Suggested Solutions for Problem Set #3

1. *You needed to upload the handwritten honor pledge. Without that, you get zero points on the entire problem set.*

2. The Taylor Rule from the textbook is $r = r_0 + r_\pi(\pi - \pi^t)$. This Taylor rule actually does a decent job of capturing how the Fed is *currently* setting interest rates.

A) Define r_0 in words. Define π^t in words. Define r_π in words. (Your answer can't be "r_0 is r-naught." That's not a definition.)

r_0 is the baseline value of the real interest rate. It is the value of the real interest rate targeted by the central bank when $\pi = \pi^t$.

π^t is the central bank's target for the inflation rate. This is the inflation rate the central bank wants the economy to have.

r_π is the responsiveness of the central bank's target for the real interest rate to changes in the inflation rate. It tells us how much the central bank changes its target for r in response to a 1 point change in the inflation rate.

B) Until the pandemic, the Fed was setting interest rates not in reaction to what the inflation rate actually was, but in reaction to the Fed's predicted value of the inflation rate some time in the future. Offer and defend an equation that captures that sort of Taylor rule.

There are a number of ways to approach this question. The essence: you need to have predicted inflation not actual inflation in the Taylor rule. A really good answer would mention the time frame as well. Is the Fed reacting today to what they predict the inflation rate will be in a week? A month? 3 months? 6 months? A year?

So for instance, if the central bank looks 2 time periods ahead, we might have

$$r_t = r_0 + r_\pi(\pi_{t+2}^{predicted} - \pi_{t+2}^t)$$

Some of you also included in your equation a model about how the Fed predicts the future inflation rate, pointing to the Phillips Curve relationship. That was an excellent response!

3.

Starting from these 3 relationships, derive the equation for the monetary policy reaction function: $u = u_0 + \phi(\pi - \pi^t)$. Be sure to show the full expression for ϕ.

- Taylor rule: $r = r_0 + r_\pi(\pi - \pi^t)$
- IS equation: $Y = \frac{A_0}{1-MPE} - \left(\frac{I_r + X_\varepsilon \varepsilon_r}{1-MPE}\right) r$, where MPE = $C_y(1-t) - IM_y$
- Okun's Law: $u = u^* - 0.4\left(\frac{Y-Y^*}{Y^*}\right) = u^* + 0.4\left(\frac{Y^*-Y}{Y^*}\right)$

Deriving the monetary policy reaction function (MPRF) is straightforward, but a bit messy.

Step (1): Plug the Taylor rule into the IS curve.

$$Y = \frac{A_0}{1-MPE} - \frac{I_r + X_\varepsilon \varepsilon_r}{1-MPE}[r_0 + r_\pi(\pi - \pi^t)]$$

$$Y = \left[\frac{A_0 - (I_r + X_\varepsilon \varepsilon_r) r_0}{1-MPE}\right] - \frac{(I_r + X_\varepsilon \varepsilon_r)}{1-MPE} r_\pi(\pi - \pi^t)$$

Step (2): Plug the equation for Y from step (1) into Okun's law and we have the MPRF

$$u = u^* + \frac{0.4}{Y^*}\left[Y^* - \left(\frac{A_0 - (I_r + X_\varepsilon \varepsilon_r)r_0}{1 - MPE} - \frac{I_r + X_\varepsilon \varepsilon_r}{1 - MPE} r_\pi (\pi - \pi^t)\right)\right]$$

$$u = \left[u^* + \frac{0.4}{Y^*}\left(Y^* - \frac{A_0 - (I_r + X_\varepsilon \varepsilon_r)r_0}{1 - MPE}\right)\right] + \frac{0.4}{Y^*}\left(\frac{I_r + X_\varepsilon \varepsilon_r}{1 - MPE}\right) r_\pi (\pi - \pi^t)$$

The first term in brackets [ ] is simply u_0.

$$u_0 = \left[u^* + \frac{0.4}{Y^*}\left(Y^* - \frac{A_0}{1-MPE} + \frac{I_r + X_\varepsilon \varepsilon_r}{1-MPE} r_0\right)\right]$$

The parameter in front of $(\pi - \pi^t)$ is ϕ.

$$\phi = \frac{0.4}{Y^*}\left(\frac{I_r + X_\varepsilon \varepsilon_r}{1-MPE}\right) r_\pi$$

Therefore! We have $u = u_0 + \phi(\pi - \pi^t)$

Let's take a moment to make sense of what we just did. First, what is u_0? The unemployment rate will be equal to u_0 when the Fed sets r equal to r_0. The larger is A_0, the smaller is u_0, and vice-versa. If A_0 is just the right size so that $Y_0 = Y^*$, then $u_0 = u^*$.

What about ϕ? ϕ is the sensitivity of unemployment to inflation as a result of this combination of demand-side (aggregate-demand-affecting) factors. It is a combination of four forces:
1) How much the Fed changes the real interest rate in response to changes in inflation, r_π.
2) How much planned expenditure changes when the interest rate changes, $I_r + X_\varepsilon \varepsilon_r$
3) How much output changes when there is an initial change in planned expenditure. That is, the multiplier, 1 / (1 – MPE)
4) The Okun's law coefficient (here, 0.4) and potential output (Y), which determine how much of an effect the change in output has on the unemployment rate.*

B. Suppose that we tweaked the standard model by allowing investment to depend on both income and interest rates: $I = I_0 + I_Y Y - I_r r$. Would this increase, decrease, or have no effect on the slope of the MPRF? Explain. In your explanation, be sure it's clear what "the slope of the MPRF" means in terms of the relationship between inflation and unemployment in the economy.

It will decrease the slope of the MPRF.

The change here is the inclusion of $I_Y Y$. Allowing investment spending (I) to change in response to a change in real GDP (Y) impacts the <u>size of the multiplier</u>, making the multiplier larger. A larger multiplier will increase the value of ϕ in the MPRF. Remember that the MPRF is graphed with π on the vertical axis and u on the horizontal axis, so the slope of the MPRF is $\frac{1}{\phi}$. A larger multiplier thus lowers the slope of the MPRF, making the MPRF flatter. For any given change in inflation (π), there will be a larger change in unemployment (u).

Question 4 (3 points) – Edited version of Final, Spring 2020, #3
For a, draw the graphs on paper, take a photo, and upload the file. For b and c, type your answers into Gradescope. If this were on paper, we would give you about a full page for a, 1/3 of a page for b, and 2/3 of a page for c.

Throughout this question, assume that expectations are <u>static</u>. Also assume the standard equations, except as noted below.

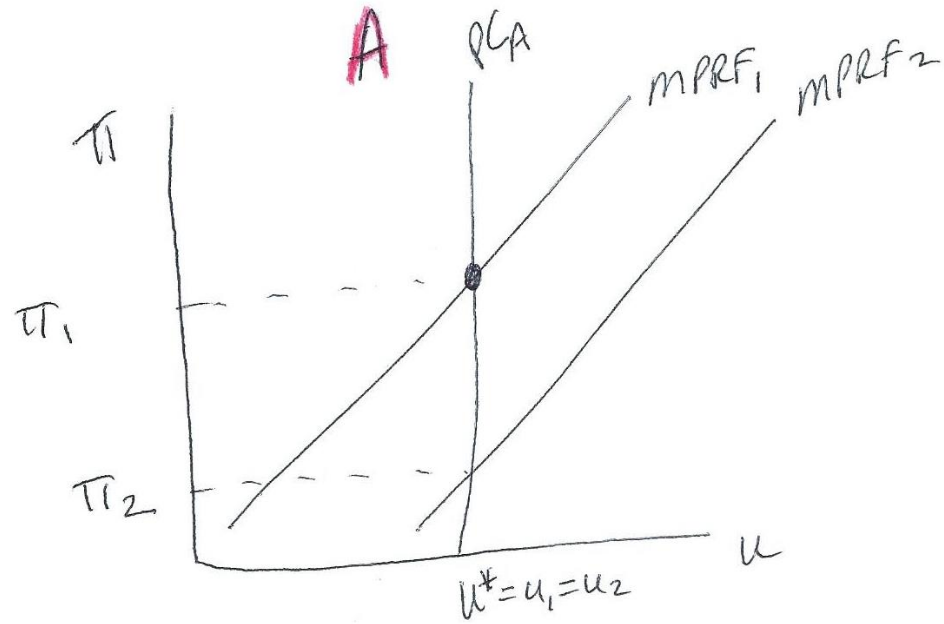
A. Draw 3 separate graphs. Above each graph, as the title, write A, B, or C. In each graph, draw a standard upward-sloping MPRF. Label it $MPRF_1$. This MPRF will be the same in each of the 3 graphs.

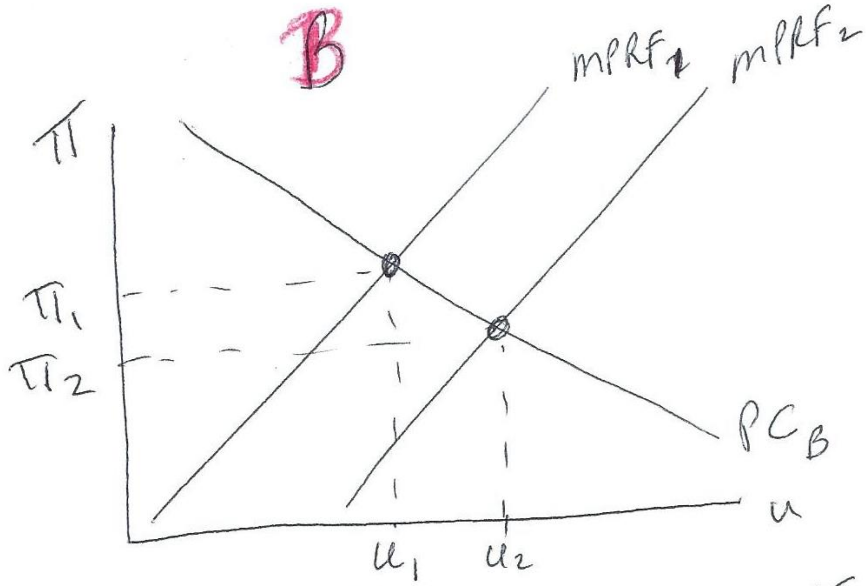
Then in each graph, draw a Phillips curve (different PC for each graph). Try to make your original equilibrium point (u_1, π_1) the same in each of your three graphs. Label the original equilibria points with u_1, π_1 in each of the 3 graphs.

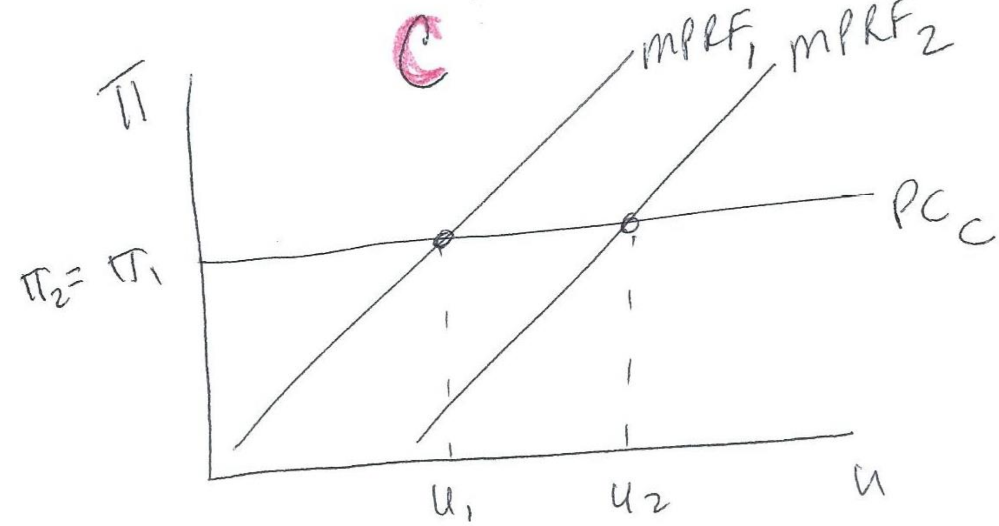
- Graph A: a Phillips curve with fully-flexible wages and prices
- Graph B: our usual Phillips curve, a PC with sticky wages and prices
- Graph C: a Phillips curve in which the inflation rate never changes

Now, in each graph, shift the MPRF so it shows the impact of a decrease in AD. Label this curve $MPRF_2$. (Remember: we are assuming static expectations.) Label the second equilibria (u_2, π_2) in each of the 3 graphs.

Graph A has a vertical Phillips Curve. Graph B has a standard downward-sloping Phillips Curve. Graph C has a horizontal Phillips Curve.


A
PC_A
$mPRF_1$
$mPRF_2$
π
π_1
π_2
u
$u^* = u_1 = u_2$


B
$mPRF_1$
$mPRF_2$
π
π_1
π_2
PC_B
u
u_1
u_2


C
$mPRF_1$
$mPRF_2$
π
PC_C
$\pi_2 = \pi_1$
u
u_1
u_2

B. In part a, you've shown the effects of a decrease in aggregate demand graphically. Here we want you to write out what the graphs show. How do the effects of the decrease in AD on unemployment and inflation vary between A, B, and C? To ease grading, organize your answer this way: u & π effects for A, then u & π effects for B, then u & π effects for C.

In graph A, the decrease in AD causes a drop in inflation from π_1 to π_2, but no change in unemployment.

In graph B, the decrease in AD causes a rise in unemployment from u_1 to u_2 and a drop in inflation from π_1 to π_2.

In graph C, the decrease in AD causes no change in inflation, but a rise in unemployment from u_1 to u_2

C. Is the Fed's reaction to the decrease in AD the same in each case, A, B, and C? Explain.

No. It is the movement along the second MPRF curve that gives us a clue as to how aggressively the Fed is responding to the initial change in AD.

In all three cases, the initial equilibrium is (u_1, π_1). The decrease in AD then causes the MPRF to shift to the right, which would normally result in an increase in unemployment.

In case A with fully flexible wages and prices, the drop in AD triggers a decrease in the inflation rate from π_1 to π_2. That big drop in inflation will trigger a big drop in the real interest rate by the Fed. Indeed, the Fed will lower interest rates so that I+GX increase sufficiently to completely offset the initial drop in AD. So case A has the largest drop in real interest rates.

In case C with no change in inflation, the Fed does not change interest rates at all. (This is essentially the analysis that comes from the Chapter 9 Keynesian Cross, or from Chapter 10 IS curve with a shift of AD and no change of r.) The inflation rate doesn't change but the full effect on output and unemployment is felt by the economy

Case B is an intermediate case. The Fed lowers interest rates, triggering increases in I&GX which partially for not completely offset the initial drop in AD. So case B has a drop in r that is in between 0 and the size of the drop in case A.

5.

Suppose a recession has hit and the economy can be described by the following (assume the standard equations apply):

$$Fed's\ target\ inflation\ rate = 2\%$$
$$initial\ expected\ inflation\ rate = 2\%$$
$$r_\pi = 0.75$$
$$r_0 = 1\%$$
$$MPE = 0.4$$
$$A_0 = 6{,}000$$
$$I_r = 40{,}000$$
$$X_\varepsilon \varepsilon_r = 20{,}000$$
$$u^* = 3\%$$
$$OLC = 0.4$$
$$Y^* = \$10{,}000\ billion\ per\ year$$
$$supply\ shocks = 0$$
$$\beta = 0.3$$

A. Suppose expectations are static. What are the short-run sticky-price equilibrium values of the inflation rate, the unemployment rate, the real interest rate set by the Fed, and real GDP?

Answers:
$\pi = 0.0137 = 1.37\%$
u = 0.0511 = 5.11%
r = 0.00526 = 0.526%
Y = 9,474

Step 1: Determine Taylor rule
Step 2: Determine IS equation; plug Taylor rule into IS
Step 3: Determine Okun's law; plug IS into Okun's law (result = MPRF)
Step 4: Determine Phillips curve
Step 5: Using MPRF & PC, solve that system of 2 equations and 2 unknowns
Step 6: Plug resulting value of π into Taylor rule
Step 7: Plug resulting value of r into IS equation

Taylor rule: $r = 0.01 + \frac{3}{4}(\pi - 0.02) = 0.02 + \frac{3\pi}{4} - 0.015 = -0.005 + \frac{(3\pi)}{4}$

IS equation: $Y = \frac{6{,}000-(40{,}000+20{,}000)\left(-0.005+\frac{3\pi}{4}\right)}{1-0.4} = \frac{6{,}000+300-45{,}000\pi}{0.6} = \frac{6{,}300-45{,}000\pi}{0.6} = 10{,}500 - 75{,}000\pi$

Okun's law to derive MPRF: $u = 0.03 + 0.4\left(\frac{10{,}000-10{,}500+75{,}000\pi}{10{,}000}\right) = 0.03 + 0.4\left(\frac{-500+75{,}000\pi}{10{,}000}\right) = 0.03 - 0.02 + 3\pi = 0.01 + 3\pi$

Phillips curve: $\pi = 0.02 - 0.3(u - 0.03) = 0.02 - 0.3u + 0.009 = 0.029 - 0.3u$

So now we have two equations in two unknowns (u & π):
MPRF: u = 0.010 + 3π
PC: π = 0.029 – 0.3u

Plugging the PC into the MPRF gives us equilibrium unemployment:
u = 0.01 + 3(0.029 – 0.3u) = 0.01 + 0.087 – 0.9u
1.9u = 0.097
u^{eq} = 0.0511 = 5.11%
And then you can plug u = 5.11% into the PC equation to derive the inflation rate: $\pi = 0.029 - 0.3(0.0511) = 0.029 - 0.0153 = 0.0137 = 1.37\%$

Or, you can start by plugging the MPRF into the PC to get equilibrium inflation:
π = 0.029 – 0.3 (0.01 + 3π) = 0.029 - 0.003 – 0.9π
1.9π = 0.026
π^{eq} = 0.0137 = 1.37%
And then you can plug π = 1.37 % into the MPRF equation: $u = 0.01 + 3(0.0137) = 0.01 + 0.0411 = 0.0511 = 5.11\%$

To find the real interest rate, we plug the equilibrium inflation rate into the Taylor rule:
r = 0.01 + 0.75(π – 0.02)
r = 0.01 + 0.75(0.0137 – 0.02)
r = 0.01 + 0.75(-0.0063)
r^{eq} = 0.00527 = 0.527 %

To find the equilibrium value of real output, we plug the equilibrium interest rate into the IS equation:
Y = 10,000 – 100,000r = 10,500 – 100,000(0.00527)
Y = 10,000 – 527
Y^{eq} = 9,473

B. Suppose instead that expectations are adaptive: $\pi^e_{t+1} = \pi_t$. In the next period, what are the new equilibrium values of the inflation rate and the unemployment rate? What real interest rate does the Fed now set? What is the new equilibrium value of real output?

We do the same 7 steps as in 5a but start with $\pi^e = 0.0137$ *rather than 0.02.*

Answers:
$\boldsymbol{\pi = 0.0104 = 1.04}\%$
u = 0.0411 = 4.11%
r = 0.00277 = 0.277%
Y = 9,723

Other than the value of π^e*, everything else we did in part (a), we simply re-use. The value of* π^e *is in the Phillips curve. Nothing in the MPRF changes. So the equation for the MPRF is still* $u = 0.010 + 3\pi$ *u*

But because inflationary expectations have decreased from 2% to 1.37%, the Phillips curve has shifted down. The new equation for the Phillips curve is $\pi = 0.0137 - 0.3(u - 0.03) = 0.0137 - 0.3u + 0.009 = 0.227 - 0.3u$

So now we have two equations in two unknowns (u & π):
MPRF: u = 0.01 + 3π
PC: π = 0.0227 – 0.3u

Plugging the PC into the MPRF gives us equilibrium unemployment:

$u = 0.01 + 3(0.0227 – 0.3u) = 0.01 + 0.0681 – 0.9u$
$1.9u = 0.0781$
$u^{eq} = 0.0411 = 4.11\%$

And then plug u = 4.11% into the PC equation to derive the inflation rate
$\pi = 0.0227 – 0.3u = 0.0227 – 0.3 (0.0411) = 0.0104 = 1.04\%$

Because inflationary expectations fell after the first period, the actual inflation rate has fallen from 1.37 to 1.04 percent. The Fed will react by decreasing the real interest rate.

To find the real interest rate, we plug the equilibrium inflation rate into the Taylor rule:
$r = 0.01 + 0.75(\pi – 0.02)$
$r = 0.01 + 0.75(0.0104 – 0.02)$
$r = 0.01 + 0.75(-0.00963)$
$r^{eq} = 0.00278 = 0.278\%$
The Fed decreases r from 0.526 to 0.278 percent. Their action will raise output.

To find the equilibrium value of real output, we plug the equilibrium interest rate into the IS equation:
$Y = 10{,}000 – 100{,}000r = 10{,}000 – 100{,}000(0.00278)$
$Y = 10{,}000 – 278$
$Y^{eq} = 9{,}722$

Question 6: The Essay (3 points total)

Consider this argument: "Dang! Have you looked at what the Fed has done to the money supply?!?! Inflation is going to soar in the U.S.!"

Follow steps 1, 2*, 3, and 4 of "The Olney Method" (the 5-step method Prof. Olney presented on 1/21 for critiquing arguments) and write a 1-page essay in which you critique the argument above. Your essay should reflect your understanding of how to critique an argument and your understanding of the quantity theory of money (Chapter 8 and video on bcourses).

There are lots of approaches to take here, so we can't provide you with "this is what you should have written." For one set of ideas, see the bcourses videos about the Quantity Theory.

You needed to follow the specs: 400 words and 1 page max, double spaced, 10-11-12 pt font, 1" margins on all sideds, your name, date, and word count in top right corner. If you didn't follow the specs, you were penalized a point so the most you got was 2 points

You needed to also submit the essay on bcourses so that bcourses could do the plagiarism check. If you didn't submit on bcourses, you got a 0 for the essay.

Beyond that you were demonstrating two things: [1] that you understand how to critique an argument, and [2] that you understand the quantity theory of money which was presented in videos that are in the Media Gallery on bcourses.

An excellent or very good job on both got you full credit for the content
An excellent or very good job on one and a not good job on the other cost you 30 – 50% of the credit for the content
A not good job on both cost you 60-80% of the credit for the content.
A clueless job cost you 100% of the credit