Econ 230B
Spring 2025
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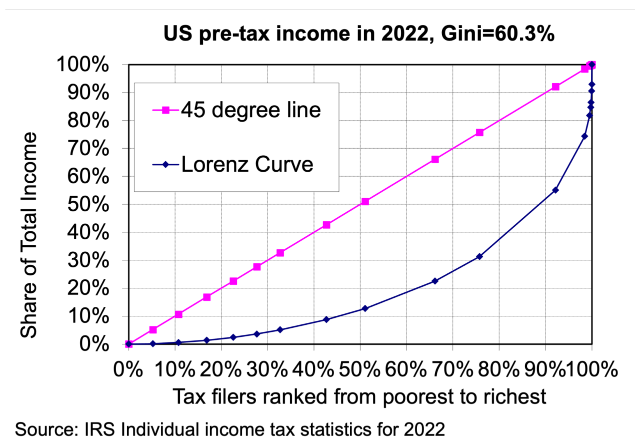
Problem Set 1 SOLUTION

1. Lorenz Curve and Gini Coefficient

The IRS posts online tabulations of the distribution of annual individual incomes based on Federal Individual Income Tax data. We will focus on statistics for year 2022 available online in Table 1.4 posted at (link here).

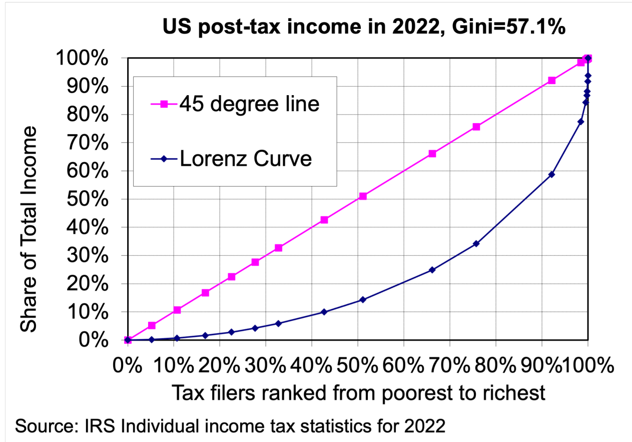
a) Use columns (1) and (2) of the excel Table 1.4 for year 2022 to draw the dots of the Lorenz curve for the Adjusted Gross Income (AGI) distribution for all returns (but excluding returns with no AGI). Feel free to use any software (such as excel, STATA, or R) for this. Connect the dots of the Lorenz curve to compute the Gini coefficient.



b) Using the interpolated Lorenz curve from a), compute the following inequality statistics: bottom 50% income share, next 40% income share, top 10% income share, and top 1% income share.

Reading from the Lorenz curve at the intersection of x=50%, x=90%, and x=99% vertical lines, we find: bottom 50% share = 12.23% , next 40% share = 39.79%, top 10% share = 47.98%, top 1% share = 21.64%

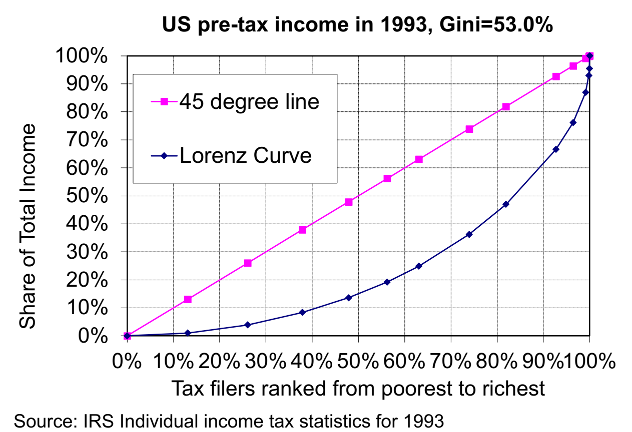
c) Redo a) and b) but using after-tax income defined as AGI minus “income tax minus credits” (the last column (148) of the table). Is after-tax income inequality higher or lower than before tax from a) and b)? Can we conclude that the US individual income tax is progressive?



bottom 50% share =13.75%, next 40% share =41.81%, top 10% share = 44.44%, top 1% share = 18.89%

Yes, income tax is progressive because it lowers inequality. Note that this computation assumes that ranking by pre-tax and post-tax income is identical.

d) Repeat the analysis of a) and b) using year 1993, the earliest year available on the IRS webpage.



Reading from the Lorenz curve at the intersection of x=50%, x=90%, and x=99% vertical lines, we find: Bottom 50% share is 15.0%, Top 10% share is 38.3%, top 1% share is 13.6%

e) Has inequality increased or decreased since 1993 by comparing d) and a-b)? Is this compelling or misleading evidence of the true trends in inequality in the United States over the last 30 years?

2022 displays much more concentration in reported income than 1993. The definition of reported income did not change substantially from 1993 to 2022 so we can confidently say that inequality for this specific measure of income increased substantially. Complete time series depicted slides shows a secular trend up since 1980. Alternative income definitions such as comprehensive national income shows a bit less of an increase.

3. Optimal Top Income Tax Rate with Income Effects

This exercise follows Saez Restud'01 section 3 (see the paper for more details).

a) In the tax reform graph drawn in class, increasing τ by $d\tau$ creates a negative substitution effect (less slope) leading to less work, and a negative income effect leading to more work. Hence, e is a mix of substitution and income effects.

b) Let $z^i(1-\tau, R)$ be the earnings supply function obtained from solving the individual utility maximization problem under a linear tax:

$$\max_{c,z} u^i(c,z) \text{ st } c = (1-\tau)z + R$$

We denote by e_u^i the uncompensated elasticity of z^i with respect to $1-\tau$ and by $\eta^i = (1-\tau)\partial z^i/\partial R$ the income effect parameter. As a simple graph shows (see Saez Restud'01 Section 3), the reform changes $1-\tau$ by $-d\tau$ and changes R by $dR = z^* d\tau$. Note, that the virtual income is defined as $R \equiv z^*\tau$ (in Saez 2001 it is written as $R \equiv \bar{z}\tau$), and as we assume that z^* is fixed it follows immediately that $dR = z^* d\tau$. Hence, we have:

$$dz^i = -d\tau \frac{\partial z^i}{\partial(1-\tau)} + z^* d\tau \frac{\partial z^i}{\partial R} = -\frac{d\tau}{1-\tau} z^i \frac{1-\tau}{z^i} \frac{\partial z^i}{\partial(1-\tau)} + \frac{d\tau}{1-\tau} z^* (1-\tau) \frac{\partial z^i}{\partial R} = -\frac{d\tau}{1-\tau} z^i e_u^i + \frac{d\tau}{1-\tau} z^* \eta^i$$

c) Recall that the elasticity e is defined as

$$e = \frac{1-\tau}{\sum z^i} \frac{\sum dz^i}{d(1-\tau)}.$$

Hence we have

$$e = \frac{1-\tau}{\sum z^i} \sum \left[\frac{1}{1-\tau} z^i e_u^i - \frac{1}{1-\tau} z^* \eta^i \right] = \frac{\sum z^i e_u^i}{\sum z^i} - \frac{z^*}{z^m} \frac{\sum \eta^i}{N}$$

with N number of top bracket taxpayers. Hence $e = \bar{e}_u - \frac{a-1}{a}\bar{\eta}$ with $\bar{e}_u$ the income weighted average of e_u^i and $\bar{\eta}$ the straight average of η^i among top bracket taxpayers.

The uncompensated elasticity is income weighted because those with higher income should count more in the response. In contrast, the income effect parameter is not an elasticity and hence should not be income weighted.