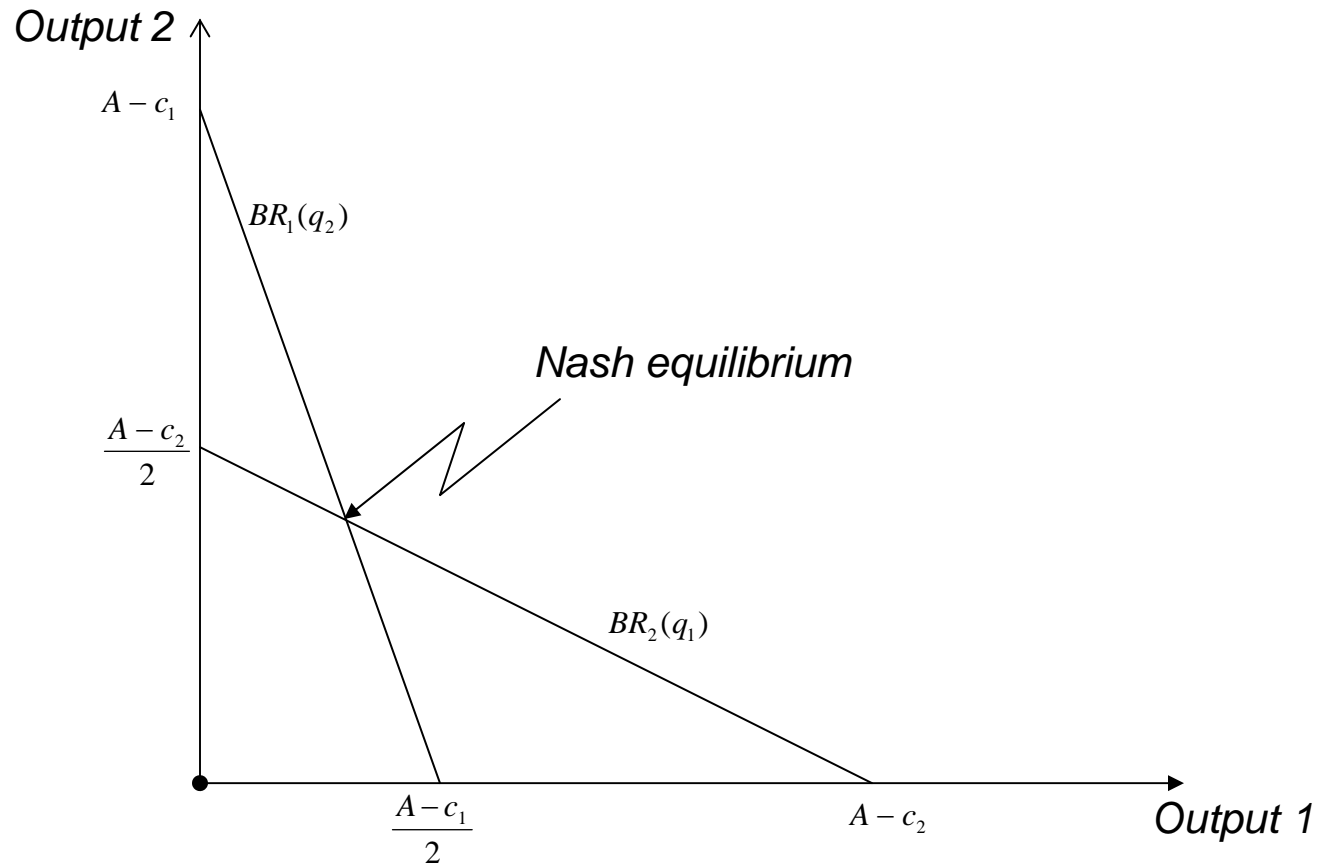
A Nash equilibrium of the Cournot's game is a pair  $(q_1^*, q_2^*)$  of outputs such that  $q_1^*$  is a best response to  $q_2^*$  and  $q_2^*$  is a best response to  $q_1^*$ .

From the figure below, we see that there is exactly one such pair of outputs

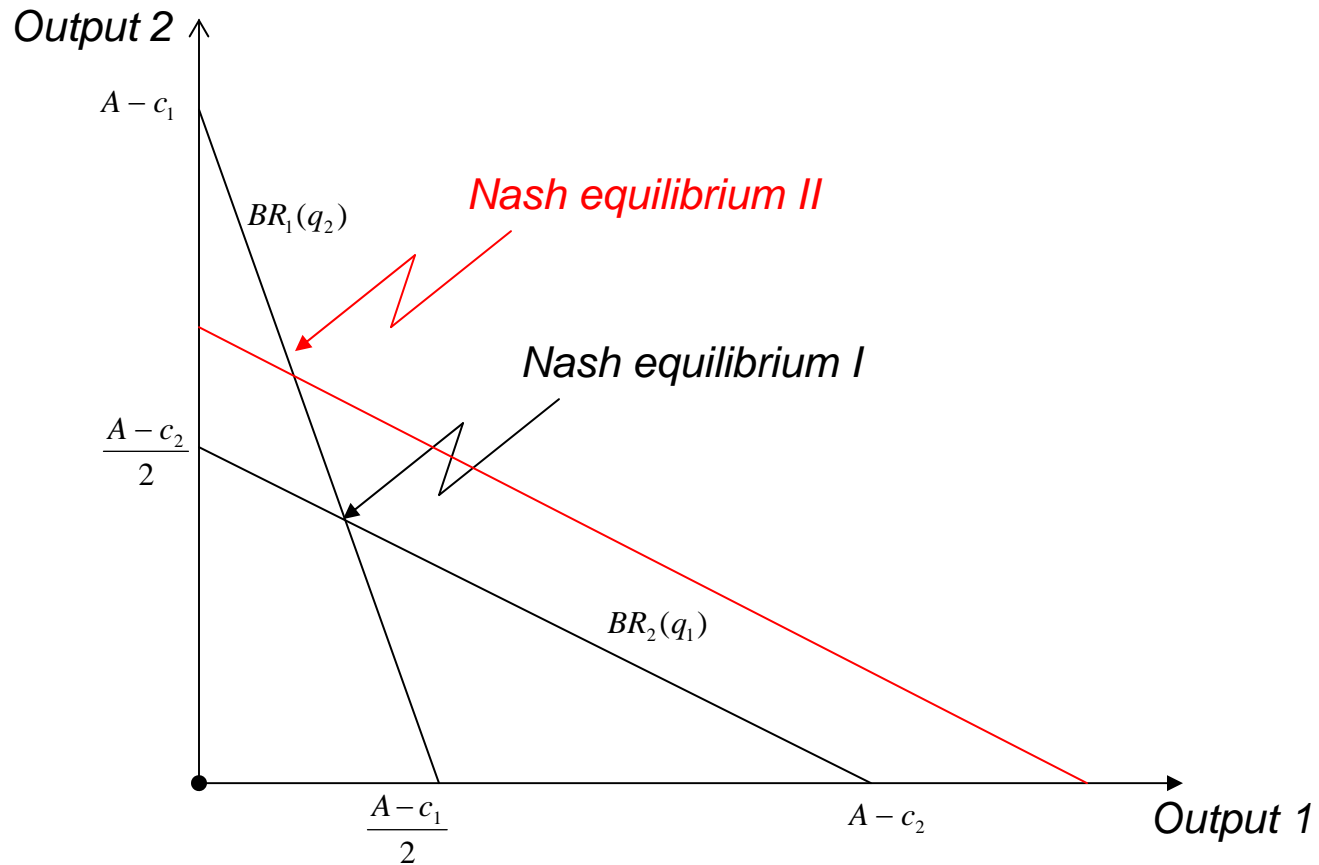
$$q_1^* = \frac{A+c_2-2c_1}{3} \quad \text{and} \quad q_2^* = \frac{A+c_1-2c_2}{3}$$

which is the solution to the two equations above.

### The best response functions in the Cournot's duopoly game



**Nash equilibrium comparative statics  
(a decrease in the cost of firm 2)**



A question: what happens when consumers are willing to pay more ( $A$  increases)?

In summary, this simple Cournot's duopoly game has a unique Nash equilibrium.

Two economically important properties of the Nash equilibrium are (to economic regulatory agencies):

- [1] The relation between the firms' equilibrium profits and the profit they could make if they act collusively.
- [2] The relation between the equilibrium profits and the number of firms.

- [1] Collusive outcomes: in the Cournot's duopoly game, there is a pair of outputs at which *both* firms' profits exceed their levels in a Nash equilibrium.
- [2] Competition: The price at the Nash equilibrium if the two firms have the *same* unit cost  $c_1 = c_2 = c$  is given by

$$\begin{aligned} P^* &= A - q_1^* - q_2^* \\ &= \frac{1}{3}(A + 2c) \end{aligned}$$

which is above the unit cost  $c$ . But as the number of firm increases, the equilibrium price decreases, approaching  $c$  (zero profits!).

**The Stackelberg model of oligopoly—a preface to extensive games**

A German economist (1906-1946)

## Stackelberg's duopoly model (1934)

How do the conclusions of the Cournot's duopoly game change when the firms move sequentially? Is a firm better off moving before or after the other firm?

Suppose that  $c_1 = c_2 = c$  and that firm 1 moves at the start of the game. We may use backward induction to find the subgame perfect equilibrium.

- First, for *any* output  $q_1$  of firm 1, we find the output  $q_2$  of firm 2 that maximizes its profit. Next, we find the output  $q_1$  of firm 1 that maximizes its profit, *given the strategy* of firm 2.

## Firm 2

Since firm 2 moves after firm 1, a strategy of firm 2 is a *function* that associate an output  $q_2$  for firm 2 for each possible output  $q_1$  of firm 1.

We found that under the assumptions of the Cournot's duopoly game Firm 2 has a unique best response to each output  $q_1$  of firm 1, given by

$$q_2 = \frac{1}{2}(A - q_1 - c)$$

(Recall that  $c_1 = c_2 = c$ ).

## Firm 1

Firm 1's strategy is the output  $q_1$  the maximizes

$$\pi_1 = (A - q_1 - q_2 - c)q_1 \quad \text{subject to} \quad q_2 = \frac{1}{2}(A - q_1 - c)$$

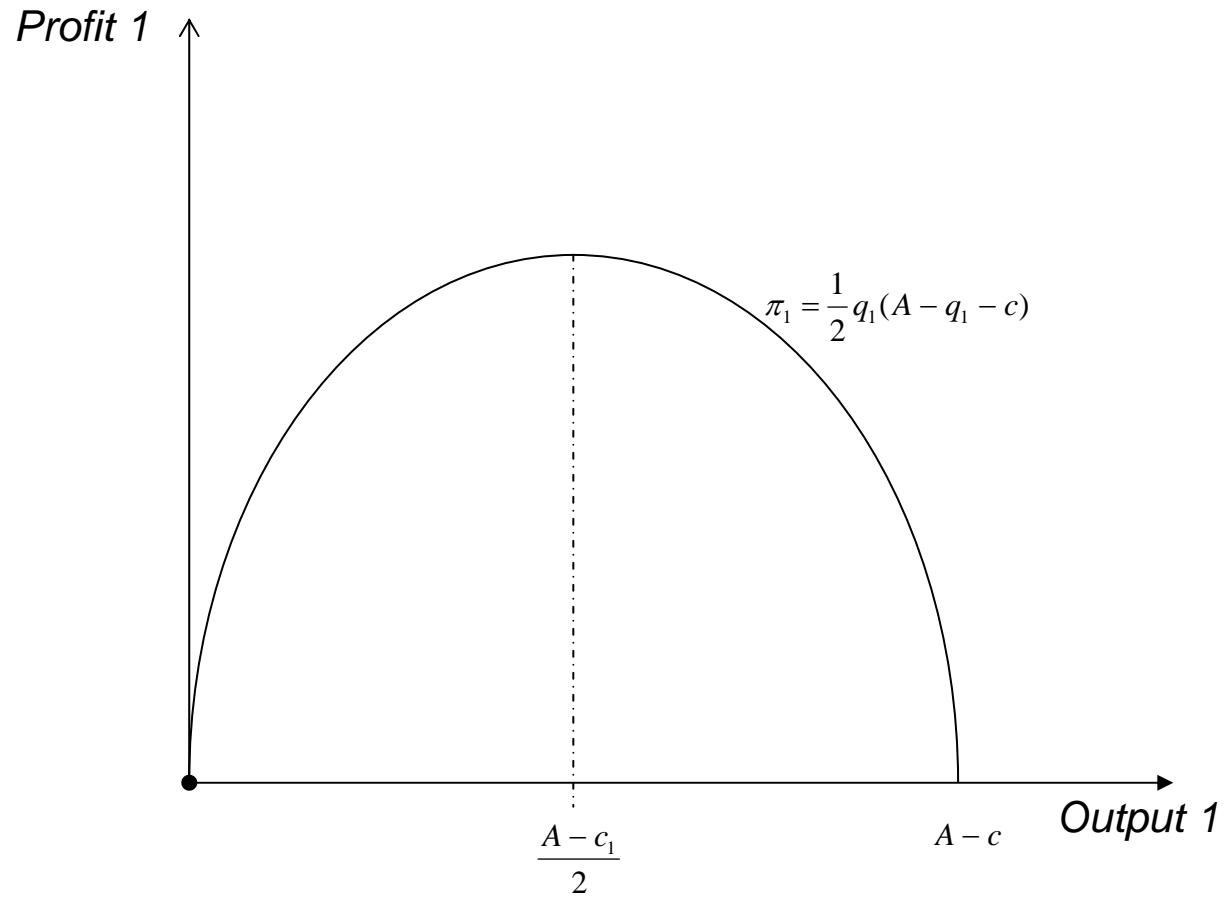
Thus, firm 1 maximizes

$$\pi_1 = (A - q_1 - (\frac{1}{2}(A - q_1 - c)) - c)q_1 = \frac{1}{2}q_1(A - q_1 - c).$$

This function is quadratic in  $q_1$  that is zero when  $q_1 = 0$  and when  $q_1 = A - c$ . Thus its maximizer is

$$q_1^* = \frac{1}{2}(A - c).$$

### Firm 1's (first-mover) profit in Stackelberg's duopoly game



We conclude that Stackelberg's duopoly game has a unique subgame perfect equilibrium, in which firm 1's strategy is the output

$$q_1^* = \frac{1}{2}(A - c)$$

and firm 2's output is

$$\begin{aligned} q_2^* &= \frac{1}{2}(A - q_1^* - c) \\ &= \frac{1}{2}\left(A - \frac{1}{2}(A - c) - c\right) \\ &= \frac{1}{4}(A - c). \end{aligned}$$

By contrast, in the unique Nash equilibrium of the Cournot's duopoly game under the same assumptions ( $c_1 = c_2 = c$ ), each firm produces  $\frac{1}{3}(A - c)$ .

### The subgame perfect equilibrium of Stackelberg's duopoly game

