

# A Theory of Optimal Inheritance Taxation

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# 1. MOTIVATION

Controversy about proper level of inheritance taxation

1) Public debate centers around equity-efficiency tradeoff

2) In economics, disparate set of models and results depending on structure of individual preferences and shocks, government objective and tools

This paper tries to connect 1) and 2) by deriving robust optimal inheritance tax formulas in terms of estimable elasticities and distributional parameters

# 1. KEYS RESULTS

We consider stochastic **heterogeneous** preferences and work abilities and **linear** inheritance taxation (for tractability)

We start with **bequests in the utility** model and **steady-state** social welfare maximization

We show that formulas carry over with

(a) social discounting

(b) dynastic utility model

Equity-efficiency trade-off is non degenerate leading to non-zero optimal inheritance taxes (except in limit cases)

We use survey data on wealth and inheritances received to illustrate the optimal formulas

# OUTLINE

## 2. **Bequests in the Utility Function**

(a) Steady-state Social Welfare Maximization

(b) Social discounting

(c) Elastic labor supply (link to Atkinson-Stiglitz, Farhi-Werning)

## 3. **Dynastic Model** (link to Chamley-Judd, Aiyagari)

## 4. **Empirical Calibration**

## 5. **Conclusions and Extensions**

## 2.1 BEQUESTS IN THE UTILITY MODEL

Infinite succession of generations each of measure one

Individuals both bequests receivers and bequest leavers

Linear tax  $\tau_B$  on bequests funds lumpsum grant  $E$  (no labor tax for simplicity initially)

Life-time budget constraint:  $c_i + b_i = y_{Li} + E + R(1 - \tau_B)b_i^r$

with  $c_i$  consumption,  $b_i$  bequests left to kid,  $y_{Li}$  inelastic labor income,  $b_i^r$  pre-tax bequests received from parent,  $R = e^{rH}$  generational rate of return on bequests

Individual  $i$  has utility  $V^i(c, \underline{b})$  with  $\underline{b} = R(1 - \tau_B)b$  net bequests

$$\max_{b_i} V^i(y_{Li} + E + R(1 - \tau_B)b_i^r - b_i, Rb_i(1 - \tau_B))$$

## 2.2 STEADY-STATE SOCIAL WELFARE

With assumptions, ergodic long-run equilibrium with a joint distribution of bequests left, received, and labor income  $(b_i, b_i^r, y_{Li})$

Government period-by-period budget constraint is  $E = \tau_B Rb$  with  $b$  aggregate (=average) bequests

Define elasticity  $e_B = \frac{1-\tau_B}{b} \frac{db}{d(1-\tau_B)}$  (keeping budget balance)

Government chooses  $\tau_B$  to maximize steady-state *SWF*:

$$\max_{\tau_B} \int_i \omega_i V^i(y_{Li} + \tau_B Rb + R(1 - \tau_B)b_i^r - b_i, Rb_i(1 - \tau_B))$$

with  $\omega_i \geq 0$  Pareto weights reflecting social preferences

Define  $g_i = \omega_i V_c^i / \int_j \omega_j V_c^j$  social marginal welfare weight for  $i$

## 2.2 OPTIMAL TAX FORMULA

Define distributional parameters:

$$\bar{b}^r = \frac{\int_i g_i b_i^r}{b} \geq 0 \quad \text{and} \quad \bar{b} = \frac{\int_i g_i b_i}{b} \geq 0$$

$\bar{b}^r < 1$  if society cares less about inheritance receivers

**Optimal inheritance tax rate:** 
$$\tau_B = \frac{1 - \bar{b}/R - \bar{b}^r \cdot (1 + \hat{e}_B)}{1 + e_B - \bar{b}^r \cdot (1 + \hat{e}_B)}$$

with  $\hat{e}_B$  the  $g_i$ -weighted elasticity  $e_B$

**Equity:**  $\tau_B$  decreases with  $\bar{b}$ ,  $\bar{b}^r$  ( $\tau_B < 0$  possible if  $\bar{b}, \bar{b}^r$  large)

**Efficiency:**  $\tau_B$  decreases with  $e_B$  (but  $\tau_B < 1$  even if  $e_B = 0$ )

$\bar{b}, \bar{b}^r$  can be estimated using distributional data  $(b_i, b_i^r, y_{Li})$  for any  $SWF$ ,  $e_B$  can be estimated using tax variation

## 2.2 INTUITION FOR THE PROOF

$$SWF = \int_i \omega_i V^i(\tau_B Rb + R(1 - \tau_B)b_i^r + y_{Li} - b_i, Rb_i(1 - \tau_B))$$

1)  $d\tau_B > 0$  increases lumpsum  $\tau_B Rb$

2)  $d\tau_B > 0$  hurts **both** bequests receivers and bequest leavers

$$\frac{dSWF}{d\tau_B} = \int_i \omega_i \left( V_c^i \cdot Rb \left[ 1 - \frac{\tau_B}{1 - \tau_B} e_B \right] - V_c^i \cdot Rb_i^r (1 + e_{Bi}) - V_b^i \cdot Rb_i \right)$$

$$\text{FOC} \Rightarrow 0 = \left[ 1 - \frac{\tau_B}{1 - \tau_B} e_B \right] - \bar{b}^r (1 + \hat{e}_B) - \frac{\bar{b}/R}{1 - \tau_B}$$

$$\Rightarrow \tau_B = \frac{1 - \bar{b}/R - \bar{b}^r \cdot (1 + \hat{e}_B)}{1 + e_B - \bar{b}^r \cdot (1 + \hat{e}_B)}$$



## 2.2 OPTIMAL TAX FORMULA

**1. Utilitarian case:**  $\omega_i \equiv 1$  and  $V^i$  concave in  $c$

(a) high  $V^i$  curvature and bequests received/left concentrated among well-off  $\Rightarrow \bar{b}, \bar{b}^r \ll 1 \Rightarrow \tau_B \simeq 1/(1 + e_B)$  [ $\tau_B = 0$  only if  $e_B = \infty$ ]

(b) low  $V^i$  curvature or bequests received/left equally distributed:  $\bar{b}, \bar{b}^r \simeq 1$  and  $\tau_B < 0$  desirable

**2. Meritocratic Rawlsian criterion:** maximize welfare of zero-receivers (with uniform  $g_i$  among zero-receivers)

$\Rightarrow \bar{b}^r = 0$  and  $\bar{b} =$  relative bequest left by zero-receivers

$\Rightarrow \tau_B = (1 - \bar{b}/R)/(1 + e_B)$ :  $\tau_B$  large if  $\bar{b}$  low

## 2.3 SOCIAL DISCOUNTING

Generations  $t = 0, 1, ..$  with social discounting at rate  $\Delta < 1$

Period-by-period budget balance  $E_t = \tau_{Bt}Rb_t$  and  $R$  constant

Government chooses  $(\tau_{Bt})_{t \geq 0}$  to maximize

$$\sum_{t \geq 0} \Delta^t \int_i \omega_{ti} V^{ti} (\tau_{Bt} R b_t + R(1 - \tau_{Bt}) b_{ti} + y_{Lti} - b_{t+1i}, R b_{t+1i} (1 - \tau_{Bt+1}))$$

Optimal policy  $\tau_{Bt}$  converges to  $\tau_B$  and economy converges

**Optimal long-run tax rate:** 
$$\tau_B = \frac{1 - \bar{b}/(R\Delta) - \bar{b}^r \cdot (1 + \hat{e}_B)}{1 + e_B - \bar{b}^r \cdot (1 + \hat{e}_B)}$$

Same formula as steady-state replacing  $R$  by  $R\Delta$

Why  $\Delta$  term? Because increasing  $\tau_{Bt}$  for  $t \geq T$  hurts bequest leavers from generation  $T - 1$  and blows up term in  $\bar{b}$  by  $1/\Delta$

## 2.3 Social Discounting and Dynamic Efficiency

If govt can transfer resources across generations with debt:

If  $R\Delta > 1$ , transferring resources from  $t$  to  $t + 1$  desirable  $\Rightarrow$   
Govt wants to accumulate assets

If  $R\Delta < 1$ , transferring resources from  $t + 1$  to  $t$  desirable  $\Rightarrow$   
Govt wants to accumulate debts

Equilibrium exists only if  $R\Delta = 1$  (Modified Golden Rule)

Suppose govt can use debt and economy is closed ( $R = 1 + F_K$ )  
then same optimal tax formula applies just setting  $R\Delta = 1$

Optimal redistribution and dynamic efficiency are **orthogonal**

## 2.3 Social Discounting and Dynamic Efficiency

1) With dynamic efficiency, timing of tax payments is neutral

2) With exogenous economic growth at rate  $G = e^{gH}$  per generation, Modified Golden rule becomes  $\Delta R G^{-\gamma} = 1$  or  $r = \delta + \gamma g$  (with  $\gamma$  “social risk-aversion”) and optimal  $\tau_B$  unchanged

3) Practically, governments do not use debt to meet Modified Golden rule and leave capital accumulation to private agents and  $R$  varies quite a bit across periods

⇒ period-by-period budget balance and steady-state  $SWF$  maximization perhaps most realistic

## 2.4 Elastic Labor Supply and Labor Taxation

Suppose labor supply is elastic with  $y_{Li} = w_i l_i$  and  $V^i(c, \underline{b}, l)$

Labor income taxed at rate  $\tau_L$

Suppose govt trades-off  $\tau_B$  vs.  $\tau_L$ , we obtain the same formula

but multiplying  $\bar{b}, \bar{b}^r$  by  $\left[1 - \frac{e_L \tau_L}{1 - \tau_L}\right] / \bar{y}_L$  with  $\bar{y}_L = g_i y_{Li} / \int_j g_j y_{Lj}$ :

$$\tau_B = \frac{1 - [\bar{b} + \bar{b}^r \cdot (1 + \hat{e}_B)] \cdot \left[1 - \frac{e_L \tau_L}{1 - \tau_L}\right] / \bar{y}_L}{1 + e_B - \bar{b}^r \cdot (1 + \hat{e}_B) \cdot \left[1 - \frac{e_L \tau_L}{1 - \tau_L}\right] / \bar{y}_L}$$

$e_L$  and  $e_B$  are budget balance total elasticity responses

If  $e_L$  high, taxing  $\tau_B$  more desirable (to reduce  $\tau_L$ )

Case for  $\tau_B > 0$  (or  $\tau_B < 0$ ) depending on  $\bar{b}, \bar{b}^r$  carries over

## 2.4 ROLE OF BI-DIMENSIONAL INEQUALITY

Two-generation model with working parents and passive kids

$$\max U^i(v(c_i, \underline{b}_i), l_i) \text{ s.t. } c_i + \frac{\underline{b}_i}{R(1 - \tau_B)} \leq w_i l_i (1 - \tau_L) + E$$

1) Atkinson-Stiglitz:  $\tau_B = 0$  if only parents utility counts

2) Farhi-Werning:  $\tau_B < 0$  if kids welfare  $u(\underline{b}_i)$  also counts

Inequality is uni-dimensional as  $(\underline{b}_i, w_i l_i)$  perfectly correlated

$$\text{Piketty-Saez: } c_i + \underline{b}_i / [R(1 - \tau_B)] \leq w_i l_i (1 - \tau_L) + E + \underline{b}_i^r$$

Inequality is two-dimensional:  $w_i l_i$  and  $\underline{b}_i^r \Rightarrow$  makes  $\tau_B$  more desirable and can push to  $\tau_B > 0$

## 2.5 Accidental Bequests or Wealth Lovers

Individuals may leave unintended bequests because of precautionary saving or wealth loving

In that case,  $\tau_B$  does not hurt welfare of bequest leavers

Same formula carries over replacing  $\bar{b}$  by  $\nu \cdot \bar{b}$  where  $\nu$  is fraction with bequest motives (and  $1 - \nu$  fraction of wealth lovers)

If all bequests are unintended and government is Meritocratic Ralwsian then  $\tau_B = 1/(1 + e_B)$

### 3.1 DYNASTIC MODEL

Same set-up as before but utility function  $V^{ti} = u^{ti}(c) + \delta V^{t+1i}$

Individual  $ti$  chooses  $b_{t+1i}, c_{ti}$  to maximize

$$EV^{ti} = u^{ti}(c) + \delta E_t V^{t+1i} \text{ s.t. } c_{ti} + b_{t+1i} = E_t + y_{Li} + Rb_{ti}(1 - \tau_{Bt})$$

$$\Rightarrow u_c^{ti} = \delta R(1 - \tau_{Bt+1}) E_t u_c^{t+1i}$$

Government has period-by-period budget  $\tau_{Bt} Rb_t = E_t$

With standard assumptions, if govt policy converges to  $(\tau_B, E)$ , economy converges to ergodic long-term equilibrium

Long-run agg.  $b$  function of  $1 - \tau_B$  with finite elasticity  $e_B$

Elasticity  $e_B = \infty$  in the limit case with no uncertainty



### 3.2 DYNASTIC MODEL: Steady State Optimum

Govt chooses  $\tau_B$  to maximize steady-state welfare

$$EV_\infty = \sum_{t \geq 0} \delta^t E[u^{ti}(\tau_B R b_t + R(1 - \tau_B)b_{ti} + y_{Li} - b_{t+1i})]$$

assuming w.l.o.g. that steady-state reached as of period 0

- 1)  $d\tau_B > 0$  increases lumpsum  $\tau_B R b_t$  for all  $t \geq 0$
- 2)  $d\tau_B > 0$  hurts bequest leavers or bequest receivers for  $t \geq 0$ ,  
no double counting except in period 0

Same formula but discounting  $\bar{b}^r$  by  $1 - \delta = 1/(1 + \delta + \delta^2 + \dots)$

$$\tau_B = \frac{1 - \bar{b}/R - (1 - \delta)\bar{b}^r \cdot (1 + \hat{e}_B)}{1 + e_B - (1 - \delta)\bar{b}^r \cdot (1 + \hat{e}_B)}$$

with  $\bar{b}^r = E[u_c^{ti} b_{ti}]/[b_t E u_c^{ti}]$  and  $\bar{b} = E[u_c^{ti} b_{t+1i}]/[b_{t+1} E u_c^{ti}]$

### 3.3 DYNASTIC MODEL: Period 0 Perspective

Govt chooses  $(\tau_{Bt})_{t \geq 0}$  to maximize period 0 utility

$$EV_0 = \sum_{t \geq 0} \delta^t E[u^{ti}(\tau_{Bt} R b_t + R(1 - \tau_{Bt}) b_{ti} + y_{Li} - b_{t+1i})]$$

Assume that  $\tau_{Bt}$  converges to  $\tau_B$ . What is long-run  $\tau_B$ ?

Consider  $d\tau_B$  for  $t \geq T$  ( $T$  large so that convergence reached)

1) Mechanical effect of  $d\tau_B$  only for  $t \geq T$

2) Behavioral effect via  $db_t$  can start before  $T$  in anticipation

Define  $e_B^{\text{pdv}} = (1 - \delta) \sum_t \delta^{t-T} \left[ \frac{1 - \tau_B}{b_t} \frac{db_t}{d(1 - \tau_B)} \right]$  the total elasticity

$$\text{Optimum: } \tau_B = \frac{1 - \bar{b}/(\delta R)}{1 + e_B^{\text{pdv}}}$$

Similar formula but double counting at all

### 3.3 DYNASTIC MODEL: Chamley-Judd vs. Aiyagari

$$\text{Optimum: } \tau_B = \frac{1 - \bar{b}/(\delta R)}{1 + e_B^{\text{pdv}}}$$

This model nests both Chamley-Judd and Aiyagari (1995)

**1) Chamley-Judd:** no uncertainty  $\Rightarrow e_B^{\text{pdv}} = \infty \Rightarrow \tau_B = 0$

**2) Aiyagari:** uncertainty  $\Rightarrow e_B^{\text{pdv}} < \infty$ :  $\bar{b} < 1$  and  $\delta R = 1 \Rightarrow \tau_B > 0$

Weaknesses of dynastic period-0 objective:

(a) forced to use utilitarian criterion [Pareto weights irrelevant]

(b) cannot handle heterogeneity in altruism: putting less weight on descendants with less altruistic ancestors is crazy

## 4. NUMERICAL SIMULATIONS

We calibrate the following general formula:

$$\tau_B = \frac{1 - \left[1 - \frac{e_L \tau_L}{1 - \tau_L}\right] \cdot \left[ (\bar{b}^r / \bar{y}_L)(1 + \hat{e}_B) + \frac{\nu}{R/G} (\bar{b} / \bar{y}_L) \right]}{1 + e_B - \left[1 - \frac{e_L \tau_L}{1 - \tau_L}\right] (\bar{b}^r / \bar{y}_L)(1 + \hat{e}_B)},$$

$e_B, e_L$  elasticity of bequests and earnings wrt to  $1 - \tau_B, 1 - \tau_L$

$\bar{b}^r, \bar{b}, \bar{y}_L$  distributional parameters depend on social objective and micro joint distribution  $(y_{Li}, b_i, b_i^r)$

$R/G = e^{(r-g)H}$  the ratio of generational return to growth

$\nu$  fraction with bequest motives [ $1 - \nu$  fraction wealth lovers]

Base parameters:  $e_B = .2, e_L = .2, \tau_L = 30\%, R/G = 1.8$  ( $r - g = 2\%$ ),  $\nu = 1$  pure bequest motives

## 4. NUMERICAL SIMULATIONS

Use joint distribution  $(y_{Li}, b_i, b_i^r)$  from survey data (SCF in the US, Enquetes Patrimoines in France) to estimate  $\bar{b}^r, \bar{b}, \bar{y}_L$

Zero-receivers ( $\sim$  bottom 50%) leave bequests 70% of average in France/US today (this was only 25% in France  $\sim$ 1900)

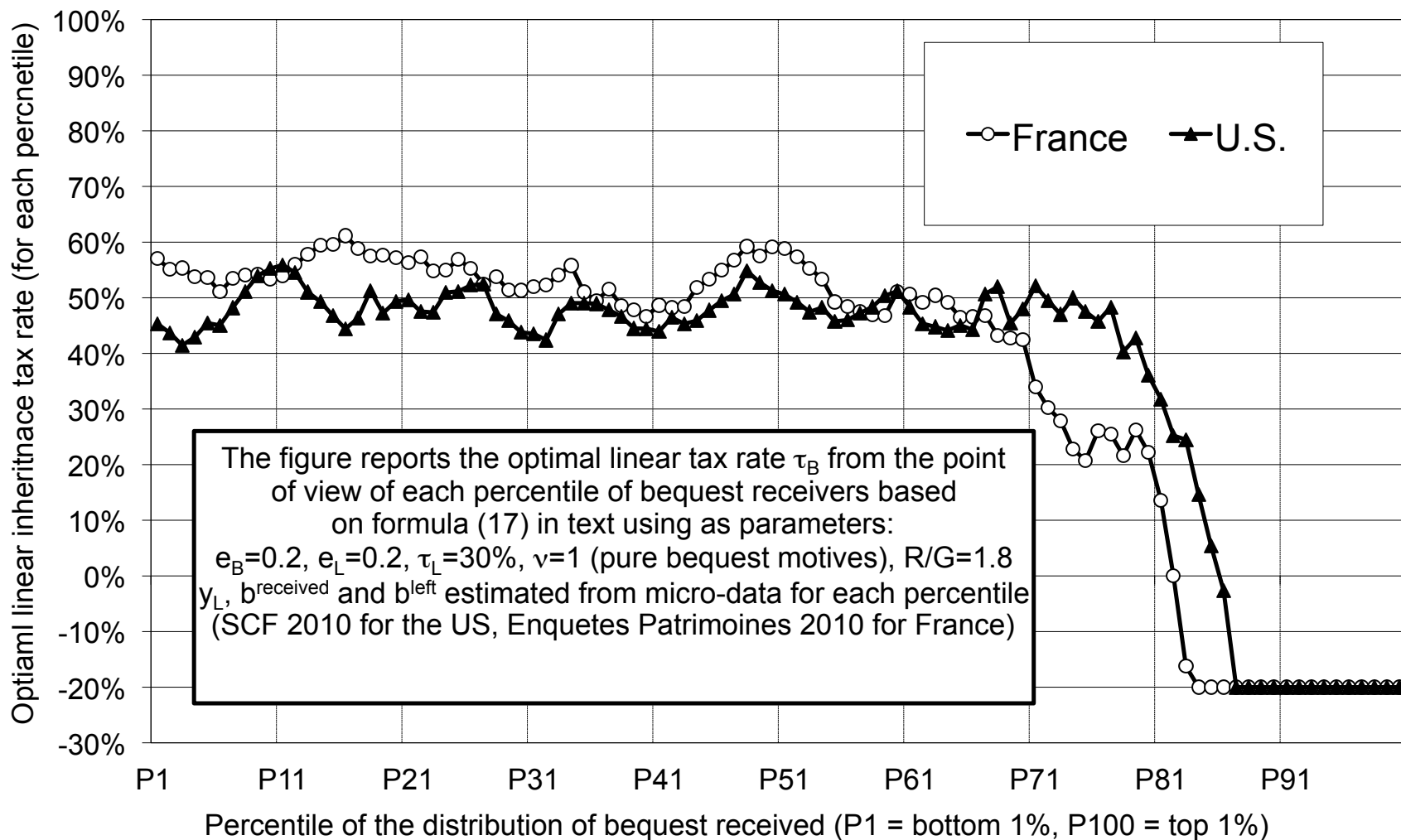
Key issue: received inheritances  $b_i^r$  under-reported in surveys

Social Welfare Objective: what is the best  $\tau_B(p)$  if I am in percentile  $p$  of bequests receivers?

Simplification: we do not take into account that  $\bar{b}^r, \bar{b}, \bar{y}_L, e_B, e_L, \tau_L$  are affected by  $\tau_B$

$\tau_B(p) \simeq 50\%$  for  $p \leq .75$ . Only top 15% receivers want  $\tau_B < 0$

**Figure 1: Optimal Linear Inheritance Tax Rate (by percentile of bequest received)**



## Optimal Inheritance Tax Rate $\tau_B$ Calibrations for the United States

	$e_B=0$	$e_B=0.2$	$e_B=0.5$	$e_B=1$
<b>1. Baseline: <math>\tau_B</math> for zero receivers (bottom 50%), <math>r-g=2\%</math> (<math>R/G=1.82</math>), <math>v=70\%</math>, <math>e_L=0.2</math></b>	70%	59%	47%	35%
<b>2. Sensitivity to capitalization factor <math>R/G=e^{(r-g)H}</math></b>				
$r-g=0\%$ ( $R/G=1$ ) or dynamic $e$	46%	38%	31%	23%
$r-g=3\%$ ( $R/G=2.46$ )	78%	65%	52%	39%
<b>3. Sensitivity to bequests motives <math>v</math></b>				
$v=1$ (100% bequest motives)	58%	48%	39%	29%
$v=0$ (no bequest motives)	100%	83%	67%	50%
<b>4. Sensitivity to labor income elasticity <math>e_L</math></b>				
$e_L=0$	68%	56%	45%	34%
$e_L=0.5$	75%	62%	50%	37%
<b>5. Optimal tax in France 1900 for zero receivers with <math>b^{\text{left}}=25\%</math> and <math>\tau_L=15\%</math></b>	90%	75%	60%	45%

This table presents simulations of the optimal inheritance tax rate  $\tau_B$  using formula (17) from the main text for France and the United States and various parameter values. In formula (17), we use  $\tau_L=30\%$  (labor income tax rate), except in Panel 5. Parameters  $b^{\text{received}}$ ,  $b^{\text{left}}$ ,  $y_L$  are obtained from the survey data (SCF 2010 for the US, Enquetes patrimoine 2010 for France, and Piketty, Postel-Vinay, and Rosenthal, 2011 for panel 5).

## CONCLUSIONS AND EXTENSIONS

**Main contribution:** simple, tractable formulas for analyzing optimal inheritance tax rates as an equity-efficiency trade-off

### Extensions:

- 1) Nonlinear tax structures (connection with NDPF)
- 2) Use same approach for optimal capital income taxation: maybe  $V(c_t, k_{t+1}[1 + r(1 - \tau_K)])$  more realistic than  $\sum \delta^t u(c_t)$
- 3) In practice, rate of return  $r$  varies dramatically across individuals and time periods, with very imperfect insurance  $\Rightarrow$  possible case for taxing capital income