**Econ 230B**

**Spring 2025**

**FINAL EXAM: Solutions**

The average grade for the final exam is 58.3 (out of 70 points). The average grade (out of 100) including all assignments is 87.4. The distribution of course grades is:
5 A+, 5 A, 3 A-, 2 B+, 2 B, 1 B-.

**True/False Questions: 40 points**

Answer all 10 questions (4 pts each). Explain your answer fully, since all the credit is based on the explanation.

Only short answers provided here. Full detailed in the class notes and relevant references.

1. If an individual maximizes utility $u(c,z)$ subject to $c = (1-\tau)z+R$ where $c$ is consumption and $z$ is earnings, then the effect on utility of a small tax rate change $d\tau$ is given by $-u_c \cdot d\tau \cdot z$ where $z$ is the earnings choice of the individual before the reform.

   **Solution:** True, this is a consequence of the envelope theorem as $z$ maximizes utility, $u_c + (1-\tau)u_z = 0$, and hence the effect $dz$ of the reform on $z$ has no first order on the utility of the person and can be ignored in the computation so that $du = -u_c \cdot d\tau \cdot z$.

2. The 1987 tax-free year in Iceland generated a positive effect on earnings of young men who were just above school age at the time of the tax-free year.

   **Solution:** This is the Sigurdsson study. True that the tax-free year in Iceland generated a positive effect on earnings of young men who were just above school age at the time of the tax-free year. However, the study finds a negative impact on earnings at age 31-40 implying that going to work right after highschool was detrimental to long-term career earning (probably due to reduced education). Therefore the statement is false on net.

3. The earnings test of the US social security retirement system is a tax on earnings above the disregard and generates substantial bunching in earnings precisely at the earnings disregard.

   **Solution:** True that the earnings test is perceived like a tax on earnings above the disregard and generates substantial bunching (as shown by Gelber, Jones, and Sacks 2013).

However, in reality it is not a pure tax as the reduction in benefits will translate into recomputed higher future benefits as if the person had claimed benefits later.

4. In addition to their traditional redistributive benefit, welfare benefits can also generate long-term benefits for children beneficiaries, which in turn can provide societal benefits.

   **Solution:** Yes, we saw two studies of this in class. The rollout of foodstamps in the 1960s across US counties showed that children more exposed to the program had better health as adults (Hoynes, Schanzenbach, Almond AER'16). Deshpande and Mueller QJE22 use welfare reform which made it harder for disabled children to keep SSI (supplemental security income for low income aged+disabled) past age 18 and finds that it increased likelihood of income related offenses and incarcerations. Both are long-term benefits to beneficiaries and both provide societal benefits (less healthy people are less productive and require more health care spending, crime is obviously a negative externalities and incarceration is very costly to society as well $50K/year).

5. If $r > n + g$, then privatizing social security is desirable.

   **Solution:** A privatized social security system funds the system with individual accounts. It delivers a return $r$ (market return on financial assets) which is higher than the return from the current pay-as-you system which is $n + g$ (population growth + real growth per capita). This is desirable from the point of view of individuals. However, this ignores the transition costs that have to be paid by some generations. If the transition cost is deferred in perpetuity with government debt, then the gain or $r$ vs. $n + g$ are entirely eaten up by higher interest payments on government debt so that it is a total wash (see problem set 2).

6. Even if perfectly enforced, a tax on capital income cannot always replicate the effects of a wealth tax.

   **Solution:** True. If returns r are the same for every individuals and across all assets, a tax at rate tk on the capital income flow is exactly equivalent to a tax at rate k on the capital stock. If r=5%, it is equivalent to tax capital stock at $\tau_k$=1% per year or to tax capital income flow at $t_k = 20\%$ per year. In practice, returns can be heterogeneous across individuals and assets. In that case, a wealth tax is different from a capital income tax. $K_{t+1,i} = R(e_{it}).(k_{t,i} - \tau(k_{t,i})))$ while $K_{t+1,i} = R(e_{it}).(k_{t,i} - t_k(k_{t,i}.(R(e_{it}) - 1))$.

Furthermore, capital income taxation might face practical challenges, as some capital income is never distributed or hard to observe.

7. Studying bunching responses at kinks or notches in the wealth tax schedule is sufficient to inform the efficiency cost of progressive wealth taxation.

   **Solution:** False. 1) Wealth is a stock and may take a long time to adjust to taxes because of fixed pre-existing asset prices and portfolio allocations. Jakobsen et al. (2020) show that bunching responses to local marginaltaxrate variations fail to capture the full long-run adjustment to a wealth tax, whereas difference-in-differences estimates do. 2) Bunching responses may reflect responses to factors other than the tax rate itself and therefore do not identify the structural elasticity of taxable wealth. Garbinti et al. (2024) document large dispersion in estimated elasticities depending on whether the kink in the wealthtax schedule coincides with changes in tax design or reporting requirements. Hence, bunching elasticities alone are difficult to interpret as structural parameters for designing optimal wealth tax policies.

8. When setting the optimal linear tax rate in an open economy, the government should only take into account the migration responses to taxes of one segment of the population, top income earners.

   **Solution:** Mostly false. A small increase in the linear tax rate dt increases out-migration of the rich, which lowers the optimal tax rate the government can set in an open economy: the larger the migration responses to taxes of the rich, the lower the tax rate will be. An increase in dt will also translate into an increase in the universal demogrant (which is determined in the equilibrium), which can in theory increase the in-migration of transfers' recipients ("welfare magnet effect"). Tax-induced migration can reduce the tax base while decreasing transfers per capita, and the government would like to know migration elasticities at each level of the income distribution. Furthermore, non labor force participants like pensioners can also move in responses to taxes, as in Kallin, Levy and Munoz (2024).

9. The decision to move from country A to country B depends only on the characteristics of countries A and B and the migration costs between them.

   **Solution:** False. The choice to move from country A to country B depends not only on the attributes of A and B and the bilateral migration cost between them, but also on the characteristics of all alternative destination countries e.g. the indirect utility derived in any potential locations. Formally, with Frechet or extreme value shocks, $P^k_{odt} = P(U^k_{odt} > U^k_{od't}, \forall d' \neq d) = \frac{\exp(V_{odt})}{\sum_{d'} \exp(V_{od't})}$.

10. VAT, corporate taxes, and payroll taxes are all trade-neutral and do not affect firms decisions to shift profits, production, or input sourcing abroad.

    **Solution:** False. Only the VAT has a border-adjustment mechanism to make it trade-neutral: both domestic and imported inputs are subject to the domestic VAT rate. Without a border-adjustment mechanism, firms have incentives to shift profits and production to low-tax countries. A border-adjustment for corporate income tax as proposed by Auerbach (2010) would include imports in the domestic corporate tax base, meaning that both domestic and foreign inputs/capital would be subject to the domestic rate, thus killing incentives to shift production or profits abroad. Exports are deducted from the base such that it's equivalent to export abroad or produce abroad in terms of taxes. Similarly, labor taxes affect domestic labor costs but not the cost of foreign production/inputs, thus incentivizing firms to shift their production or input sourcing to low-tax and low regulation countries.

**PROBLEM (30 pts):**

Consider an economy where the government sets a flat tax at rate $\tau$ on earnings to raise revenue. We assume that the economy is static: the total population remains constant and equal to $N$ over years and there is no overall growth in earnings.

Individual $i$ earns $z_i = z_i^0(1-\tau)^e$ where the tax rate is $\tau$. $z_i^0$ is independent of taxation and is called potential income. $e$ is a positive parameter equal for all individuals in the economy. The government wants to set $\tau$ so as to raise as much tax revenue as possible.

a) (4 pts) What is the parameter $e$? Show that the tax rate maximizing total tax revenue is equal to $\tau^* = 1/(1+e)$.

**Solution:**

$e$ is the elasticity of income with respect to the net-of-tax rate $1-\tau$. There are no income effects, so this elasticity is both compensated and uncompensated.

Total tax $T = \tau \sum_i z_i = \tau(1-\tau)^e \sum_i z_i^0$.

FOC in $\tau$ gives $\tau^* = 1/(1+e)$.

b) (4 pts) The government does not know $e$ perfectly and thus requests the help of an economist to estimate $e$. The government can provide individual data on earnings for two consecutive years: year 1 and year 2. In year 1, the tax rate is $\tau_1$. In year 2, the tax rate is *decreased* to level $\tau_2$. Suppose that the government can provide you with two cross-section random samples of earnings of the same size $n$ for each year. This is *not* panel data.

How would you proceed to estimate $e$ from this data? Provide a formula for your estimate $\hat{e}$.

**Solution:**

$$\hat{e} = \frac{(1/n)\sum_i \log(z_{i2}) - (1/n)\sum_i \log(z_{i1})}{\log(1-\tau_2) - \log(1-\tau_1)}$$

obtained by OLS regression $\log(z_{it}) = \alpha + e\log(1-\tau_t) + \epsilon_{it}$

c) (4 pts) Suppose now that the economy is experiencing exogenous economic growth from year to year at a constant rate $g > 0$. The population remains constant at $N$. How is the estimate $\hat{e}$ biased because of growth? Suppose you know $g$, how would you correct $\hat{e}$ to obtain a consistent estimate of $e$? (provide an exact formula of this new estimate).

**Solution:**

Assuming that incomes are multiplied by $e^g > 1$ because of growth from year 1 and year 2, previous $\hat{e}$ is biased upward. To get consistent estimate of $e$, need to subtract the growth rate from the numerator:

$$\hat{e} = \frac{(1/n)\sum_i \log(z_{i2}) - (1/n)\sum_i \log(z_{i1}) - g}{\log(1-\tau_2) - \log(1-\tau_1)}$$

d) (4 pts) Suppose now that you do not know $g$ but that the government gives you a new cross-section of data for year 0 in which the tax rate was equal to $\tau_1$ as in year 1. Using data on year 0 and year 1, provide an estimate of $g$ and the corresponding regression specification.

**Solution:**

$$\hat{g} = (1/n)\sum_i \log(z_{i1}) - (1/n)\sum_i \log(z_{i0})$$

obtained by OLS regression $\log(z_{it}) = \alpha + g\,t + \epsilon_{it}$

e) (4 pts) We now assume again that there is no growth. Suppose that the parameter $e$ differs across individuals and is equal to $e_i$ for individual $i$. Assume that there are $N$ individuals in the economy. Individual $i$ earns $z_i = (1-\tau)^{e_i} z_i^0$. As above, $z_i^0$ is not affected by taxation.

As in question 1, express the tax rate maximizing tax revenue $\tau^{**}$ as a function of the $e_i$ and the realized incomes $z_i$. Show that the tax rate $\tau^{**}$ can be expressed as $\tau^{**} = 1/(1+\bar{e})$ where $\bar{e}$ is an average of the $e_i$'s with suitable weights. Give an analytic expression of these weights and provide an economic explanation.

**Solution:**

Total tax $T = \tau \sum_i z_i = \tau \sum_i (1-\tau)^{e_i} z_i^0$.

FOC: $\sum_i z_i - \tau \sum_i e_i (1-\tau)^{e_i - 1} z_i^0$

implies $\sum_i z_i = [\tau/(1-\tau)] \sum_i e_i (1-\tau)^{e_i} z_i^0$

that is, $\sum_i z_i = [\tau/(1-\tau)] \sum_i e_i z_i$

Let us note $\bar{e} = \sum_i e_i z_i / \sum_i z_i$ the average elasticity weighted by incomes (high incomes have a disproportionate effect on total elasticity), we have:

$\tau/(1-\tau) = 1/\bar{e}$, that is, $\tau = 1/(1+\bar{e})$.

f) (6 pts) Suppose now that the parameter $e$ is the same for all individuals and that the government redistributes the tax collected as a lump-sum to all individuals. I note $R$ this lump-sum which is equal to average taxes raised. Suppose that the level of this lump-sum $R$ affects labor supply through income effects. More precisely, the earnings of individual $i$ are given by $z_i = (1-\tau)^e z_i^0(R)$. The potential income $z_i^0(R)$ now depends (negatively) on the lump-sum $R$.

Calculate the compensated elasticity and show that it is larger than $e$.

Suppose that the government still wants to set $\tau$ so as to raise as much taxes as possible in order to make the lump-sum $R$ as big as possible. Should the government set the tax rate $\tau$ higher or lower than $\tau^* = 1/(1+e)$ obtained in question a)?

**Solution:** The compensated elasticity $e^c$ is given by the Slutsky equation $e^c = e^u - \eta$ where $e^u$ is the uncompensated elasticity and $\eta$ the income effect parameter. With $z_i = (1-\tau)^e z_i^0(R)$, it is easy to compute $e^u$ (change $1-\tau$ keeping $R$ constant) and $\eta$ (change $R$ keeping $1-\tau$ constant):

$e^u = ((1-\tau)/z_i) \cdot \partial z_i/\partial(1-\tau) = e$ is constant

$\eta = (1-\tau)\partial z_i/\partial R = (1-\tau)^{1+e} dz_i^0/dR < 0$

Hence $e^c = e - (1-\tau)^{1+e} dz_i^0/dR > e$

Total tax $T = \tau \sum_i z_i = \tau \sum_i (1-\tau)^{e_i} z_i^0(R)$.

FOC: $\sum_i z_i - [\tau/(1-\tau)] e \sum_i z_i + \tau \sum_i (1-\tau)^{e_i} (z_i^0)'(R) \partial R/\partial\tau = 0$

but last term is zero because at the optimum, $R$ is maximized and thus $\partial R/\partial\tau = 0$. Therefore, the FOC is the same as in a) and $\tau = 1/(1+e)$ as in a).

g) (4 pts) Suppose now that the behavioral response to taxes comes entirely from tax avoidance and evasion, i.e., real earnings are $z_i^0$ no matter what the tax rate is but that individuals report only $z_i = z_i^0 \cdot (1-\tau)^e$ when the tax rate is $\tau$. Is the revenue maximizing tax rate still $1/(1+e)$ (as in a)) under this scenario?

**Solution:** The revenue maximizing tax rate is still $1/(1+e)$ in a narrow sense (i.e., if tax avoidance/evasion is taken as given). In a broader sense however, the government can change tax design and enforcement to reduce the elasticity $e$ and therefore increase the revenue maximizing rate.