Wealth Inequality and Collective Action*

Pranab Bardhan
University of California, Berkeley

Maitreesh Ghatak
London School of Economics

Alexander Karaivanov
Simon Fraser University

January 2006

Abstract

We analyze the effect of inequality in the distribution of endowment of a private input (briefly, wealth) that is complementary in production with a collective input on efficiency. The collective input is the outcome of a collective action problem (e.g., contribution to pure or impure public goods and extraction from common-property resources). In an environment where transactions costs prevent the efficient allocation of the private input across individuals, and the collective action problem is resolved in a decentralized manner, we characterize the optimal second-best distribution of wealth. We show that while efficiency increases with greater equality *within* the group of contributors and non-contributors, in some situations there is an optimal degree of inequality *between* the groups, thereby locating Olson's original insight in a more general framework.

1 Introduction

Does inequality adversely affect resolution of collective action problems? Evidence ranging from the micro-level (Wade, 1994; Alesina and La Ferrara, 2000; Bardhan, 2000; Dayton-Johnson, 2000; and Khwaja, 2004) to the macro level (Knack and Keefer 1997, Banerjee, 2004; Banerjee, Iyer, and Somanathan, 2005) strongly suggests that the propensity of individuals to join groups, to participate in social activities, to cooperate in various collective action problems, or the provision of public goods and services is negatively related to inequality. Collective action problems arise in a variety of contexts. Consider the following examples from recent micro-level empirical studies that link inequality and collective action: maintenance of infrastructure projects in rural areas[1], measures of social capital (such as the propensity of individuals to join groups)[2], cooperation in water allocation and field channel

*A previous version of this paper was circulated under the title "Inequality, Market Imperfections, and the Voluntary Provision of Collective Goods". We thank Jean-Marie Baland, Abhijit Banerjee, Tim Besley, Pierre-Andre Chiappori, Avinash Dixit, Eliana La Ferrara, Hannes Mueller, Paul Seabright, and several seminar audiences for helpful comments. However, the responsibility for all errors and shortcomings lies only with us.

[1] Khwaja (2004) finds a negative relationship in his study of 132 community-maintained infrastructure projects in rural areas of Northern Pakistan.

[2] Using survey data on group membership and data on U. S. localities, Alesina and La Ferrara (2000) find that, after controlling for many individual characteristics, participation in social activities is significantly lower in more unequal localities. See also La Ferrara (2003).

maintenance in agriculture[3], extraction of common property resources (henceforth, CPR) such as forest resources[4], and community level forest conservation efforts[5]. Some of these collective action problems involve positive externalities (pure or impure public goods) and some of them include negative externalities (tragedy of the commons). What is missing is a theoretical framework that links inequality to efficiency across a range of collective action problems such as those mentioned above. The goal of this paper is to provide such a framework.

The link between inequality and collective action suggests an alternative mechanism, compared to those that emphasize borrowing constraints (e.g., Galor and Zeira, 1993, Banerjee and Newman, 1993) or political economy (Alesina and Rodrik, 1994), that could explain the relationship between initial wealth inequality and the pace and pattern of economic development.[6] Indeed, empirical studies that show reducing inequality (e.g., through land reform) can have large positive productivity have suggested that a part of this is likely to be due to better resolution of collective action problems (Banerjee, Gertler and Ghatak, 2002).

While our framework will be relevant to the examples mentioned above, for the sake of a concrete benchmark we will take the example of a collective action problem in agriculture. There is inequality in the distribution of endowment of a private input (land). Producers use this private input and a collective good (say, irrigation water) to produce a private good (say, rice). The private and collective inputs are complements in the production function. Water use by one farmer creates a negative externality by making less available for others (like in a CPR) but effort toward maintenance generates positive externalities (as in a public good).

The assumption of decreasing returns, a standard one in most economic contexts, implies that the more scarce an input is in a given production unit, the higher is its marginal return. As a result, one would expect a more equal distribution of this input across production units that will improve efficiency. If the market for this input operated well, then forces of arbitrage would make sure it is allocated equally to maximize efficiency. However there is considerable evidence suggesting that the market for inputs such as land or capital does not operate frictionlessly and the wealth of an individual determines how much of that input she can use in her production unit.[7]

But in the presence of collective action problems inequality of private endowments such as land or wealth may pull in the opposite direction. Indeed, in his pioneering work on collective action, Olson (1965) makes a strong case in favor of inequality since the greater the interest in the collective good of any single member, the greater the likelihood that this member will get such a significant proportion of

[3] Using data from India and Mexico respectively, Bardhan (2000) and Dayton-Johnson (2000) find that the Gini coefficient for inequality of landholding among irrigators has a significant negative effect on cooperation in water allocation and field channel maintenance.

[4] Baland et al (2006) study the link between firewood collection and inequality using data from Nepal.

[5] Somanathan et al (2006) study community management of forests in the Indian Central Himalayas and find that greater equality in land ownership increases conservation of pine forests.

[6] See Benabou (1996) and Banerjee and Duflo (2003) for empirical evidence on this question.

[7] Evans and Jovanovic (1989) analyzed panel data from the National Longitudinal Survey of Young Men (NLS), which surveyed a sample of 4000 men in the US between the ages of 14-24 in 1966 almost every year between 1966-81, and found that entrepreneurs are limited to a capital stock of no more than one and one-half times their wealth when starting a new venture. There is also a large literature showing that small farms are more efficient than large farms in agricultural sector of developing countries (see Berry and Cline, 1979).

the total benefit from the collective good that he will gain from seeing that the good is provided, even if he has to pay all of the cost himself. This role of "elites" has been confirmed in field studies, such as Wade's authoritative study on irrigation systems in South Indian villages.

We can interpret the "size" of a player as her endowment of the private input if it is complementary with the collective good in production.[8] Olson considered pure public goods only, which indeed have the property that only the largest (richest) player contributes. Our paper is concerned with the following questions. Is this property pointed out by Olson true for a more general class of collective goods that include both impure public goods and CPRs? If we look at welfare instead of the level of the provision of the collective good, is it possible that some degree of inequality may yield a higher level of joint surplus than perfect equality? Furthermore, is it possible for the allocation under some degree of inequality to Pareto-dominate the allocation under perfect equality? If more than one player is involved in the provision of the public good, how would inequality *within* the class of contributors affect efficiency and how would inequality between the class of contributors and non-contributors affect efficiency?

Since the private and collective inputs are complementary in our framework, the marginal return from contributing to the public good is increasing in the amount of the private input an agent has and which we are going to refer to as "wealth" in the rest of the paper. As a result, there will exist a threshold level of the amount of the private input such that only agents who have a level of wealth higher than this threshold will participate in providing the collective good while those with a lower level will free ride on the former group.[9] This means that redistributions that increase the wealth of the richer players at the expense of non-contributing poorer players would achieve a greater amount of the public good, and *other things being constant*, this should increase joint surplus. In our framework this is how Olson's original argument shows up. However, his argument focuses only on the total amount of the public good and not on joint surplus. In particular, the gain from increasing the size of the collective input has to be measured against the cost arising from worsening the allocation of the private input in the presence of decreasing returns.

We show that the amount of contribution towards the provision (in the case of common property resources this has to be interpreted as extraction) of the collective input is a concave function of the private input endowment for most well-known production functions (e.g., the Cobb-Douglas and the CES) and also that the equilibrium level of joint surplus (of both contributing and non-contributing players) is a concave function of the wealth distribution and hence displays inequality aversion. In addition, the total amount of the collective input is a concave function of the wealth distribution *among contributing players*. This means that initial asset inequality lowers the total provision of (pure and impure) public goods, and lowers the total extraction from the CPR. We provide a precise characterization of what the optimal distribution of wealth that maximizes joint surplus is in the case of imperfect convertibility between the private input and contribution to the collective input. We show that the joint surplus maximizing wealth distribution under private provision of the public good involves equalizing the wealth levels within the group of all non-contributing players at some positive level and

[8] In Olson (1965) this is the share of the total benefit to the group that accrues to an individual player.

[9] Baland and Platteau (1997) provide some very interesting examples where richer agents tend to play a leading role in collective action in a decentralized setting. For example, in rural Mexico the richer members of the population take the initiative in mobilizing labor to manage common lands and undertake conservation measures such as erosion control.

also within the group of all contributing players. The contrast with the conclusions of Olson is quite sharp. The key assumptions leading to our result are: market imperfections that prevent the efficient allocation of the private input across production units, and some properties of the production function that are shared by widely used functional forms such as Cobb-Douglas and CES under decreasing returns to scale. With constant returns to scale, the joint surplus within the group of contributors is independent of the distribution of wealth as in the well known distribution neutrality theorem in the public economics literature.[10]

Next, we consider the question of inter-group inequality. In particular, does perfect equality among *all* players maximize joint surplus? We provide a limited answer to this question. It turns out that perfect equality among all players (i.e. inter-group equality in addition to intra-group equality) is not always optimal. If wealth was equally distributed among all, the average wealth of contributors is low and this could reduce the level of the collective good. In contrast concentrating all wealth in the hand of one person will maximize the average wealth of contributors, but will involve significant losses due to the assumed decreasing returns in the individual profit function with respect to wealth. The optimal distribution of wealth characterized above achieves a compromise between these two opposing forces. Under constant returns to scale, where the latter factor is absent, we get the result that an allocation with *some degree of inequality can Pareto-dominate the allocation under perfect equality*, i.e. it not only achieves higher joint surplus but also higher individual surplus for each agent.

Related Literature

Following Olson's pioneering work, the public economics literature has addressed the question of inequality among contributing players in some detail, but mostly in the context of pure public goods. For example, the well known distribution-neutrality result mentioned above applies to the provision of *pure* public goods. These are public goods where individual contributions are perfect substitutes in the production of the public good and everyone gets the same benefit from the public good irrespective of the level of their contributions. Then in a Nash equilibrium the wealth distribution within the set of contributors does not matter for the amount of public goods provision. The intuition behind this result is explained very clearly in Bergstrom, Blume and Varian (1986). Suppose after the redistribution every player adjusts his contribution to the public good by exactly the same amount as his change in wealth and leaves the consumption of the private good unchanged. In that case the amount of the public good is the same as before and so the initial allocation is still available to all players. Those who have lower wealth because of redistribution have a restricted budget set and would clearly prefer the previous allocation if it is still available. The budget set of those who have higher wealth because of redistribution expands, but not in the neighborhood of the original choice. In particular, now the extra options available to the player which are not dominated by options in the previous budget set involve a lower level of the public good compared to what she would receive if she did not contribute before, and higher levels of the private good. But she did contribute before, and so she is also better off with her previous choice.

Subsequent work has shown that the neutrality result depends crucially on the individual contributions being perfect substitutes in the production of the public good, the linearity of the resource

[10] Some of the contributions to the theoretical literature related to this result are Warr (1983), Bergstrom, Blume and Varian (1986), Cornes and Sandler (1996) and Itaya, de Meza and Myles (1997).

constraints, the absence of corner solutions, and the "pureness" of the public good (i.e., the benefit received by a player must depend only on the total level of contributions, but not on her own contribution (see Cornes and Sandler, 1996, pp. 184-190 and p. 539; Bergstrom, Blume and Varian, 1986, Cornes and Sandler, 1994).[11]

In this paper we consider three points of departure from the distribution-neutrality framework.

First, we adopt the framework of a generalized collective good of which pure and impure public goods with positive externality (e.g., roads, canal irrigation, law and order, public R&D, public health and sanitation) are particular cases. We also analyze collective goods with negative externality (e.g., forestry, fishery, grazing lands, surface or groundwater irrigation).

Second, another point of departure from the standard literature on voluntary provision of public goods is that we look not only at the level of provision of the collective good in question, but also the total surplus from the good, net of costs.

Third, the distribution-neutrality result assumes that the contributions towards the public good and the private input are fully convertible.[12] In practice, particularly in the building of rural infrastructure in developing countries, the contribution towards the public good often takes the form of labor. To fix ideas, let us think of the private input as capital. Then this assumption bypasses an important issue of economic inequality: labor is not freely convertible into capital. Typically, labor and capital are not perfect substitutes in the production technology, and because of credit market imperfections capital does not flow freely from the rich to the poor to equate marginal returns. We take this more plausible scenario as our starting point and examine the effect of distribution of wealth among members of a given community on allocative efficiency in various types of collective action problems (involving public goods as well as common property resources or CPR) in the presence of missing and imperfect capital markets. This is particularly important in less developed countries where the life and livelihood of the vast masses of the poor crucially depend on the provision of above-mentioned public goods and the local CPR (particularly when it is not under commonly agreed-upon regulations[13]), and where markets for land and credit are often highly imperfect or non-existent. In poor countries where property rights are often ill-defined and badly enforced, even usual private goods have sometimes certain public good features attached to them, and due to ongoing demographic and market changes the traditional norms and regulations on the use of CPR are often getting eroded. In such contexts inequality of the players may play a special role.

There has also been some recent theoretical work that links wealth inequality to efficiency in the commons problem (e.g., Baland and Platteau, 1997, Dayton-Johnson and Bardhan, 2002). The key distinguishing features of our framework are: first, we allow for a range of collective action problems,

[11] Baland, Dagnelie, and Ray (2003) consider whether inequality in the shares of the benefit that players receive from a public good is good or bad for efficiency might depend on whether the contributions of the players are substitutes or complements in the production function of the public good.

[12] The distribution neutrality literature is couched in the framework of a consumer choosing to allocate a given level of income between her private consumption and contribution to a pure public good. We adopt the framework of a firm using a private input and a public input to produce some good. While not exactly equivalent, formally these frameworks are very similar and what we call the private input is similar to the private consumption good in the distribution neutrality literature.

[13] Notice that agreeing upon such regulations is itself a collective action problem.

ranging from private provision of pure/impure public goods to the tragedy of the commons; and second, we explicitly distinguish between contributors and non-contributors, and correspondingly, between intra-group inequality and inter-group inequality.

The plan of the paper is as follows. In section 2 we present the formal model and in section 3 we analyze the decentralized equilibrium. In section 4 we discuss the implications of relaxing some of our main assumptions. In section 5 we provide some concluding observations. The appendix contains all formal proofs.

2 The Model

There are $n > 1$ players. Each player uses two inputs, k_i and z_i, to produce a final good. The input k_i is a purely *private* good, such as land, capital, or managerial inputs. We assume that there is no market for this input and so a player is restricted to choose $k_i \leq w_i$ where w_i is the exogenously given endowment of this input of player i. While we will focus on this interpretation, there is an alternative one which views w_i as capturing some characteristic of a player, such as a skill or a taste parameter.[14] In contrast, z_i is a *collective* good in the sense that it involves some externalities, positive or negative. We assume that each player chooses some action x_i which can be thought of as her effort that goes into using a common property resource or contributing towards the collective good. Let $X \equiv \sum_{i=1}^{n} x_i$ be the sum total of the actions chosen by the players. The individual actions aggregate into the collective input in the following simple way $z_i = bx_i + cX$. The profit (surplus) function of player i is[15]:

$$\pi_i = f(w_i, z_i) - x_i$$

Note that the input x_i appears twice in the profit function, once on its own as a private input, and once in combination with the quantities used or supplied by other agents. This implies that the private return to a player always exceeds the social return as long as $b > 0$. The input X can be a good (e.g., R&D, education) or a bad (e.g., any case of congestion or pollution). This formulation allows each player to receive a different amount of benefit from the collective input which depends on the action level they choose. In contrast, for pure public goods every player receives the same benefits irrespective of their level of contribution. This case, as well as many others (involving both positive and negative externalities) appear as special cases of our formulation as we will see shortly. Following the distribution neutrality literature we assume that the cost of supplying x_i units of the collective input, is simply x_i and that the production function, f exhibits non-increasing returns with respect to the private and the collective inputs x_i and z_i.

Let $\mathbf{w} = (w_1, w_2, .., w_n)$ denote the vector of wealth levels of the players. Assume that the w_i are arranged in descending order of magnitude, i.e., $w_1 > w_2 > .. > w_n > 0$ and let $W \equiv \sum_{i=1}^{n} w_i$ denote the total amount of wealth of the n players. [16] We make the following assumption about the production

[14] The assumption that the market for the private input does not exist at all, while stark, is not crucial for our results. All that is needed is that the amount a person can borrow or the amount of land she can lease in depends positively on how wealthy she is. Various models of market imperfections, such as adverse selection, moral hazard, costly state verification or imperfect enforcement of contracts will lead to this property.

[15] We will also refer to $\Pi = \sum_{i=1}^{n} \pi_i$ as joint surplus or joint profits later in the paper.

[16] Following the literature on distribution neutrality, we are assuming linear costs.

function:

Assumption 1: *$f(w,z)$ is a strictly increasing, strictly concave function that is twice continuously differentiable with respect to both arguments, $f_{12} \geq 0$ for all (w,z), $\lim_{w\to 0} f_2(w,z) = 0$ and it satisfies the Inada endpoint conditions. In addition $f(w,z) = -D$ for $z < 0$, where D is a very large number.*

We also make an assumption about the parameters b and c:

Assumption 2

$$b \geq 0 \quad \textit{and} \quad b + cn > 0 \tag{A2}$$

Notice that we allow c to be positive, negative or zero. Only when $c < 0$ does the second inequality in (A2) become relevant ensuring that $|c|$ is not too large. This implies that if a social planner chooses the level of the collective input, she would choose a positive level of x_i for at least one player. In addition the same inequality also turns out to be a condition on the reaction functions of the players which ensures the stability of the equilibrium. When $c = 0$ we have the case of a pure private good - there are no externalities. For $b = 0$ and $c > 0$ we have the case of pure public goods, i.e. the one on which most of the existing literature has focused. For $b > 0$ and $c > 0$ we have the case of impure public goods as defined by Cornes and Sandler (1996). For $b > 0$ and $c < 0$ we have a version of the commons problem: by increasing her action relative to those of the others an individual gains.

We begin our analysis by the following result which shows how the choice of x_i by player i depends on how much wealth she has:

Lemma 1: *$\gamma(w) \equiv \arg\max_{z\geq 0}\{f(w,z) - z\}$ is strictly positive for $w > 0$ and is an increasing function.*

This property follows directly from the complementarity between w_i and z_i and the diminishing returns with respect to z. An increase in w_i raises the marginal return of z_i relative to its marginal cost which is assumed to be constant and equal to 1. To restore equilibrium at the individual level, the amount of the collective input must increase. This is a very intuitive property and is plausible in the context of agriculture (e.g., Wade, 1994) and elsewhere (e.g., DiPasquale and Glaeser, 1999 show that there are significant connections between homeownership and a variety of social capital and citizenship measures.)

This property is not satisfied if w and z_i are substitutes in the production function. We will discuss this case in section 4. Several widely used in economics functional forms that display complementarity among the inputs satisfy the conditions stipulated in this lemma. These include the following:

(a) the Cobb-Douglas production function, $f(w,z) = w^{\alpha} z^{\beta}$ with $\alpha + \beta \leq 1$. In this case $\gamma(w) = \beta^{\frac{1}{1-\beta}} w^{\frac{\alpha}{1-\beta}}$ which is clearly an increasing function;

(b) the generalized CES production function, namely, $f(w,z) = (\delta w^{\rho} + (1-\delta) z^{\rho})^{\frac{k}{\rho}}$ under parameter restrictions that ensure non-increasing returns to scale ($k \leq 1$), and complementarity between w and z ($\rho < k$). We work out the details of this case in the appendix. For $k = 1$ (constant returns to scale) it is possible to solve for $\gamma(w)$ explicitly, which turns out to be $\left(\frac{\delta}{\{1-(1-\delta)^{\frac{1}{1-\rho}}\}}\right)^{\frac{1}{\rho}} (1-\delta)^{\frac{1}{1-\rho}} w$.

An important question from the economic point of view is that while clearly both a rich and a poor individual would increase their contribution to the collective input if their wealth increases by the same amount, who would want to expand their contribution to the collective input more? The answer to this question is crucial for determining the effect of the distribution of wealth on the total amount of the collective good, X and joint profits. This effect depends on the curvature of γ, which in turn depends on the third-order properties of the production function. Intuitively, the question is whether diminishing returns with respect to z would kick in at a faster or a slower rate at a higher wealth level. This would determine whether for a richer person a relatively small or large increase in z would restore her individual optimum compared to a poorer person. It turns out that all widely used functional forms in economics where the inputs are complements to each other have the property that diminishing returns kick in at the same or faster rate the higher is the wealth level. This implies that γ is linear or strictly concave. We make the following technical assumption about the production function which we show is equivalent to γ having this property.

Assumption 3

$$h(w,z) \equiv \frac{\partial f(w,z)}{\partial z} \text{ is quasi-concave.}$$

The following lemma proves that h being (weakly) quasi-concave is equivalent to γ being (weakly) concave:

Lemma 2: *Suppose Assumption 1 holds.* $\gamma(w)$ *is concave if and only if* $h(w,z) \equiv \frac{\partial f(w,z)}{\partial z}$ *is quasi-concave.*

For the Cobb-Douglas and CES production functions, $\gamma(w)$ is strictly concave under decreasing returns to scale ($\alpha + \beta < 1$ for Cobb-Douglas and $k < 1$ for CES) and linear under constant returns to scale ($\alpha + \beta = 1$ or $k = 1$). This property is confirmed in some empirical studies: for example, Baland et al (2006) find this "concavity effect" in the context of their study of firewood collection in rural Nepal. In particular, they find that the amount of firewood extracted increases with the income of the family, but less than proportionally.

The following lemma provides additional characterization of the class of production functions for which γ is linear:

Lemma 3: *Suppose Assumption 1 holds. If* $f(w,z)$ *is homogeneous of degree* 1 *then* $\gamma(w) = Aw$ *where* A *is a positive constant.*

This result follows from the fact that if the production function is homogeneous of degree 1 then if both the wealth of a player and the amount of the collective input received by her are increased proportionally, the marginal return from contributing remains unchanged. The Cobb-Douglas production $w^{\alpha}z^{\beta}$ with $\alpha + \beta = 1$ and the CES production function $(\delta w^{\rho} + (1-\delta)z^{\rho})^{\frac{k}{\rho}}$ with $k = 1$ are examples of production functions that are homogeneous of degree 1 and we have already verified that $\gamma(w)$ is linear in these cases.

3 The Decentralized Equilibrium

We characterize the decentralized equilibrium in the following two steps. First, for a given distribution of the private input $\mathbf{w} = \{w_i\}_{i=1..n}$, we solve for the optimal contributions of each agent, $\hat{x}_i$, the total contribution X, and the joint surplus, Π. Second, we look for the distributions of w_i, which maximize the total contribution and joint profits to be able to analyze the effects of inequality on these two variables.

3.1 Characterization of the Nash Equilibrium for a Given Distribution of the Private Input

Let us consider the decentralized Nash equilibrium allocation. Player i takes the contribution of other players, X_{-i}, as given and solves:

$$\max_{x_i \geq 0} \pi^i = f(w_i, bx_i + cX) - x_i$$

The first-order conditions are

$$f_2(w_i, bx_i + cX)(b + c) - 1 \leq 0$$

$$x_i \geq 0$$

together with the standard Kuhn-Tucker complementary slackness condition. Let the function $g(w_i) > 0$ denote the solution to[17]

$$f_2(w_i, g(w_i))(b + c) = 1$$

Notice that under Assumptions 1-3 $g(w)$ is increasing and concave in w.

Suppose $m \leq n$ players contribute in equilibrium. By the assumed complementarity between w_i and z_i in the production function, for a given value of z_i, $f_2(w_i, z_i)$ is increasing in w_i. Therefore, $g(w_i)$ is increasing. Also, by Assumption 1, $\frac{\partial z_i}{\partial x_i} = b + c > 0$. Therefore irrespective of the value of c (in particular, even if it is negative) for a given level of contribution of other players, X_{-i}, the richest player has the greatest marginal profit from contributing, followed by the second richest and so on. For the case of pure public goods, i.e., $b = 0$, the amount of the collective input enjoyed by each player is the same, whether the player contributes or not. Therefore, only the richest player will contribute, i.e., $m = 1$. So long as $b > 0$, the amount of the collective input enjoyed by each player is different, i.e., $bx_i + cX \neq bx_j + cX$. Therefore, even if the second richest player's marginal return from contributing is less than that of the richest player, she can contribute less and enjoy a lower level of the collective input and thereby attain an interior equilibrium. As a result the set of contributing players will consist of the richest m ones, where $n \geq m \geq 1$.

Let us denote by $\hat{x}_i$ the optimal contribution of player i and let $X = \sum_{i=1}^{m} \hat{x}_i$. We assume that the wealth of the richest player exceeds some threshold level so that $x_1 > 0$. Given $z_1 = g(w_1) > 0$ and Assumption 2, we will have $x_1 = \dfrac{g(w_1)}{b + c} > 0$.

[17] This function would be identical to $\gamma(.)$ defined in the previous section if $b + c = 1$ and it obviously inherits the properties of γ discussed above.

The above inequality implies that $m \geq 1$. Then, the equilibrium conditions for the optimal individual contributions are as follows:

$$\hat{x}_i = \begin{cases} \dfrac{g(w_i) - cX}{b}, \ i = 1, .., m \\ 0, \ i = m+1, .., n. \end{cases} \tag{1}$$

$$X(m) = \frac{\sum_{i=1}^{m} g(w_i)}{b + mc} \tag{2}$$

$$g(w_m) \geq \frac{c \sum_{i=1}^{m} g(w_i)}{b + mc} > g(w_{m+1}) \tag{3}$$

Condition (1) states that, for all contributing agents, the first order condition must hold as equality. Condition (2) is equivalent to $X(m) = \sum_{i=1}^{m} \hat{x}_i$. It states that the total contribution must have a value that is consistent with the individual maximization problems of all m contributors, and is obtained by adding up the m first-order conditions from (1). The third condition is the most interesting one, as it determines the size of the contributing group, m. To see why it should hold, note that the $m+1$-th agent would not contribute if $f_2(w_{m+1}, cX(m))(b+c) < 1$, since any further contribution has a marginal benefit that is lower than marginal cost. This condition is equivalent to $cX(m) > g(w_{m+1})$, which is exactly what the second inequality in (3) states. By the same logic, the m-th agent would not be contributing if $g(w_m) < cX(m-1) = \frac{c \sum_{i=1}^{m-1} g(w_i)}{b+(m-1)c}$, which can be rearranged as $g(w_m) < \frac{c \sum_{i=1}^{m} g(w_i)}{b+mc}$. Thus, if she is to contribute, the condition $g(w_m) \geq \frac{c \sum_{i=1}^{m} g(w_i)}{b+mc}$ must be satisfied. The number of contributing agents, m is the smallest integer for which (3) is satisfied. If the left inequality in (3) holds for $m = 1, 2..n$, then all agents contribute.

The following lemma, together with the fact that g is increasing, ensures the existence of a unique value of m that solves (3):

Lemma 4: *If $k+1 \leq m$, then the function $s(k) = \dfrac{c \sum_{i=1}^{k} g(w_i)}{b + kc}$ is increasing in k. If $k > m$ the function $s(k)$ is decreasing in k.*

Several useful observations follow directly from (1)-(3):

Observation 1. For the case of a pure public good ($b = 0$, $c \geq 0$) (3) cannot hold for $m > 1$ as that would imply

$$g(w_m) \geq \frac{\sum_{i=1}^{m} g(w_i)}{m}$$

which is impossible given that $w_1 > w_2 > .. > w_n$ by assumption and Lemma 1 showing that $g(.)$ is increasing. Thus for pure public goods only the richest player contributes. This has the implication that even when the difference in the wealth between the richest player and second richest player is arbitrarily small, the former provides the entire amount of the public good.

Observation 2. For the case of pure private goods ($c = 0$), there is no interdependence across players and all of them will contribute.

Observation 3. For those player who contribute in equilibrium, the condition (1) can be rewritten in the form of a reaction function:

$$\hat{x}_i = \frac{1}{b+c}\{g(w_i) - cX_{-i}\}, \ i = 1, .., m$$

where $X_{-i} \equiv \sum_{j=1,j\neq i}^{m} \hat{x}_j$. We show in the appendix that the condition for stability of the Nash equilibrium is $b + cm > 0$ which is ensured by Assumption 2. The contributions of players are strategic complements for $c < 0$ and strategic substitutes for $c > 0$. Formally, this follows from the fact that $\frac{\partial^2 \pi^i}{\partial x_i \partial X_{-i}} = c f_{22}(w_i, bx_i + cX)(b + c)$. Intuitively, the reason is that the contributions of various players are perfect substitutes in the payoff function, and in the case of public goods (commons) an increase in the contribution of others is similar to an increase (decrease) in the player's own contribution, which reduces (increases) the marginal return of her contribution due to diminishing returns in the collective input (i.e., z).

3.2 Effect of Wealth Inequality on Total Contributions and Joint Profits

From (2),

$$X = \frac{\sum_{i=1}^{m} g(w_i)}{b + mc} \equiv \tilde{g}(\mathbf{w})$$

Under Assumptions 1-3 $X = \tilde{g}(\mathbf{w})$ is the sum of m concave functions and as such is a concave function itself. Moreover, as these functions are identical and receive the same weight, if we hold the number of contributors constant the total contribution is maximized when all contributing agents have equal amounts of the private input. Therefore we have:

> **Proposition 1**: *Suppose Assumptions 1-3 are satisfied. If $g(w_i)$ is strictly concave in w_i then X is strictly concave in $\mathbf{w}$ and is maximized when all contributing agents have equal amounts of the private input. If $g(w_i)$ is linear in w_i then X is linear in $\mathbf{w}$.*

Recall that Assumption 3 implies that diminishing returns with respect to the collective input used by the i-th individual set in at a faster rate at a higher wealth level, and so the optimal level of the collective input is a concave function of the wealth level (Lemma 2). Proposition 1 follows from this assumption, and the fact that the collective input used by the i-th individual is a linear function of the individual's own contribution and the contribution of other players.

To see this more clearly consider the two player version of the game where player 1 has wealth $w + \varepsilon$ and player 2 has wealth $w - \varepsilon$ where $\varepsilon > 0$. From the first order condition of the two players:

$$\begin{aligned} g(w + \varepsilon) &= (b + c)x_1 + cx_2 \\ g(w - \varepsilon) &= cx_1 + (b + c)x_2 \end{aligned}$$

Therefore, the reaction functions are:

$$\begin{aligned} x_1 &= \frac{1}{b + c}\{g(w + \varepsilon) - cx_2\} \\ x_2 &= \frac{1}{b + c}\{g(w - \varepsilon) - cx_1\} \end{aligned}$$

Consider the effect of an increase in ε. The direct effect is to increase x_1 and reduce x_2. For the case of positive externalities ($c > 0$) the indirect effects move in the same direction, while for the case of negative externalities, the indirect effects move in the opposite direction. For example, in the former case, a reduction in x_2 stimulates a further increase in x_1 and an increase in x_1 leads to a further

decrease in x_2. The stability condition ensures that indirect effects in the successive rounds shrink in terms of size. Linearity of the reaction functions implies that the total effects of a change in ε on x_1 and x_2 are linear combinations of the direct effects on the two players. The fact that the reaction functions of both players have the same slope in our set up implies additionally that the direct effects of redistribution on the contribution of the two players receive the same weight in $\frac{dX}{d\varepsilon}$ and so it follows directly from the concavity of $g(.)$ that X is decreasing in ε.

The effect of wealth inequality on X has implications which are quite different from those available so far in the public economics literature. Our analysis shows that greater equality *among those who contribute towards the collective good* will increase the value of X. Therefore a more equal wealth distribution among contributors will increase the equilibrium level of the collective input. In addition, any redistribution of wealth from non-contributors to contributors that does not affect the set of contributors will also increase X.[18] In terms of the two-player example, this implies that as long as both players contribute, any inequality in the distribution of wealth reduces X. But with sufficient inequality if one player stops contributing then any further increases in inequality will increase X.

Let us now turn to the normative implications of changes in the distribution of wealth. Under the first-best, which can be thought of a centralized equilibrium where players choose their contributions to maximize joint surplus, the first-order condition for player i is:

$$f_2(w_i, bx_i + cX)(b + nc) \leq 1.$$

The difference with the decentralized equilibrium is that now individuals look at the social marginal product of their contribution to the collective input, i.e., $f_2(w_i, bx_i + cX)(b + nc)$ as opposed to the private marginal product, i.e., $f_2(w_i, bx_i + cX)(b + c)$. Then it follows directly that those who will contribute will contribute more (less) than in the decentralized equilibrium if $c > 0$ ($c < 0$). Also, the number of contributors will be higher (lower) than in the decentralized equilibrium if $c > 0$ ($c < 0$).

Therefore, for the case of positive externalities, the total contribution in a decentralized equilibrium is less than the efficient (i.e., joint surplus maximizing) level. Conversely, for the case $c < 0$, total contributions exceed the socially efficient level. From this one might want to conclude that greater inequality among contributors increases efficiency in the presence of negative externalities and reduces efficiency if there are positive externalities.[19] Indeed, the literature on the effect of wealth (or income) distribution on collective action problems have typically focused on the size of total contributions. However, that is inappropriate as the correct welfare measure is joint profits.

In the presence of decreasing returns to scale the distribution of the private input across firms will have a direct effect on joint profits irrespective of its effect on the size of the collective input. In particular, greater inequality will reduce efficiency by increasing the discrepancy between the marginal returns to the private input across different production units. In the case of negative externalities, these two effects of changes in the distribution of the private input work in different directions, while in the case of positive externalities, they work in the same direction. Now we proceed to formally analyze this issue.

[18] In the above formula for X, holding m constant a redistribution from non-contributors to contributors will increase w_i $(i = 1, 2, .., m)$ with the increase being strict for some i.

[19] Note however that a sufficiently large degree of inequality among contributors may reduce X below the first-best level in the $c < 0$ case.

Using the conditions (1)-(3) agent i's surplus is:

$$\pi_i(w_i, x_i, X) = f(w_i, g(w_i)) - \frac{g(w_i) - cX}{b},\ i = 1..m \text{ (contributors)}$$

$$\pi_i(w_i, x_i, X) = f(w_i, cX),\ i = m+1..n \text{ (non-contributors)}$$

Joint surplus is given by:

$$\Pi = \sum_{i=m+1}^{n} f(w_i, cX) + \sum_{i=1}^{m} f(w_i, g(w_i)) - \frac{\sum_{i=1}^{m} g(w_i)}{b + mc}.$$

Let us denote the joint surplus of contributing players by $\Pi^c \equiv \sum_{i=1}^{m} f(w_i, g(w_i)) - \frac{\sum_{i=1}^{m} g(w_i)}{b+mc}$ and that of non-contributors by $\Pi^n \equiv \sum_{i=m+1}^{n} f(w_i, cX)$.

First consider the effect of distribution of wealth among non-contributors. This is trivial, since $f(w_i, cX)$ is concave, and hence $\sum_{i=m+1}^{n} f(w_i, cX)$ is concave as well. Therefore perfect equality of wealth among non-contributors will maximize their joint profits. Note that even if $f(w, z)$ is homogeneous of degree 1, this is still true.

Next, let us consider the effect of distribution of wealth among contributors. Let

$$\tilde{\pi}(w) \equiv f(w, g(w)) - \frac{g(w)}{b + mc}.$$

Notice that $\Pi^c = \sum_{i=1}^{m} \tilde{\pi}(w_i)$. The following lemma helps characterize the effect of wealth inequality on Π^c :

> **Lemma 5**: *Suppose Assumptions 1-3 hold and $c \geq 0$, or $c < 0$ but $|c|$ small. If $g(w)$ is concave then $\tilde{\pi}(w) \equiv f(w, g(w)) - \frac{g(w)}{b+mc}$ is concave. If $f(w, z)$ is homogeneous of degree one then $\tilde{\pi}(w)$ is linear in w.*

The intuition behind this result is the following. In the absence of externalities (i.e., $c = 0$) if we want to find the effect of a change in w on the profit of a player, we can focus only on the direct effect and ignore the indirect effect via the envelope theorem. As a result, the second derivative of the profit function also depends only on the direct effect through w. In the presence of externalities, we must take into account the indirect effect of w on X that affects other players. This residual term, which is a fraction of X (namely, $\frac{1}{b+c} - \frac{1}{b+mc} = \frac{(m-1)c}{(b+c)(b+mc)}$) increases joint profits for $c > 0$ and reduces them for $c < 0$ compared to the case where $c = 0$. Since we already know that X is concave, in the former case this reinforces the concavity of the joint profit function but in the latter case the effect goes the other way. As a result, for $c < 0$ a sufficient condition to ensure concavity of $\tilde{\pi}(w)$ is $|c|$ to be small.

Lemma 5 implies immediately that for $c \geq 0$ and for $c < 0$ but $|c|$ small, $\Pi^c = \sum_{i=1}^{m} \tilde{\pi}(w_i)$ is concave in the wealth of contributors so that greater equality will increase joint profits. As a result, perfect equality of wealth among contributors maximizes their joint surplus. For the special case where $f(w, z)$ is homogeneous of degree one Π^c is linear in the wealth of the contributors. In this case a redistribution of wealth among contributors will not affect joint surplus. However, equalizing wealth among non-contributors will still maximize Π^n. For $c < 0$ but $|c|$ large (while continuing to satisfy Assumption 1) we cannot determine the curvature of Π^c in general.

It turns out that for the Cobb-Douglas case, with decreasing returns, $\tilde{\pi}(w)$ is strictly concave even if $c < 0$ and $|c|$ not necessarily very small. In this case, $\tilde{\pi}(w) = w^{\frac{\alpha}{1-\beta}}[(b+c)\beta]^{\frac{1}{1-\beta}}\left(\frac{1}{\beta(b+c)} - \frac{1}{b+mc}\right)$. In order for total contribution to be positive, we need $\tilde{\pi}(w_1) > 0$. This inequality holds if and only if $b(1-\beta) > (\beta - m)c$, which holds for $c \geq 0$ or $c < 0$ and $|c| < \frac{b(1-\beta)}{m-\beta}$. We assume this to be true - otherwise no player ever contributes. Hence $\tilde{\pi}(w)$ is concave in w for $\alpha + \beta < 1$. In the case of constant returns, i.e., $\alpha + \beta = 1$, $\tilde{\pi}(w)$ is linear.

Given the initial wealth distribution $\mathbf{w}$, there is some $m \geq 1$ such that players with wealth $w \geq w_m$ contribute and those with $w \leq w_{m+1}$ do not. For this value of m, it is clear by the concavity of Π, that the wealth distribution maximizing joint surplus should have $w_i = \widehat{w}$ for all $i = 1, .., m$ and $w_j = \tilde{w} < \widehat{w}$ for all $j = m+1$,..,n. subject to the following two conditions:

$$(n-m)\tilde{w} + m\widehat{w} = W$$

$$g(\widehat{w}) \geq \frac{cmg(\widehat{w})}{b+mc} > g(\tilde{w}).$$

The first of the above conditions can be rewritten as:

$$\tilde{w} = \frac{W - m\widehat{w}}{n-m}$$

Using this, the expression for joint profits becomes:

$$\Pi = (n-m)\ f(\frac{W-m\widehat{w}}{n-m}, \frac{cmg(\widehat{w})}{b+mc}) + m\left[f(\widehat{w}, g(\widehat{w})) - \frac{g(\widehat{w})}{b+mc}\right]. \tag{4}$$

Also, the total contribution is $X = m\frac{g(\widehat{w})}{b+mc}$. The following result characterizes the joint surplus maximizing wealth distribution for a given m.

> **Proposition 2**: *Suppose Assumptions 1-3 are satisfied, $c \geq 0$ and if $c < 0$, $|c|$ is small. For a given m the joint profit maximizing wealth distribution under private provision of the public good involves equalizing the wealth levels of all non-contributing players to $\tilde{w} > 0$ and also those of all contributing players to $\widehat{w} > \tilde{w}$.*

This result shows that maximum joint surplus is achieved for both contributors and non-contributors, if there is no intra-group inequality. This is a direct consequence of joint profit of each group being concave in the wealth levels of the group members. The contrast with the conclusions of both Olson and the distribution neutrality literature is quite sharp. The key assumptions leading to the result are, market imperfections that prevent the efficient allocation of the private input across production units, and some technical properties of the production function that are shared by widely used functional forms such as Cobb-Douglas and CES under decreasing returns to scale. With constant returns to scale, the joint profits within the group of contributors are independent of the distribution of wealth as in the distribution neutrality theorem.

In the above result we did not talk about inter-group inequality. Formally, we took m as given while considering alternative wealth distributions. An obvious question to ask is, what is the joint-profit maximizing distribution of wealth when we can also choose the number of contributors, m. For example, does perfect equality among all players maximize joint surplus? This turns out to be a

difficult question. Below we provide a partial answer to this question for the case of both positive and negative externalities. Let us first look at the case of positive externalities ($c > 0$). Suppose all players are contributing when wealth is equally distributed. Then from Proposition 2 we know that limited redistribution that does not change the number of contributors cannot improve efficiency. This immediately suggests the following result:

> **Corollary to Proposition 2**: *Suppose all players contribute under perfect equality. Then if after a redistribution all players continue to contribute joint profits cannot increase.*

But suppose we redistribute wealth from one player to the other $n-1$ players up to the point where this player stops contributing. Recall that when the group size is $m < n$, $X = m\frac{g(\widehat{w})}{b+mc}$. It is obvious that an increase in the average wealth of contributing players keeping the number of contributors fixed will increase X. It turns out that an increase in m holding the average wealth of contributors constant will always increase X.[20] However, if we simultaneously decrease m from n to $n-1$ and increase the average wealth of contributors, it is not clear whether X will go up or not. If X goes down then we can unambiguously say that joint profits are lower due to this redistribution (for $c > 0$) since the effect of this policy on the efficiency of allocation of the private input across production units is definitely negative. However, if X goes up then there is a trade off: the increase in X benefits all players (since $c > 0$), including the player who is too poor to contribute now, but this has to be balanced against the greater inefficiency in the allocation of the private input.

To analyze the effect of wealth distribution on joint profits when some players do not contribute we restrict attention to the comparison between joint profits under perfect equality (i.e., when all players have wealth $\overline{w} \equiv \frac{W}{n}$) and the wealth distribution that is obtained by a redistribution that leads to k contributing and $n-k$ non-contributing players. From the discussion above, we know that under our assumptions all players contribute under perfect equality. We focus on studying only the efficient wealth distributions, i.e. ones which achieve maximum joint surplus. Since any intra-group inequality among the contributors and non-contributors reduces joint surplus we assume that all k contributors have wealth $\overline{w} + \frac{\varepsilon}{k}$ and all $n-k$ non-contributors have wealth $\overline{w} - \frac{\varepsilon}{n-k}$ after the redistribution. Let us denote by Π^E the joint surplus under perfect equality and with $\Pi^I(\varepsilon)$ the one under the unequal wealth distribution of the above type. Let also total wealth be normalized to $n\overline{w}$. A player stops contributing if

$$g(\overline{w} - \frac{\varepsilon}{n-k}) < cX = \frac{kcg(\overline{w} + \frac{\varepsilon}{k})}{b+kc} \tag{5}$$

Let ε^* be defined as the indifference point between contributing and not contributing, i.e. the solution to

$$g(\bar{w} - \frac{\varepsilon^*}{n-k}) = \frac{kcg(\bar{w} + \frac{\varepsilon^*}{k})}{b+kc} \tag{6}$$

Let $\tilde{\varepsilon}$ denote the degree of inequality maximizing $\Pi^I(\varepsilon)$ subject to $\varepsilon \geq \varepsilon^*$, i.e. when there are non-contributors in equilibrium[21]. The level of joint surplus when there are k contributors each with wealth

[20] Formally, this is because $\frac{m}{b+mc}$ is increasing in m. The intuition is, the new entrant to the group of contributor will contribute a positive amount, which would reduce the incentive of existing contributors to contribute due to diminishing returns. However, in the new equilibrium X must go up, as otherwise the original situation could not have been an equilibrium.

[21] Clearly, if $\varepsilon < \varepsilon^*$ (5) cannot hold (since g is an increasing function and all players contribute).

$\overline{w} + \frac{\varepsilon}{k}$ and $(n-k)$ non-contributors each with wealth $\overline{w} - \frac{\varepsilon}{n-k}$ is:

$$\Pi^I(\varepsilon) = kf\left(\overline{w} + \frac{\varepsilon}{k}, g(\overline{w} + \frac{\varepsilon}{k})\right) + (n-k)f\left(\overline{w} - \frac{\varepsilon}{n-k}, \frac{kc}{b+kc}g(\overline{w} + \frac{\varepsilon}{k})\right) - \frac{kg(\overline{w} + \frac{\varepsilon}{k})}{b+kc}.$$

Differentiating with respect to ε we get:

$$\begin{aligned}\frac{d\Pi^I(\varepsilon)}{d\varepsilon} &= \left[f_1\left(\overline{w} + \frac{\varepsilon}{k}, g(\overline{w} + \frac{\varepsilon}{k})\right) - f_1\left(\overline{w} - \frac{\varepsilon}{n-k}, \frac{kc}{b+kc}g(\overline{w} + \frac{\varepsilon}{k})\right)\right] \\ &+ f_2\left(\overline{w} - \frac{\varepsilon}{n-k}, \frac{kc}{b+kc}g(\overline{w} + \frac{\varepsilon}{k})\right)\frac{(n-k)c}{b+kc}g'(\overline{w} + \frac{\varepsilon}{k}) + \\ &+ g'(\bar{w} + \frac{\varepsilon}{k})\left[\frac{1}{b+c} - \frac{1}{b+kc}\right] \qquad (7)\end{aligned}$$

where we used the fact that $f_2(\bar{w} + \frac{\varepsilon}{k}, g(\bar{w} + \frac{\varepsilon}{k})) = \frac{1}{b+c}$ by the definition of g. The following lemma helps us characterize the optimal degree of inequality:

Lemma 6: $\frac{\partial \Pi^I(\varepsilon^*)}{\partial \varepsilon} < 0$ *implies that* $\frac{\partial \Pi^I(\varepsilon)}{\partial \varepsilon} < 0$ *for all* $\varepsilon \geq \varepsilon^*$ *and so* $\tilde{\varepsilon} = \varepsilon^*$. *Conversely, if* $\frac{\partial \Pi^I(\varepsilon^*)}{\partial \varepsilon} > 0$ *then* $\tilde{\varepsilon} > \varepsilon^*$.

The above lemma implies that if $\frac{\partial \Pi^I(\varepsilon^*)}{\partial \varepsilon} < 0$ then we have a corner solution, i.e. $\Pi^I(\varepsilon)$ is maximized at $\tilde{\varepsilon} = \varepsilon^*$. Now we are ready to prove:

Proposition 3

(a) *For pure public goods ($b = 0$ and $c > 0$) perfect equality among the agents is never joint profit maximizing.*

(b) *For pure private goods ($b > 0$ and $c = 0$) perfect equality is always joint profit maximizing.*

We noted a special property of pure public goods in the previous section (Observation 1), namely, even if the difference in the wealth between the richest player and the second richest player is arbitrarily small, the former provides the entire amount of the public good with everyone else free riding on her. This property is the key to explain why perfect equality is not joint profit maximizing in this case. Start with a situation where all players except for one have the same wealth level, and this one player has a wealth level which is higher than that of others by an arbitrarily small amount. As a result this player is the single contributor to the public good. A small redistribution of wealth from other players to this player, keeping the average wealth of the other players constant, will have three effects on joint profits: the effect due to the worsening of the allocation of the private input, the effect of the increase in X on the payoff of the non-contributing players, and the effect of the increase in X on the payoff of the single contributing player. The result in the proposition follows from the fact that the first effect is negligible since by assumption the extent of wealth inequality is very small, the second effect is positive, and the third effect can be ignored by the envelope theorem. It should also be noted that this result goes through for both constant and decreasing returns to scale. The second part of

Proposition 3 follows from the fact that when $c = 0$ a player will always choose $x_i > 0$ however small her wealth level. Then all players are contributors so long as they have non-zero wealth and it follows directly from Proposition 2 that perfect equality will maximize joint profits.

For the case of impure public goods ($b, c > 0$ under decreasing returns to scale we can provide only a local characterization:

Proposition 4

Consider the case of impure public good subject to decreasing returns to scale, i.e. $c > 0$ and suppose that Assumptions 1-3 hold. Then:

(a) Given c, n there exist some $b_1, b_2 > 0$ such that for all $b \in [\bar{b}_2, \infty)$ perfect equality is always joint profit maximizing, whereas for $b \in [0, \bar{b}_1]$ perfect equality is never joint profit maximizing.

(b) Given b, n there exists some $\bar{c} > 0$ such that for all $c \in (0, \bar{c}]$ perfect equality is always joint profit maximizing.

Two opposing forces are at work in this case - the "decreasing returns to scale" effect calling for equalizing the wealth of agents and the "dominant player" effect due to the positive externality calling for re-distribution towards the richest players as there is a positive effect on the payoffs of the non-contributing players. Each of the two effects can dominate the other depending on the parameter values. The direct effect of an increase in the richer player contribution on her own payoff can once again be ignored by the envelope theorem.

While we cannot provide a full characterization of the case of decreasing returns, due to the existence of two opposing forces, we can provide some illustrative examples using the Cobb-Douglas production function $f(w, z) = w^{\alpha} z^{\beta}$ for a two player game. In Figures 1 and 2 we plot how the difference between joint profits under perfect equality and under inequality (where the degree of inequality is chosen to maximize joint profits given than only one player contributes) vary with b and c for several alternative sets of values of α and β. As we can see from the figures: (a) there is a unique $\bar{b}$ such that $\Pi^E \geq \Pi^I(\tilde{\varepsilon})$ for $b \geq \bar{b}$ and $\Pi^E < \Pi^I(\tilde{\varepsilon})$ for $b < \bar{b}$; and (b) there is a unique $\bar{c}$ such that $\Pi^E \geq \Pi^I(\tilde{\varepsilon})$ for $c \leq \bar{c}$ and $\Pi^E < \Pi^I(\tilde{\varepsilon})$ for $c > \bar{c}$.

Under constant returns to scale, we know from Lemma 5 that joint profits are linear in the total wealth of contributors. Given that joint profits are higher under some degree of inequality for pure public goods compared with joint profits under perfect equality, in this particular case one would expect this property to be true for $c > 0$ and $b > 0$. This conjecture turns out to be correct:

Proposition 5: *If the production function displays constant returns to scale then:*

(a) In the pure private good case ($c = 0$) joint profits are independent of the wealth distribution.

(b) Perfect equality is never joint profit maximizing for impure public goods (i.e., $c > 0$). Moreover, it is possible to have inequality Pareto-dominate perfect equality.

The first part of the proposition follows from the fact that under constant returns to scale joint profits are linear in total wealth (see Lemma 5). The logic of the result in (b) is similar to that of the

pure public goods case. In that case the richest person is the only contributor even when the wealth difference between him and the second richest player is very small and therefore a small amount of inequality does not result in large losses due to the inefficient allocation of the private input. With constant returns to scale and for impure public goods, the difference between the wealth level of the contributors and non-contributors need not be very small. However, joint profits of contributors depend only on their total wealth and not how it is distributed. As a result, creating some inequality from the point where only player is exactly indifferent between contributing and not, to the point where she strictly prefers not to contribute, involves a small loss due the inefficient allocation of the private input. Unlike the pure public goods case, this could involve a significant amount of inequality with respect to the perfectly equal wealth distribution. The second part of Proposition 5 (b) demonstrates the striking possibility that under some circumstances it is possible to have some degree of inequality among agents Pareto-dominate the allocation under perfect equality. If we think of a two player set up, starting with perfect equality if we redistribute wealth from one player to the other, the poorer player is initially *strictly better off* than the rich player because she is free-riding on the rich player who contributes most of the good and bears a large share of the costs. This is the starkest possible demonstration of what Olson called the "exploitation of the great by the small". However, if we continue increasing inequality eventually the loss of the private input offsets the gain from free riding on the provision of the public good for the poorer player. This makes it possible that the two players get the same level of surplus at some positive level of inequality and that this surplus is higher than the level they get at perfect equality.

Finally, we turn to the case of negative externalities, i.e., $c < 0$. We show that:

Proposition 6

Consider the case of commons, i.e. $c < 0$ *and suppose that Assumptions 1-3 hold. Then there exist two critical values of* c, $c_1 < c_0 < 0$ *such that:*

(a) For $c \in [c_0, 0)$ *perfect equality is a local maximum of the joint profit function.*

(b) For $c \in (-\frac{b}{n}, c_1)$ *perfect equality is never joint profit maximizing.*

In this case, Assumption 1 implies that all agents contribute. Notice that then we can write the joint profits function as:

$$\Pi = k\hat{\pi}(w_1) + (n-k)\hat{\pi}(w_2) + \frac{(n-1)c}{(b+c)}X$$

where $\hat{\pi}(w) \equiv f(w, g(w)) - \frac{g(w)}{b+c}$, w_1 is the wealth of rich players and w_2 is the wealth level of poor players. Notice also that $\hat{\pi}'(w) = f_1(w, g(w)) + \left(f_2(w, g(w)) - \frac{1}{b+c}\right) g'(w) = f_1(w, g(w))$ from the definition of $g(w)$. Therefore given the definition of $g(w)$ and the concavity of f, $\hat{\pi}$ is strictly concave. On the other hand, the term $\frac{(n-1)c}{(b+c)}X$ is convex given that $c < 0$ and Proposition 1. Intuitively, joint profits is the sum of individual profits ignoring the externality of a player's action on others, and the sum total of the externality terms. The former is concave in the wealth distribution but in the case of negative externalities, the latter is convex. For c small enough (in absolute value) the decreasing returns to scale effect dominates, i.e. joint profits are maximized at perfect equality but for c large

(in absolute value) the "cost of negative externality" term, which is convex, dominates and so greater inequality leads to higher joint profits.

4 Extensions

4.1 Convertibility between the private input and contribution to the collective input

It is important for our result that x_i and w_i are different types of goods and one cannot be freely converted into the other. Suppose the individual can freely allocate a fixed amount of wealth between two uses, namely, as a private input and as her contribution to the collective input. This is the formulation chosen by the literature on distribution-neutrality (e.g., Warr, 1983; Bergstrom, Varian and Blume, 1986; Cornes and Sandler, 1996; and Itaya et al., 1997). This literature focuses on pure public goods, i.e., where $z_i = cX$. For ease of comparability, let us consider this case first. Let k_i denote the amount of the private input chosen by player i. Then player i's decision problems is to maximize $f(k_i, cX)$ with respect to k_i and x_i subject to the budget constraint $k_i + x_i \leq w_i$. The first-order condition of an individual who contributes a positive amount in equilibrium is

$$f_1(k_i, cX) = cf_2(k_i, cX), \qquad i = 1, 2, .., m.$$

As $k_i = w_i - x_i$ from the budget constraint of the individual, and $x_i + X_{-i} = X$ for all $i = 1, 2, .., m$, this condition implicitly defines the following function:

$$w_i - x_i = h(X).$$

Summing across all players who contribute in equilibrium, we get $X + mh(X) = W$. This equation can be solved for X which therefore depends only on total wealth, W and not on its distribution. Joint profits will also be independent of the distribution of wealth.

The above formulation is similar to that of a consumer allocating a fixed amount of money to alternative goods in order to maximize utility. An alternative formulation to capture free convertibility between k_i and x_i is to pose the problem as that of a firm maximizing profits by choosing inputs which can be sold or purchased from the market at a given price. One could think of k_i as capital which has a given price r such that a firm that has an excess of it (relative to its endowment w_i) can sell it to other firms, and a firm that has a shortage of it can buy it at the same price, say r. Similarly, one can think of x_i as labor that can be used to contribute towards the collective input, or sold in the labor market at price w.[22] Now the first order condition of a contributing player, i, is

$$f_1(k_i, cX) = \frac{r}{w} cf_2(k_i, cX), \qquad i = 1, 2, .., m.$$

This condition is the same as in the previous formulation, except for the multiplicative constant $\frac{r}{w}$ and so the distribution neutrality result goes through.

[22] In our framework labor is not directly used in production. We can think of another sector which uses labor. Alternatively, we can extend the basic model by adding labor as a third input. The distribution neutrality result will go through.

Turning now to impure public goods, i.e., where $b > 0$, the first order condition for player i according to the first formulation is:

$$f_1(k_i, bx_i + cX) = (b + c)f_2(k_i, bx_i + cX), \qquad i = 1, 2, .., m.$$

It is clear that in general the distribution neutrality result will not go through now. It will go through for some special cases, such as the case where $f(w, z)$ is homothetic. In this case, the values of k_i and z_i at a point of individual optimum satisfies the condition

$$\frac{k_i}{bx_i + cX} = A$$

where A is a positive constant. It is readily verified that the distribution neutrality result holds in this case. Our analysis shows that in this case, relaxing the assumption of perfect convertibility of the private input and the contribution to the collective input implies that the distribution neutrality result no longer holds. Specifically, greater equality among contributors always improves efficiency for impure public goods (i.e., $c > 0$) while for collective inputs subject to negative externalities, the effect of inequality on efficiency is ambiguous. In the latter case, we characterize conditions under which we can sign the effect of inequality on efficiency. Our results do not depend on the production functions being homothetic, but in the general case even with free convertibility, distribution neutrality can break down if the collective input is not a pure public good, as is well recognized in the literature (see for example, Bergstrom, Varian and Blume, 1986 and Cornes and Sandler, 1996).

4.2 Substitutability between the private and the collective input

Above, we assumed that the private input and the public good are complements in the production function. In this section we examine the implications of these being substitutes. For simplicity, we examine the case where w and z are perfect substitutes: $\pi(w_i, x_i, X) = f(w + bx_i + cX) - x_i$, where f is increasing and strictly concave and b and c satisfy Assumption 2. The first order conditions for the agent's problem is:

$$(b + c)f'(w_i + bx_i + cX) \leq 1$$

with strict equality when $x_i > 0$. Let us denote by w^* the solution to $f'(w) = \frac{1}{b+c}$, which exists and is unique given the above assumptions. In contrast to the complements case, it is now the poorest player who has the highest marginal product of contributing. In the pure public good case ($b = 0$) the poorest player will be the only contributor if $w_n < w^*$ and if $w_n \geq w^*$ the public good will not be provided at all.

As before, joint surplus goes up if wealth is equally distributed among non-contributors. Also, we cannot say for sure whether the optimal distribution of wealth involves perfect equality, or some inequality among the contributor (the poorest agent) and the rest. This is clearly seen for the case of the pure public good ($b = 0$). For simplicity, suppose there are two players with wealth levels $w_1 = \overline{w} + \varepsilon$ and $w_2 = \overline{w} - \varepsilon$ and, in addition assume for simplicity that $c = 1$. Now joint profits are: $\Pi(\varepsilon) = f(w^*) - \{w^* - (\overline{w} - \varepsilon)\} + f(\overline{w} + \varepsilon + w^* - (\overline{w} - \varepsilon)) = f(w^*) - w^* + \overline{w} + f(2\varepsilon + w^*) - \varepsilon$ and so $\Pi'(\varepsilon) = 2f'(2\varepsilon + w^*) - 1$. We know that $f'(2\varepsilon + w^*) - 1 \leq 0$ since by definition $f'(w^*) - 1 = 0$ but whether $2f'(2\varepsilon + w^*) - 1 \leq 0$ or > 0 cannot be determined *a priori*. For the intuition behind this, notice that,

those who choose $x_i > 0$, i.e., the poorest players, use the efficient amount of the input. Other players have more than the efficient level of the input in their production units. Any redistribution from the poor to the rich players does not affect the profit of the former as they exactly compensate for this by increasing their contribution. Since rich players have more than the efficient level of the input in their firms, normally a transfer of an additional unit of wealth would reduce joint profits since the marginal gain to the rich player is less than the marginal cost to the poor player. But every extra unit of wealth received by the rich player increases the input received by her firm by *twice* the amount because of the increase in the effort by the poor player and as a result it is not clear whether joint profits increase or decrease.

4.3 Complementarity between the individual relative contribution and the total contribution

Above we studied the case where the player's own contribution and the total contribution of all players are perfect substitutes in determining the benefit from the collective input enjoyed by a player, z_i. In this section we consider an alternative formulation where they can be complements:

$$z_i = \left(\frac{x_i}{X}\right)^{\theta} X^{\gamma}$$

where $0 \leq \theta \leq 1$ and $0 \leq \gamma \leq 1$. According to this specification, each player not only gains from the total contribution, but her gains are greater, the her contribution is relative to the total. This induces people to choose a higher level of x_i which benefits others through the term X^{γ}. But it also reduces how much others can enjoy the collective good by a congestion effect captured by the term $\left(\frac{x_i}{X}\right)^{\theta}$. If the latter effect is unimportant compared to the former, then we have a public good and indeed for $\theta = 0$ we have the textbook case of a pure public good. But if it is the other way round then the congestion effect dominates the beneficial externality effect and in the limit, for $\theta = 1$ we have the textbook case of the commons. When these two effects exactly balance each other out ($\theta = \gamma$), we have the case of the a pure private good.

Analytically, this case turns out to be quite hard to characterize even when we assume a specific form of the production function, namely Cobb-Douglas, and consider a two player game. We show that if we compare the allocations under perfect equality (both players have the same level of wealth) and perfect inequality (one player has all the wealth and the other player has nothing) joint surplus is always higher under perfect equality for non-negative externalities (i.e., $\theta \geq \gamma$). However, if there are substantial negative externalities then under some parameter values joint surplus will be higher under perfect inequality. The intuition for this result lies in the fact that when the negative externality problem is very severe then under perfect equality the players choose their actions related to the collective input at too high a level relative to the joint surplus maximizing solution. Perfect inequality converts the model to a one player game and hence eliminates this problem. On the other hand due to joint diminishing returns to the private input and the collective input, joint surplus is lower under perfect inequality compared to perfect equality if there were no externalities. What this result tells us is that perfect inequality is desirable only when the negative externality problem is severe and when the extent of diminishing returns is not too high.

If, instead of comparing the allocations under perfect equality and perfect inequality, we consider the effects of a continuous change in inequality on total contributions and joint profits, the results are not clear-cut. We prove that in the case of commons its total use (X) decreases with increasing wealth inequality and joint profits per unit of total contributions (i.e., Π/X), or what one may call the average rate of return on the collective input, increases with inequality. But the absolute level of joint profits may increase or decrease with inequality. Numerical simulations suggest that joint profits in general decrease with inequality, except for the case of substantial negative externalities. In the case of public goods (pure and impure), we prove that the average rate of return on the public good input decreases with inequality. But as the extent of positive externalities become large (approaching the pure public goods case) the total amount of public good provision (and the absolute amount of the joint profits) may increase with inequality. However there exists a range of moderate presence of positive externalities such that total contributions as well as joint profits decrease with inequality.

5 Concluding Remarks

In this paper we analyze the effect of inequality in the distribution of endowment of private inputs that are complementary in production with collective inputs (e.g., contribution to public goods such as irrigation and extraction from common-property resources) on efficiency in a simple class of collective action problems. In an environment where transaction costs prevent the efficient allocation of private inputs across individuals, and the collective inputs are provided in a decentralized manner, we characterize the optimal second-best distribution of the private input. We show that while efficiency increases with greater equality *within* the group of contributors and non-contributors, in some situations there is an optimal degree of inequality *between* the groups.

Some limitations of our model suggest several directions of potentially fruitful research. First, our model is static. It is important to extend it to the case where both the wealth distribution and the efficiency of collective action are endogenous. For example, it is possible to have multiple stationary states with high (low) wealth inequality leading to low (high) incomes to the poor due to low (high) level of provision of public goods, which via low (high) mobility can sustain an unequal (equal) distribution of wealth. Also, in the dynamic case it could be interesting to analyze the effects of inequality on the sustainability of cooperation in a situation of repeated games. Second, technological non-convexities and differential availability of exit options seriously affect collective action in the real world, while our model ignores them.[23] For example, the public good may not be generated if the total amount of contribution is below a certain threshold (e.g. Cornes, 1993). This is the case for renewable resources like forests or fishery where a minimum stock is necessary for regeneration, or in the case of fencing a common pasture. Third, the empirical literature suggests that even when the link between inequality and collective action is consistent with our results, the mechanisms involved may be quite different in some cases. For example, transaction costs in conflict management and costs of negotiation may be higher in situations of higher inequality. Fourth, following the public economics literature, in this paper

[23] The model of Dayton-Johnson and Bardhan (2002) examines the effect of inequality on resource conservation with two periods and differential exit options for the rich and the poor in the case when technology is linear. Baland and Platteau (1997) discuss the effect of non-convexities of technology in a static model.

we focus mainly on the free-rider problem arising in a collective action setup. Hence, the main issue is the sharing of the costs of collective action. But there is another problem, often called the "bargaining problem", whereby collective action breaks down because the parties involved cannot agree on the sharing of the benefits.[24] Inequality matters in this problem as well. For example, bargaining can break down when one party feels that the other party is being unfair in sharing the benefits (there is ample evidence for this in the experimental literature on ultimatum games). More generally, social norms of cooperation and group identification may be difficult to achieve in highly unequal environments. Putnam (1993) in his well-known study of regional disparities of social capital in Italy points out that "horizontal" social networks (i.e., those involving people of similar status and power) are more effective in generating trust and norms of reciprocity than "vertical" ones. Knack and Keefer (1997) also find that the level of social cohesion (which is an outcome of collective action) is strongly and negatively associated with economic inequality. Finally, we focus only on the voluntary provision of public goods and do not consider the possibility that the players might elect a decision maker who can tax them and choose the level of provision of the collective good. The role of inequality in such a framework is an important topic for future research.[25]

6 Appendix

Proof of Lemma 1: Consider the first order condition, $f_2(w,z)-1=0$. By Assumption 1, $f_2(w,z)>0$ for all $w>0$ and $\lim_{w\to 0} f_2(w,z)=0$. Therefore $\gamma(w)>0$ for all $w>0$. By concavity, a global maximum exists and $f_{22}(w,z)<0$. By definition, $f_2(w,\gamma(w))-1=0$. Notice that under our assumptions $\gamma(w)$ is differentiable, and hence continuous. In particular, $\frac{d\gamma(w)}{dw}=-\frac{f_{12}}{f_{22}}>0$.■

Proof of Lemma 2: By the definition of $h(w,z)$, $h(w,\gamma(w))=1$. Totally differentiating with respect to w we get $h_1+h_2\frac{d\gamma(w)}{dw}=0$, or, $\frac{d\gamma(w)}{dw}=-\frac{h_1}{h_2}$. Notice that $h_1=\frac{\partial^2 f(w,z)}{\partial z\partial w}>0$ (as w and z are complements) and $h_2=\frac{\partial^2 f(w,z)}{\partial z^2}<0$ (by strict concavity). Differentiating once again with respect to w we get:

$$\frac{d^2\gamma(w)}{dw^2}=-\frac{h_1^2h_{22}+h_2^2h_{11}-2h_1h_2h_{12}}{h_2^3}.$$

The condition $\frac{d^2\gamma(w)}{dw^2}\leq 0$ is equivalent to the determinant $\begin{vmatrix} 0 & h_1 & h_2 \\ h_1 & h_{11} & h_{12} \\ h_2 & h_{12} & h_{22} \end{vmatrix}$ being ≤ 0 which in turn is equivalent to $h(w,z)$ being quasi-concave (see Theorem 21.20 of Simon and Blume (1994)).■

Proof of Lemma 3: Since $f(w,z)$ is homogeneous of degree 1, $f_2(w,z)$ is homogeneous of degree 0. If $\lambda>0$, $f_2(\lambda w,\lambda\gamma(w))=f_2(w,\gamma(w))$. Since by definition $f_2(w,\gamma(w))=1$, so $f_2(\lambda w,\lambda\gamma(w))=f_2(w,\gamma(w))=1$. Then it must be true that $\gamma(\lambda w)=\lambda\gamma(w)$ which means $\gamma(w)=Aw$ where $A>0$ is a constant.■

[24] See for example, Elster (1989).

[25] Olszewski and Rosenthal (2004) address this question for pure public goods within the framework of the distribution neutrality literature.

Proof of Lemma 4: Since agent $k+1$ contributes a positive amount by assumption, $g(w_{k+1}) > \frac{c\sum_{i=1}^{k} g(w_i)}{b+kc}$. Straightforward algebra shows that this is equivalent to the inequality $\frac{c\sum_{i=1}^{k+1} g(w_i)}{b+(k+1)c} > \frac{c\sum_{i=1}^{k} g(w_i)}{b+kc}$. The second part of the lemma can proved in the same way.■

Proof of Lemma 5: Totally differentiating with respect to w we get:

$$\frac{\partial\tilde{\pi}(w)}{\partial w} \equiv f_1(w,g(w)) + \left(f_2(w,g(w)) - \frac{1}{b+mc}\right) g'(w).$$

>From the definition of $g(w)$ and the first-order condition of a contributing player, $f_2(w,g(w)) = \frac{1}{b+c}$. Substituting in we get

$$\frac{\partial\tilde{\pi}(w)}{\partial w} \equiv f_1(w,g(w)) + \frac{(m-1)c}{(b+c)(b+mc)} g'(w).$$

Totally differentiating once again with respect to w :

$$\frac{\partial^2\tilde{\pi}(w)}{\partial w^2} \equiv f_{11}(w,g(w)) + f_{12}(w,g(w))g'(w) + \frac{(m-1)c}{(b+c)(b+mc)} g''(w).$$

>From the proof of Lemma 1, $g'(w) = -\frac{f_{12}}{f_{22}}$. Therefore $f_{11}\ f_{11} + f_{12}g'(w) = \frac{f_{11}f_{22}-f_{12}^2}{f_{22}} < 0$ since $f(w,z)$ is concave. Therefore $\frac{\partial^2\tilde{\pi}(w)}{\partial w^2}$ is negative if one of the following holds: (i) $g(w)$ is concave and $c > 0$; (ii) $c = 0$ or (iii) $c < 0$ and $|c|$ small. The second part of the lemma follows from the fact that if $f(w,z)$ is homogeneous of degree one then $g(.)$ is linear and $\tilde{\pi}(w) = f(\lambda w, \lambda g(w)) - \frac{g(\lambda w)}{b+mc} = \lambda\left[f(w,g(w)) - \frac{g(w)}{b+mc}\right]$ is linear as well.■

Proof of Lemma 6: From Lemma 5, we know that the joint profit of contributing players is concave in ε. Also, it can be directly verified that the joint profit of non-contributors is concave in ε. Differentiating the terms in (7) that relate to non-contributing players and using the superscript n to denote these players we get

$$\begin{aligned}
&\frac{1}{n-k} f_{11}^n - \frac{c}{b+kc} g'(\overline{w} + \frac{\varepsilon}{k}) f_{12}^n + \\
&f_2^n\left(\overline{w} - \frac{\varepsilon}{n-k}, \frac{kc}{b+kc} g(\overline{w} + \frac{\varepsilon}{k})\right) \frac{1}{k} \frac{(n-k)c}{b+kc} g''(\overline{w} + \frac{\varepsilon}{k}) \\
&+ \frac{(n-k)c}{b+kc} g'(\overline{w} + \frac{\varepsilon}{k}) \left\{ -\frac{1}{n-k} f_{21}^n + f_{22}^n \frac{c}{b+kc} g'(\overline{w} + \frac{\varepsilon}{k}) \right\}.
\end{aligned}$$

This expression is negative since all the terms are negative. Therefore $\Pi^I(\varepsilon)$ is concave in ε and so

$$\frac{\partial\Pi^I(\varepsilon)}{\partial\varepsilon} \lesseqgtr 0 \text{ as } \varepsilon \gtreqless \tilde{\varepsilon}.$$

The claim in the lemma follows directly from the above.■

Proof of Proposition 2: For a given value of m it follows from the concavity of the profit functions of both contributors and non-contributors that there should not be any intra-group heterogeneity. Also, $\widehat{w} > \tilde{w}$ given that contributors must be richer than non-contributors (see (1)-(3)). It is never optimal to set $\tilde{w}$ at a very low level given the Inada endpoint conditions, namely, $\lim_{\tilde{w}\to 0} f_1(\tilde{w}, cX) = \infty$. Since

$\widehat{w} > \tilde{w}$, it would never be optimal to make $\widehat{w}$ arbitrarily small, since that would mean $\tilde{w}$ would be even smaller and almost all of W would be left unused.■

Proof of Proposition 3:

(a) If $b = 0$ (6) implies that $\varepsilon^* = 0$ i.e. any degree of inequality can be sustained in an equilibrium with non-contributors. Consider the derivative in (7) evaluated at $\varepsilon = 0$ (i.e. around the point of perfect equality). We have that:

$$\frac{d\Pi^I(0)}{d\varepsilon} = \frac{d\Pi^I(\varepsilon^*)}{d\varepsilon} = f_2\left(\overline{w}, \frac{kc}{b+kc}g(\overline{w})\right)\frac{(n-k)c}{b+kc}g'(\overline{w}) + \\ + g'(\bar{w})\left[\frac{c(k-1)}{(b+c)(b+kc)}\right] > 0$$

as all the terms are positive. Then by Lemma 6, $\tilde{\varepsilon} > \varepsilon^* = 0$, i.e., perfect equality is never joint profit maximizing in the case of pure public goods.

(b) In the case of pure private goods ($c = 0$), (3) is clearly satisfied for any redistribution of wealth among the agents, i.e. all of them always contribute. But then it follows directly from Proposition 2 that greater inequality reduces joint profits.

Proof of Proposition 4:

(a) Differentiating both sides of (6) with respect to b we get:

$$g'(\bar{w} - \frac{\varepsilon^*}{n-k})(-\frac{1}{n-k})\frac{\partial \varepsilon^*}{\partial b} = \frac{kcg'(\bar{w} + \frac{\varepsilon^*}{k})\frac{1}{k}\frac{\partial \varepsilon^*}{\partial b}(b+kc) - kcg(\bar{w} + \frac{\varepsilon^*}{k})}{(b+kc)^2}$$

i.e.

$$\frac{\partial \varepsilon^*}{\partial b} = \frac{kcg(\bar{w} + \frac{\varepsilon^*}{k})}{cg'(\bar{w} + \frac{\varepsilon^*}{k})(b+kc) + g'(\bar{w} - \frac{\varepsilon^*}{n-k})\frac{1}{n-k}(b+kc)^2} > 0$$

Therefore, $w_1 \equiv \bar{w} + \dfrac{\varepsilon^*}{k}$ is increasing in b and $w_2 \equiv \bar{w} - \dfrac{\varepsilon^*}{n-k}$ is decreasing in b. Given the definition of ε^*, and the fact that $f_2(z, g(z)) = \dfrac{1}{b+c}$ we get

$$f_2\left(\overline{w} - \frac{\varepsilon^*}{n-k}, \frac{kc}{b+kc}g(\overline{w} + \frac{\varepsilon^*}{k})\right) = f_2\left(\overline{w} - \frac{\varepsilon^*}{n-k}, g(\overline{w} - \frac{\varepsilon^*}{n-k})\right) = \frac{1}{b+c}$$

Therefore (7) evaluated at ε^* can be written as:

$$\frac{d\Pi^I(\varepsilon^*)}{d\varepsilon} = \frac{(n-1)c}{(b+c)(b+kc)}g'(w_1) + f_1(w_1, g(w_1) - f_1(w_2, g(w_2)). \tag{8}$$

Note that when $b \to \infty$, $\varepsilon^* \to (n-1)\bar{w}$. Therefore, $w_1 \to \dfrac{n}{k}\bar{w}$ and $w_2 \to 0$. From the Inada conditions $f_1(w_2, g(w_2)) \to \infty$ and thus $\dfrac{d\Pi^I(\varepsilon^*)}{d\varepsilon} = -\infty < 0$ while the other two terms in the above expression are finite and non-negative. Since the function $\Pi^I(\varepsilon^*)$ is continuous, this proves that there exists some $\bar{b}_2 > 0$ (which would, in general, depend on n, k and c) such that for $b \in [\bar{b}_2, \infty)$, $\dfrac{d\Pi^I(\varepsilon^*)}{d\varepsilon} < 0$, implying

that $\Pi^I(\varepsilon)$ is maximized at ε^* (the minimum degree of inequality needed to have non-contributing agents in equilibrium), i.e. the problem has a corner solution. Let

$$\Pi^C(\bar{w}+\frac{\varepsilon}{k},\bar{w}-\frac{\varepsilon}{n-k}) \equiv k[f(\bar{w}+\frac{\varepsilon}{k},g(\bar{w}+\frac{\varepsilon}{k}))-\frac{g(\bar{w}+\frac{\varepsilon}{k})}{b+nc}]+$$
$$+(n-k)[f(\bar{w}-\frac{\varepsilon}{n-k},g(\bar{w}-\frac{\varepsilon}{n-k}))-\frac{g(\bar{w}-\frac{\varepsilon}{n-k})}{b+nc}]$$

denote joint profits when there are k agents with wealths $\bar{w}+\frac{\varepsilon}{k}$ and $n-k$ agents with wealths $\bar{w}-\frac{\varepsilon}{n-k}$, all of whom contribute. From the definition of $\varepsilon^*, \Pi^C(\bar{w}+\frac{\varepsilon^*}{k},\bar{w}-\frac{\varepsilon^*}{n-k})=\Pi^I(\varepsilon^*)$. Also, $\Pi^E>\Pi^C(\bar{w}+\frac{\varepsilon^*}{k},\bar{w}-\frac{\varepsilon^*}{n-k})$ from Proposition 2. Thus perfect equality maximizes joint surplus for large enough values of b. From Proposition 3, part (a) and the continuity of the joint profit function, we know that there exists $\bar{b}_1>0$ such that if $b\in[0,\bar{b}_1]$ such that perfect equality is never joint profit maximizing.

(b) The proof is very similar to that of part (a). Differentiating both sides of (6) with respect to c we get:

$$g'(\bar{w}-\frac{\varepsilon^*}{n-k})(-\frac{1}{n-k})\frac{\partial\varepsilon^*}{\partial c}=\frac{[kcg'(\bar{w}+\frac{\varepsilon^*}{k})\frac{1}{k}\frac{\partial\varepsilon^*}{\partial c}+kg(\bar{w}+\frac{\varepsilon^*}{k})](b+kc)-k^2cg(\bar{w}+\frac{\varepsilon^*}{k})}{(b+kc)^2}$$

i.e.

$$\frac{\partial\varepsilon^*}{\partial c}=\frac{-bkg(\bar{w}+\frac{\varepsilon^*}{k})}{kcg'(\bar{w}+\frac{\varepsilon^*}{k})(b+kc)+g'(\bar{w}-\frac{\varepsilon^*}{n-k})\frac{1}{n-k}(b+kc)^2}<0.$$

The above implies that $w_1\equiv\bar{w}+\frac{\varepsilon^*}{k}$ is decreasing in c and $w_2\equiv\bar{w}-\frac{\varepsilon^*}{n-k}$ is increasing in c. At $c=0$ we have (assuming $g(0)=0$), $\varepsilon^*=(n-k)\bar{w}$, $w_1=\frac{n}{k}\bar{w}$ and $w_2=0$. Now let us look at (8) once again. From the Inada conditions $f_1(w_2,g(w_2))=\infty$ and thus $\frac{d\Pi^I(\varepsilon^*)}{d\varepsilon}=-\infty<0$ as the other two terms in (8) are finite and non-negative (the first term is actually equal to 0). Since f is concave by assumption, $f_1(z,g(z))$ is decreasing in z and as w_1 is decreasing in c the second term above is increasing in c. Similarly, as $-f_1(z,g(z))$ is increasing in z and w_2 is increasing in c, the third term is increasing in c as well. The latter imply that $f_1(w_1,g(w_1)-f_1(w_2,g(w_2))<0$ and thus the left hand side increases towards zero for $c\to\infty$. By a continuity argument, the above shows that there exists some $\bar{c}>0$ (depending on n,k,b) such that for $c\in[0,\bar{c}]$, $\frac{d\Pi^I(\varepsilon^*)}{d\varepsilon}<0$, i.e. $\Pi^I(\varepsilon)$ is maximized at ε^* (the minimum degree of inequality needed to have non-contributing agents) as. But $\Pi^E>\Pi^C(\bar{w}+\frac{\varepsilon^*}{k},\bar{w}-\frac{\varepsilon^*}{n-k})=\Pi^I(\varepsilon^*)$ and so perfect equality maximizes joint surplus for small c.■

Proof of Proposition 5:

(a) The result follows immediately from Lemma 5.

(b) Since we assume constant returns to scale, it follows from Lemma 3 that $g(w)=Aw$, where A is a positive constant. A player stops contributing if

$$A(\overline{w}-\frac{\varepsilon}{n-k})<cX=\frac{kcA(\overline{w}+\frac{\varepsilon}{k})}{b+kc}$$

Consider the derivative in (7) evaluated at ε^*. The second and third term are clearly positive. Because of constant returns to scale, $f_1(\lambda w, \lambda z) = f_1(w, z)$ for $\lambda > 0$. Notice also that from the definition of ε^* :

$$\frac{kc}{b+kc}\left(\overline{w} + \frac{\varepsilon^*}{k}\right) = \overline{w} - \frac{\varepsilon^*}{n-k}.$$

>From this equation we can solve for ε^* :

$$\varepsilon^* = \frac{b\overline{w}(n-k)}{cn+b}.$$

But then the first term is:

$$\begin{aligned} & f_1\left(\overline{w} + \frac{\varepsilon^*}{k}, g(\overline{w} + \frac{\varepsilon^*}{k})\right) - f_1\left(\overline{w} - \frac{\varepsilon^*}{n-k}, \frac{kc}{b+kc} g(\overline{w} + \frac{\varepsilon^*}{k})\right) = \\ = \; & f_1\left(\overline{w} + \frac{\varepsilon^*}{k}, g(\overline{w} + \frac{\varepsilon^*}{k})\right) - f_1\left(\frac{kc}{b+kc}\left(\overline{w} + \frac{\varepsilon^*}{k}\right), \frac{kc}{b+kc} g(\overline{w} + \frac{\varepsilon^*}{k})\right) = 0 \end{aligned}$$

This implies that $\Pi^I(\varepsilon)$ is increasing in a neighborhood of ε^* and thus it achieves a maximum for some $\bar{\varepsilon} > \varepsilon^*$, i.e. $\max_{\varepsilon \in [\varepsilon^*, \bar{w}(n-k)]} \Pi^I(\varepsilon) > \Pi^I(\varepsilon^*)$. Now let us prove that inequality is always joint profit maximizing. Joint profits under perfect equality when $f(w, z)$ is homogeneous of degree one is

$$\begin{aligned} \Pi^E(\bar{w}) &= n[f(\bar{w}, g(\bar{w})) - \frac{g(\bar{w})}{b+nc}] \\ &= n\bar{w}[f(1, A) - \frac{A}{b+nc}]. \end{aligned}$$

Also, joint profits under an unequal wealth distribution such that there are k agents with wealths $\bar{w} + \dfrac{\varepsilon^*}{k}$ and $n-k$ agents with wealths $\bar{w} - \dfrac{\varepsilon^*}{n-k}$ in which only the former group contributes is:

$$\begin{aligned} \Pi^I(\varepsilon^*) &= k[f(\bar{w} + \frac{\varepsilon^*}{k}, g(\bar{w} + \frac{\varepsilon^*}{k})) - \frac{g(\bar{w} + \frac{\varepsilon^*}{k})}{b+kc}] + (n-k) f(\bar{w} - \frac{\varepsilon^*}{n-k}, \frac{kc}{b+kc} g(\bar{w} + \frac{\varepsilon^*}{k})) \\ &= k f(\bar{w} + \frac{\varepsilon^*}{k}, A(\bar{w} + \frac{\varepsilon^*}{k})) + (n-k) f(\overline{w} - \frac{\varepsilon^*}{n-k}, A\left(\overline{w} - \frac{\varepsilon^*}{n-k}\right)) - \frac{kA(\bar{w} + \frac{\varepsilon^*}{k})}{b+kc} \\ &= k\left(\bar{w} + \frac{\varepsilon^*}{k}\right)\left(f(1, A) - \frac{A}{b+kc}\right) + (n-k)\left(\overline{w} - \frac{\varepsilon^*}{n-k}\right) f(1, A) \\ &= n\bar{w} f(1, A) - k\left(\bar{w} + \frac{\varepsilon^*}{k}\right)\frac{A}{b+kc}. \end{aligned}$$

Using the value of ε^* we get:

$$\frac{k\bar{w} + \frac{b\overline{w}(n-k)}{cn+b}}{b+kc} = \frac{k(b+cn) + b(n-k)}{(b+kc)(b+cn)}\overline{w} = \frac{n(b+kc)}{(b+kc)(b+cn)}\overline{w} = \frac{n}{(b+cn)}\overline{w}.$$

i.e. $\Pi^I(\varepsilon^*) = \Pi^E$. Therefore, $\max\limits_{\varepsilon} \Pi^I(\varepsilon) > \Pi^I(\varepsilon^*) = \Pi^E$ and thus some degree of inequality (with $\varepsilon > \frac{b(n-k)}{b+nc}\overline{w}$) is joint profit maximizing.

For the second part of Proposition 5 (b), it is sufficient to provide an example. Suppose $f(w, z)$ has the Cobb-Douglas, constant returns to scale form $f(w, z) = w^\alpha z^{1-\alpha}$ and there are two agents in the economy[26] with endowments of the private input $w + \varepsilon$ and $w - \varepsilon$, where $\varepsilon \in [0, w]$. By Lemma 3 we

[26] We have actually proven the proposition for any $f(w, z)$ satisfying Assumptions 1-3 and any redistribution in which k agents obtain $w + \varepsilon$ and $n-k$ obtain $w - \varepsilon$ but the expressions corresponding to (??) and (??) are much less tractable which is why we chose to present the result for a Cobb-Douglas function.

have $g(w) = Aw$ and $f(w, g(w)) = A^{1-\alpha}w$, where $A = [(b+c)(1-\alpha)]^{\frac{1}{\alpha}}$. Under perfect equality each player obtains a surplus of:

$$\pi^E = wA^{1-\alpha}[1 - \frac{A^\alpha}{b+2c}] = wA^{1-\alpha}[\frac{c+\alpha(b+c)}{b+2c}] \tag{9}$$

Let, as in the proof of Proposition 5, $\varepsilon^* = \frac{bw}{b+2c}$ be the degree of inequality at which the poorer player is just indifferent between contributing and not contributing. Thus, for $\varepsilon \in (\varepsilon^*, w]$ only the richer player (i.e. the one with endowment $w+\varepsilon$) would contribute and her profits would be given by:

$$\pi^{rich}(\varepsilon) = A^{1-\alpha}(w+\varepsilon) - \frac{A(w+\varepsilon)}{b+c} = A^{1-\alpha}\alpha(w+\varepsilon)$$

$$\pi^{poor}(\varepsilon) = (w-\varepsilon)^\alpha(w+\varepsilon)^{1-\alpha}(\frac{Ac}{b+c})^{1-\alpha}$$

using the expression for π_i obtained previously. First notice that, evaluating the above expressions at $\varepsilon = \varepsilon^*$ it is possible to have:

$$\pi^{rich}(\varepsilon^*) < \pi^{poor}(\varepsilon^*) \tag{10}$$

as it is equivalent to $\alpha < \frac{c}{b+c}$. Clearly, for $\varepsilon > \varepsilon^*$ $\pi^{rich}(\varepsilon)$ is increasing in ε. We can verify directly that $\frac{\partial \pi^{poor}(\varepsilon)}{\partial \varepsilon} < 0$ for $\varepsilon > \varepsilon^*$ if $\alpha > \frac{c}{b+2c}$. In addition as $\varepsilon \to w$, $\pi^{rich}(\varepsilon)$ goes to some positive value, while $\pi^{poor}(\varepsilon)$ goes to 0. Combining these results with (10) we see that there can exist some level of inequality $\varepsilon_0 \in (\varepsilon^*, w)$ such that:

$$\pi^{rich}(\varepsilon_0) = \pi^{poor}(\varepsilon_0) \tag{11}$$

Using the expressions obtained above we can solve for ε_0 to get:

$$\varepsilon_0 = \frac{w(1-B)}{1+B}$$

where $B = \alpha^{\frac{1}{\alpha}}(\frac{b+c}{c})^{\frac{1-\alpha}{\alpha}}$. Finally, it is easy to verify that the condition

$$\pi^{rich}(\varepsilon_0) = \pi^{poor}(\varepsilon_0) > \pi^E$$

is equivalent to $\frac{2\alpha(b+2c)}{c+\alpha(b+c)} > 1 + B$. As long as this condition, and $\frac{c}{b+2c} < \alpha < \frac{c}{b+c}$ hold simultaneously, we have an example where inequality Pareto-dominates perfect equality. For the case $\alpha = 1/2$ the first condition is equivalent to $c^2 > b^2$, and the second one is equivalent to $c > b > 0$, i.e. if the latter is true inequality is Pareto dominating.■

Proof of Proposition 6: From Assumption 1 we know that $f_2(w, z) = \infty$ as z approaches 0 from above and also, that $f(w, z) = -D$ for $z < 0$, where D is a very large number. Therefore all agents contribute in equilibrium, i.e. $m = n$. Also we know that X is maximized when wealths are equalized as it is equal to $\frac{\sum g(w_i)}{b+nc}$. The individual contributions then equal:

$$x_i = \frac{g(w_i) - cX}{b} = \frac{(b+(n-1)c)}{b(b+nc)}g(w_i) - \frac{c}{b(b+nc)}\sum_{j\neq i} g(w_j) > 0$$

as $c<0$ and $b+(n-1)c>b+nc>0$ by Assumption 2. Since all agents contribute joint surplus equals:

$$\Pi=\sum f(w_i,g(w_i))-\frac{\sum g(w_i)}{b+nc}.$$

Let us start at perfect equality, i.e. $w_i=w$ and consider a redistribution giving k of the agents $w+\frac{\varepsilon}{k}$ and the rest $w-\frac{\varepsilon}{n-k}$, $\varepsilon>0$. We then have:

$$\Pi=k[f(\overline{w}+\frac{\varepsilon}{k},g(\overline{w}+\frac{\varepsilon}{k}))-\frac{g(\overline{w}+\frac{\varepsilon}{k})}{b+nc}]+(n-k)[f(\overline{w}-\frac{\varepsilon}{n-k},g(\overline{w}-\frac{\varepsilon}{n-k}))-\frac{g(\overline{w}-\frac{\varepsilon}{n-k})}{b+nc}].$$

Let us see how a change in ε affects joint profits:

$$\begin{aligned}\frac{\partial\Pi}{\partial\varepsilon} &= f_1(\overline{w}+\frac{\varepsilon}{k},g(\overline{w}+\frac{\varepsilon}{k}))+\\ &+g'(\overline{w}+\frac{\varepsilon}{k})[f_2(\overline{w}+\frac{\varepsilon}{k},g(\overline{w}+\frac{\varepsilon}{k}))-\frac{1}{b+nc}]\\ &-f_1(\overline{w}-\frac{\varepsilon}{n-k},g(\overline{w}-\frac{\varepsilon}{n-k}))-\\ &g'(\overline{w}-\frac{\varepsilon}{n-k})[f_2(\overline{w}-\frac{\varepsilon}{n-k},g(\overline{w}-\frac{\varepsilon}{n-k}))-\frac{1}{b+nc}]\end{aligned}$$

We have $f_2(z,g(z))=\frac{1}{b+c}$ from the first-order conditions. So:

$$\begin{aligned}\frac{\partial\Pi}{\partial\varepsilon} &= [f_1(\overline{w}+\frac{\varepsilon}{k},g(\overline{w}+\frac{\varepsilon}{k}))-f_1(\overline{w}-\frac{\varepsilon}{n-k},g(\overline{w}-\frac{\varepsilon}{n-k}))]+\\ &+\frac{(n-1)c}{(b+c)(b+nc)}[g'(\overline{w}+\frac{\varepsilon}{k})-g'(\overline{w}-\frac{\varepsilon}{n-k})]\end{aligned}\tag{12}$$

Evaluating the above at $\varepsilon=0$, we have that

$$\frac{\partial\Pi}{\partial\varepsilon}|_{\varepsilon=0}=0$$

i.e. $\varepsilon=0$ is a critical point for the joint surplus function. Denote $w_1=\overline{w}+\frac{\varepsilon}{k}$ and $w_2=\overline{w}-\frac{\varepsilon}{n-k}$.The second derivative of Π is:

$$\begin{aligned}\frac{\partial^2\Pi}{\partial\varepsilon^2} &= \frac{1}{k}[f_{11}(w_1,g(w_1))+f_{12}(w_1,g(w_1))g'(w_1)]+\\ &+\frac{1}{n-k}[f_{11}(w_2,g(w_2))+f_{12}(w_2,g(w_2))g'(w_2)]\\ &+\frac{(n-1)c}{(b+c)(b+nc)}[\frac{1}{k}g''(w_1)+\frac{1}{n-k}g''(w_2)].\end{aligned}$$

At $\varepsilon=0$ the above equals:

$$\frac{n}{k(n-k)}[f_{11}(\overline{w})+f_{12}(\overline{w})g'(\overline{w})+\frac{(n-1)c}{(b+c)(b+nc)}g''(\overline{w})].$$

The first term within the brackets is negative (recall from the proof of Lemma 5 that f_{11} $f_{11}+f_{12}g'(w)=\frac{f_{11}f_{22}-f_{12}^2}{f_{22}}<0$ as $f(z,w)$ is concave) but the second term is positive. Therefore we cannot sign the derivative in general. For $c\to 0$, however, we know it is going to be negative by the concavity of f,

i.e. $\varepsilon = 0$ is a local maximum. Recall that by Assumption 2, $b + nc > 0$, or $c > -\frac{b}{n}$. Suppose c is large enough in absolute value such that $b + nc$ is close enough to 0. Then the last term within the square brackets becomes arbitrarily large and so $\dfrac{\partial^2 \Pi}{\partial \varepsilon^2} > 0$ i.e. $\varepsilon = 0$ is a local minimum. Therefore by a continuity argument, if c is close to zero, i.e. for all c in some interval $[c_0, 0)$ perfect equality is locally joint profit maximizing. If however c is large in absolute value, i.e. $c \in [-\dfrac{b}{n}, c_1)$ and so $b + cn$ close to 0, the second term above is arbitrarily large and therefore joint profits are maximized at some positive degree of inequality.■

Stability of Equilibrium

The stability condition in Assumption 2 that $b + nc \geq 0$ can be derived from a simple adjustment mechanism of the following form:

$$\frac{dx_i}{dt} = \mu_i(\hat{x}_i - x_i(t)),\ i = 1, 2, .., m$$

where μ_i are positive constants, $x_i(t)$ is the actual value of x_i at time t, and $\hat{x}_i$ is the reaction function as given by (1). Given that reaction functions are linear in our model, the condition for stability is equivalent to the following determinant of order m

$$\begin{vmatrix} b+c & c & & c \\ c & b+c & & c \\ & & .. & \\ . & . & & . \\ c & c & & b+c \end{vmatrix}$$

being positive definite. Performing some simple operations to make all elements in the first row (or column) except for the first two to be equal to zero, we can prove by induction that the value of this determinant is equal to $b^{m-1}(b + mc)$.

The CES Example

For the CES production function:

$$f(w, z) = (\delta w^\rho + (1 - \delta) z^\rho)^{\frac{k}{\rho}}$$

we show that if $0 < \rho < k \leq 1$ then $\gamma(w)$ is increasing and concave. First we need to ensure that f is concave and w and z are complements. The condition for non-increasing returns is $k \leq 1$, since $f(\lambda w, \lambda z) = \lambda^k f(w, z)$. The condition for $f_{12} > 0$ is $k > \rho$. The first order condition of maximization is:

$$(\delta w^\rho + (1 - \delta)\gamma(w)^\rho)^{\frac{k}{\rho}-1} \gamma(w)^{\rho-1} = \frac{1}{k(1 - \delta)(b + c)}.$$

Differentiating with respect to w and using the first order condition we get:

$$\gamma'(w) = \frac{(k - \rho)\delta w^{\rho-1}\gamma(w)}{(1 - k)(1 - \delta)(\gamma(w))^\rho + \delta(1 - \rho)w^\rho}.$$

As $k > \rho$ by assumption the numerator is positive. Also, the denominator is positive as $1 - k \geq 0$ and $\rho \in (0, 1)$ and $\delta \in (0, 1)$. Therefore $\gamma(.)$ is increasing. Observe that $\frac{w\gamma'(w)}{\gamma(w)} = \frac{(k-\rho)\delta w^\rho}{(1-k)(1-\delta)(\gamma(w))^\rho + \delta(1-\rho)w^\rho} \leq$

1 since the numerator is less than the second term in the denominator (which follows from $k \leq 1$). Differentiating the expression for $\gamma'(w)$, the sign of $\gamma''(w)$ turns out to be the same as that of the following expression:

$$(1-\rho)\left\{(1-k)(1-\delta)w^{\rho-2}\gamma(w)^{\rho+1} + \delta w^{2\rho-2}\gamma(w)\right\}\left\{\frac{w\gamma'(w)}{\gamma(w)} - 1\right\}.$$

This expression is non-negative under our assumptions, and the fact that $\frac{w\gamma'(w)}{\gamma(w)} \leq 1$. For $k = 1$, $\frac{w\gamma'(w)}{\gamma(w)} = 1$ and so the expression is equal to 0. Therefore $\gamma(w)$ is concave, and strictly so for $k < 1$.■

References

[1] Alesina, A. and D. Rodrik (1994) : "Distributive Politics and Economic Growth" *Quarterly Journal of Economics,* vol. 109, no. 2, pp. 465-490.

[2] Alesina, A. and E. La Ferrara (2000): "Participation in Heterogeneous Communities", *Quarterly Journal of Economics*; vol. 115, no. 3, pp. 847 – 904.

[3] Baland, J.-M. and Platteau, J.-P. (1997) : Wealth inequality and efficiency in the commons : I. The unregulated Case, *Oxford Economic Papers* 49, pp. 451-482.

[4] Baland, J.-M., O. Dagnelie, and D. Ray (2003) : "Inequality and Efficiency in Joint Projects", Mimeo. Facultés Universitaires Notre-Dame de la Paix, Namur and New York University.

[5] Baland, J.M., P. Bardhan, S. Das, D. Mookherjee, and R. Sarkar (2006): "Inequality, Collective Action and the Environment: Evidence from Firewood Collection in Nepal", forthcoming in J. M. Baland, P. Bardhan and S. Bowles (Ed.) *Inequality, Collective Action and Environmental Sustainability*, Princeton University Press.

[6] Bandiera, O., I. Barankay and I. Rasul (2005), "Cooperation in Collective Action", *Economics of Transition*, 13(3), p. 473-98.

[7] Banerjee, A. (2004): "Who is Getting the Public Goods in India? Some evidence and some speculation," in Basu, K. (ed.), *The Indian Economy: Up to 1991 and Since*, Cambridge: MIT Press.

[8] Banerjee, A., P. Gertler, and M. Ghatak (2002) : "Empowerment and Efficiency: Tenancy Reform in West Bengal" *Journal of Political Economy*, Vol. 110, No. 2, pp. 239-280.

[9] Banerjee, A. and E. Duflo (2003): "Inequality and Growth: What Can the Data Say?", *Journal of Economic Growth* 8, pp. 267-99.

[10] Banerjee, A. and A. Newman (1993) : "Occupational Choice and the Process of Development," *Journal of Political Economy* 101, 274-298.

[11] Banerjee, A., L. Iyer and R. Somanathan (2005): "History, Social Divisions and Public Goods in Rural India", *Journal of the European Economic Association*, Papers and Proceedings, Vol. 3, No. 2-3.

[12] Bardhan, P. (2000) : "Irrigation and Cooperation: An Empirical Analysis of 48 Irrigation Communities in South India". *Economic Development and Cultural Change 48,* pp. 847-65.

[13] Benabou, R. (1996) : "Inequality and Growth", *NBER Macroeconomics Annual.*

[14] Bergstrom, T., L. Blume and H. Varian (1986) : "On the Private Provision of Public Goods", *Journal of Public Economics*, **29,** p.25-49**.**

[15] Berry, R.A. and Cline, W.R. (1979): *Agrarian Structure and Productivity in Developing Countries.* Geneva, ILO

[16] Cornes, R. (1993): "Dyke Maintenance and other Stories: Some Neglected Types of Public Good", *Quarterly Journal of Economics*, Vol. 107, 259-71.

[17] Cornes, R. and T. Sandler (1994) : "The Comparative Static Properties of the Impure Public Good Model" *Journal of Public Economics*, **54,** p.403-21**.**

[18] Cornes, R. and T. Sandler (1996) : *The Theory of Externalities, Public Goods and Club Goods*, Cambridge University Press, Second Edition.

[19] Dayton-Johnson, J. (2000), "The Determinants of Collective Action on the Local Commons: A Model with Evidence from Mexico", *Journal of Development Economics.*

[20] Dayton-Johnson, J. and P. Bardhan (2002) : "Inequality and Conservation on the Local Commons: A Theoretical Exercise". *Economic Journal* 112*,* pp. 577-602.

[21] DiPasquale, D. and E. Glaeser (1999) : "Incentives and Social Capital: Are Homeowners Better Citizens?", *Journal of Urban Economics* 45 (1999): 354-384.

[22] Elster, J. (1989): *The Cement of Society*, Cambridge University Press, Cambridge.

[23] Evans, D. and B. Jovanovic (1989) : "An Estimated Model of Entrepreneurial Choice under Liquidity Constraints". *Journal of Political Economy* 97, pp. 808-27.

[24] Galor, O. and J. Zeira (1993) : "Income Distribution and Macroeconomics." *Review of Economic Studies,* Vol. 60*,* No. 1*,* pp.35-52.

[25] Itaya, J., D. de Meza and G.D. Myles (1997) : "In Praise of Inequality : Public Good Provision and Income Distribution", *Economics Letters*, 57, p.289-296.

[26] Khwaja, Asim I. (2004): "Can Good Projects Succeed in Bad Communities?", Working Paper, Kennedy School of Government, Harvard University.

[27] Knack, S. and P. Keefer (1997) : "Does social capital have an economic payoff? A cross-country investigation", *Quarterly Journal of Economics*; vol. 112, no. 4, pp. 1251-1288.

[28] La Ferrara, E. (2003), "Solidarity in Heterogeneous Communities" in Phillipe van Parijs (ed), *Cultural Diversity vs. Economic Solidarity*, Franqui Science Library, Brussels.

[29] Olson, M. (1965) : *The Logic of Collective Action: Public Goods and the Theory of Groups*. Cambridge Mass.: Harvard University Press.

[30] Olszewski, W. and H. Rosenthal (2004): "Politically Determined Income Inequality and the Provision of Public Goods", *Journal of Public Economic Theory* 6, pp. 707-735.

[31] Putnam, R. (1993) : *Making Democracy Work: Civic Traditions in Modern Italy,* Princeton University Press, Princeton, NJ.

[32] Simon, C.P. and L. Blume (1994): *Mathematics for Economists*. New York and London: W.W. Norton.

[33] Somanathan, E., R. Prabhakar and B.S. Mehta (2006): "Collective Action for Forest Conservation: Does Heterogeneity Matter?" forthcoming in J. M. Baland, P. Bardhan and S. Bowles (Ed.) *Inequality, Collective Action and Environmental Sustainability*, Princeton University Press.

[34] Wade, Robert (1994): *Village Republics: Economic Conditions for Collective Action in South India*, Cambridge.

[35] Warr, P. G. (1983) : "The Private Provision of a Public Good is Independent of the Distribution of Income", *Economics Letters*, 13, p.207-211.

Figure 1: Difference in Joint Surplus Under Perfect Equality and Optimal Inequality

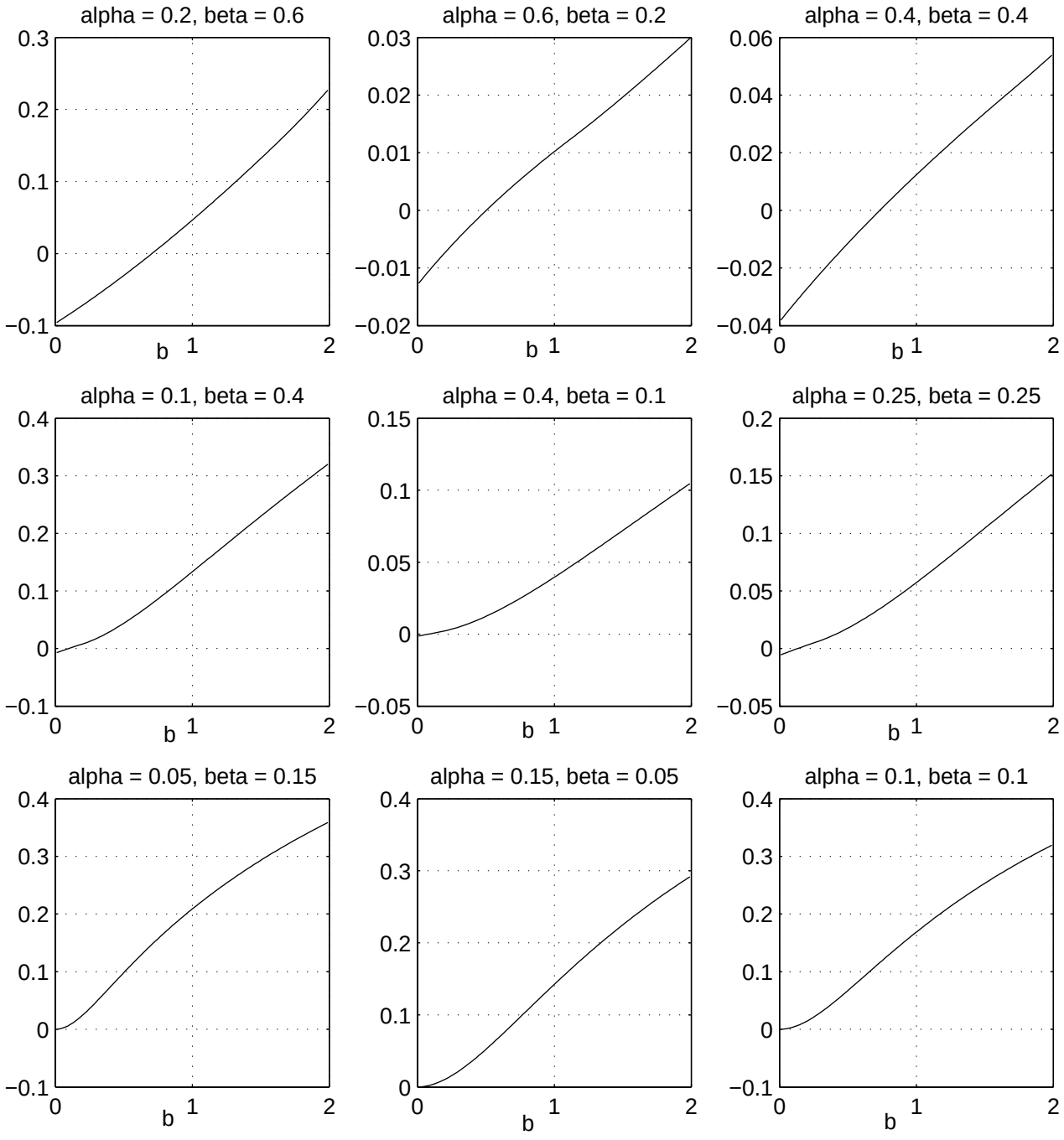

Figure 2: Difference in Joint Surplus Under Perfect Equality and Optimal Inequality