Econ 230B
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Problem Set 2 Solution

2. Social Security Privatization

a) $b = (1+n)\tau w$
$c_1^t + c_2^t/(1+r) = (1 - \tau(r-n)/(1+r))w$

b)

$$\max_s u(w - d - s) + \delta u((1+r)s + (1+n)d)$$

implies FOC:

$$u'(w - d - s) = \delta(1+r)u'((1+r)s + (1+n)d)$$

Taking the total derivative on both sides gives:

$$\delta u''(c_2)(1+r)[(1+r)ds + (1+n)dd] = -u''(c_1)[dd + ds]$$

$$\frac{ds}{dd} = -\frac{\delta u''(c_2)(1+r)(1+n) + u''(c_1)}{\delta u''(c_2)(1+r)^2 + u''(c_1)}$$

which is between 0 and 1 when $n < r$ and equal to 1 when $n = r$.

c) Generation $t_0 - 1$ is hurt. Generation t_0 onward gets a higher return on savings r (instead of n) and hence are better off.

d) Debt per capita a_t evolves according to $(1+n)a_{t+1} = (1+r)a_t - T_t$, where T_t is payment made by each old person in generation t.

To keep debt constant, $a_{t+1} = a_t = a_{t_0} = d$, we need $T_t = (r-n)d$.

So the maximization problem for any future generation will be:

$$\max_s u(w(1-\tau) - s) + \delta u((1+r)(s + \tau w) - (r-n)d)$$

Using the fact that $\tau w = d$, this is equivalent to:

$$\max_s u(w - d - s) + \delta u((1+r)s - (1+n)d)$$

which is exactly the same maximization problem as in b) showing the welfare of future generations is not affected.

e) Consider the initial path (w_t, k_t, r_t) with the PAYG system. Suppose the system is switched to the funded system as in d). Then, as in d), we can show that taking (w_t, r_t) as given and as in the PAYG system, the savings decision of the individual remains the same, so that the savings decision s_t will be such that $k_{t+1} = s_t/(1+n)$ and hence the wage rate and the interest rate will indeed be as the individual expect.

So the initial macro-economic equilibrium PAYG path (w_t, k_t, r_t) remains an equilibrium in the reformed system. So indeed nothing is changed in the general equilibrium.

3. Bunching at kink points

a)

$$\max wh - T(wh) - \frac{h^{1+k}}{1+k}$$

FOC h: $w(1-T') = h^k$ hence $h = w^{1/k}(1-T')^{1/k}$ and $z = wh = w^{1+1/k}(1-T')^{1/k}$:
Hence, three cases depending on size of w:
+ if $w \leq \bar{z}^{k/(k+1)}$ then $z = w^{1+1/k}$. This is the first bracket.
+ if $\bar{z}^{k/(k+1)} \leq w \leq \bar{z}^{k/(k+1)}/(1-\tau)^{1/(k+1)}$ then $z = \bar{z}$. This is bunching at $\bar{z}$.
+ if $\bar{z}^{k/(k+1)}/(1-\tau)^{1/(k+1)} \leq w$ then $z = w^{1+1/k}(1-\tau)^{1/k}$. This is the second bracket.

b) Elasticity is $1/k$.
Fraction bunching is $\int_{w_1}^{w_2} f(w)dw$ where $w_1 = \bar{z}^{k/(k+1)}$ and $w_2 = \bar{z}^{k/(k+1)}/(1-\tau)^{1/(k+1)}$

c) Histogram attached (created with matlab).
Histogram shows bunching at \$10,000 which is $\bar{z}$.

d) All individuals with w in (w_1, w_2) bunch at $\bar{z}$.

Absent the tax rate τ, those with wage w_1 would earn $\bar{z}$ and those with wage w_2 would earn $w_2^{1+1/k} = \bar{z}/(1-\tau)^{1/k}$.

Excess bunching is 193 individuals (with earnings exactly equal to \$10,000). There are also 193 individuals on the left of the kink with earnings between \$10,000-\$827 and \$10,000-\$1. So, absent the kink, those bunching taxpayers would have spread across a band of width \$827 approximately.

Hence \$827$= \bar{z}[1 - 1/(1-\tau)^{1/k}]$.

which translates into $e = 1/k = \log(1 - 827/10000)/\log(1-0.3) = 0.24$ which is very close to the 0.25 I have used to simulate the data.

e) In principle, Blomquist et al. 2021 critique could apply but in this case it does not because I chose a uniform density for the skill distributions which in turn generates a very smooth earnings density (absent the kink in the budget set).