Economics 250a Lecture 12 Search Theory 3

Outline

I. DMP search and matching model

II. Flinn's minimum wage model

III. empirical analysis of matching: duration dependence and the matching function

References

Christopher Flinn (2006). "Minimum Wage Effects on Labor Market Outcomes Under Search, Matching and Endogenous Contract Rates" Econometrica 74 (July 2006), pp. 1013-1062.

Kory Koft, Fabian Lange, Matt. Notowidigdo and Lawrence Katz. "Long Term Unemployment and the Great Recession: The Role of Composition, Duration Dependence and Non-Participation." JOLE 34 (S1, January 2016)

I. Basic DMP (Recap from Lecture 11)

- L workers: uL unemployed, vL job vacancies.

- match function M(uL, vL) with CRS so $M(uL, vL) = vL \cdot M(\frac{u}{v}, 1)$.

- $\theta \equiv v/u$

 $-q \equiv M/vL = M(\frac{u}{v}, 1) = M(\frac{1}{\theta}, 1) = q(\theta)$ = vacancy filling rate

- $\theta q(\theta) = M/uL$ = unemployed exit hazard

- we expect $q(\theta)\to\infty$ as $\theta\to0$ and $q(\theta)\to0$ at as $\theta\to\infty$ (see Figure at end)

- we expect $\theta q(\theta) \to 0$ as $\theta \to 0$ and $\theta q(\theta) \to \infty$ as $\theta \to \infty$

Example:

 $M(uL, vL) = mLu^{1/2}v^{1/2} \Longrightarrow q(\theta) = m\theta^{-1/2} \text{ and } \theta q(\theta) = m\theta^{1/2}$

Notice that the constant m raises both the vacancy filling rate and the unemployment exit hazard.

1. Beveridge curve

BC defines set of u, v such that job creation = job destruction

- job destruction rate = $\delta(1-u)L$
- job creation rate = $\theta q(\theta) \times uL$
- equating we get:

$$u = \frac{\delta}{\delta + \theta q(\theta)} = \frac{\delta}{\delta + (v/u)q(v/u)}$$

For example, set $M = mLu^{1/2}v^{1/2}$ then we get

$$v = \frac{\delta^2 (1-u)^2}{m^2 u}.$$

There is some interest in the idea that govt' policies and/or market characteristics (such as the density of vacancies and job searchers per square mile) could affect m and shift the BC. 2. Vacancy creation

Vacancy creation depends on the wage w and on the ease of filling jobs, which is given by $q(\theta)$. The wage, in turn, depends on the size of the match surplus when a job is created (which will depend on θ) and on the share received by workers. So we are going to have 2 equations in (w, θ) : one from the vacancy creation side, and the other that is from the wage-determination side, allowing us to express everything in terms of an equilibrium level of θ . Then in (u, v)space there will be a positively sloped line: $v = \theta^* u$ that intersects the BC at the equilibrium level of tightness.

2a. Vacancy creation conditional on wages

V = value of unfilled vacancy

J = value of a filled vacancy (i.e., a job) to firms

$$rV = -c + q(\theta)(J - V)$$

assuming V = 0 we get:

 $J = c/q(\theta) = \text{cost per period} \times \text{expected time to fill}$

What is J? With output p, wage w, and job destruction rate δ , interest rate r we get:

$$J \quad = \quad \frac{p-w}{r+\delta}$$

so in equilibrium we must have

$$w = p - (r + \delta)c/q(\theta) \quad (a)$$

2b. Wages conditional on tightness

Finally we have to determine how wages are set, once a match is formed. Note that wages do not affect the job creation rate, so the game here is to figure out how tighter labor market raises w, which we can use in combination with equation (a) to find the equilibrium θ from the vacancy creation side.

A representative worker has value of unemployment U and value W(w) of job with wage w:

$$rU = b + \theta q(\theta)(W(w) - U)$$

$$rW(w) = w + \delta(U - W(w)).$$

The second equation implies:

$$W(w) = \frac{w}{r+\delta} + \frac{\delta}{r+\delta}U$$

Using this plus the equation for rU we get:

$$rU = \frac{(r+\delta)}{r+\delta+\theta q(\theta)}b + \frac{\theta q(\theta)}{r+\delta+\theta q(\theta)}w (*)$$

which is a weighted average of b and w. Notice that the weight on w is larger when $\theta q(\theta) =$ hazard rate to a new job is higher. Finally, the gain to a worker of having a job versus being unemployed is W(w) - U and using the expression $W(w) = \frac{w}{r+\delta} + \frac{\delta}{r+\delta}U$ we get:

$$W(w) - U = \frac{w - rU}{r + \delta}$$

Now when the worker and the firm "match" the total surplus created (holding constant U) is

$$S = W(w) - U + J$$

= $\frac{w - rU}{r + \delta} + \frac{p - w}{r + \delta}$
= $\frac{p - rU}{r + \delta}$

which does not depend on w. Nash bargaining gives:

$$w = argmax_w (\frac{w - rU}{r + \delta})^{\beta} (\frac{p - w}{r + \delta})^{1 - \beta}$$

the solution is

$$w = rU + \beta(p - rU)$$

= $(1 - \beta)rU + \beta p$

Note that the negotiated wage is higher when the worker has a better "fallback option", rU.

Now notice that rU is endogenous, and actually depends on w. In equilibrium we have to use equation (*) above (which expresses rU in terms of w) and the wage determination equation to get w in terms of θ . We get:

$$w = \frac{(1-\beta)(r+\delta)}{r+\delta+\beta\theta q(\theta)}b + \frac{\beta(r+\delta)+\beta\theta q(\theta)}{r+\delta+\beta\theta q(\theta)}p \quad (b)$$

Equation (b) says that the wage is a weighted average of b and p, with weights that depend on θ :

$$w = (1 - A(\theta))b + A(\theta)p$$
$$A(\theta) = \frac{\beta(r + \delta) + \beta\theta q(\theta)}{r + \delta + \beta\theta q(\theta)}$$

The weight $A(\theta)$ equals β when $\theta q(\theta) = 0$ (the weakest possible labor market), and equals 1 when $\theta q(\theta) \to \infty$ (the best possible labor market).

Now we have 2 equations in θ and w, (a) and (b), that we can combine to summarize the "demand side" of the labor market. Collecting the equations we have:

$$w = p - (r + \delta)c/q(\theta) \quad (a)$$

$$w = (1 - A(\theta))b + A(\theta)p \quad (b)$$

Notice that $\theta \to 0$, $q(\theta) \to \infty$ and equation (a) has w = p. Higher values of θ lower the right hand side of (a), since $q(\theta)$ is decreasing in θ . On the other hand, as $\theta \to 0$, $\theta q(\theta) \to 0$ and equation (b) has w = b. Higher values of θ raise the right hand side of (b), since $A(\theta)$ is increasing in θ , eventually getting to w = p. So the two equations have to cross at some intermediate value of θ with some wage between b and p. See figure at the end of the lecture. The solution satisfies:

$$G(\theta) \equiv \frac{q(\theta)}{r+\delta+\beta\theta q(\theta)} = \frac{c}{(p-b)(1-\beta)}$$

The function $G(\theta)$ is decreasing in θ . If we call the solution $\theta^*(p, b, c, \beta, r, \delta)$ it's clear that lower values of c and higher values of p-b lead to higher equilibrium values for θ^* . See the figure at the end of the lectures.

3. Combing BC and Vacancy creation

To use an S-M model to think about the equilibrium levels of frictional u and v in the market, we simply plot the Beveridge curve and the line points $v = \theta^* u$. Any shift in θ^* rotates the v-creation curve.

II. Flinn's search-matching model with a minimum wage

Flinn presents a search-matching model to model the effect of the minimum wage. The model introduces a couple of things relative to the basic S-M model. First, there is an idiosyncratic "match effect" for each potential worker-firm match, so with rent-splitting the wage varies across jobs reflecting a fraction of this match, and not all matches lead to a job. Second, with the match effect there is room for a minimum wage to cause a spike in the bargained wage. An obstacle for reading Flinn's paper is that he uses very different notation than other authors in the literature. Nevertheless, the building blocks are similar to those in the basic S-M model. Here I will use Flinn's notation and try to point out how it maps to the standard notation.

Notation:

 $-\theta$ = productivity of the match, d.f. $G(\theta)$; otherwise all workers and firms homogenous

 $-\rho = \text{discount rate (what everyone else calls } r)$

 $-\eta = job$ destruction rate (what everyone else calls δ)

-b = flow utility while searching

 $-\lambda = \text{arrival rate of offers} - (\text{this is } \theta q(\theta) \text{ in standard S-M notation})$

- as in the basic S-M model there is no on the job search.

Basic Setup with no minimum wage:

-firm's flow profit if employing a worker with match θ at wage w is $\theta-w$. So value function for a match with productivity θ is

$$J = \frac{\theta - w}{\rho + \eta}$$

as in the standard model. But now some jobs are more valuable to firms than others.

- value functions for worker $V_n, V_e(w)$ if searching or employed at wage w
- reservation wage w^* will satisfy $V_n = V_e(w^*)$
- Bellman equations for a worker:

$$\rho V_n = b + \lambda \int_{w^*} (V_e(w) - V_n) f(w) dw$$
$$(\rho + \eta) V_e(w) = w + \eta V_n$$

where f(w) is the density of wages. Some manipulation establishes:

$$\rho V_n = w^*$$

$$V_e(w) - V_n = \frac{w}{\rho + \eta} - \frac{\rho}{\rho + \eta} V_n = \frac{w - \rho V_n}{\rho + \eta}$$

Notice that the value to the worker of a job paying w relative to remaining unemployed is same as in the basic S-M model above.

Wage Determination.

When a searching worker meets a firm and the value of the match is θ , they conduct Nash bargaining, leading to a wage that maximizes:

$$\Omega(w) = (V_e(w) - V_n)^{\alpha} \left(\frac{\theta - w}{\rho + \eta}\right)^{1 - \alpha}$$
$$= \left(\frac{w - \rho V_n}{\rho + \eta}\right)^{\alpha} \left(\frac{\theta - w}{\rho + \eta}\right)^{1 - \alpha}$$

which leads to rent-split with worker share α :

$$w = \rho V_n + \alpha (\theta - \rho V_n)$$

= $(1 - \alpha) \rho V_n + \alpha \theta.$

Now from the relations above:

$$V_e(w) - V_n = \frac{w - \rho V_n}{\rho + \eta} = \frac{\alpha(\theta - \rho V_n)}{\rho + \eta}.$$

Finally we can rewrite the expression for ρV_n :

$$\rho V_n = b + \lambda \int_{w^*} (V_e(w) - V_n) f(w) dw$$

= $b + \lambda \int_{\theta^*} \frac{\alpha(\theta - \rho V_n)}{\rho + \eta} dG(\theta)$
= $b + \frac{\lambda \alpha}{\rho + \eta} \int_{\rho V_n} (\theta - \rho V_n) dG(\theta)$

which can be solved for ρV_n , given $b, \lambda, \alpha, \rho, \eta$ and $G(\theta)$. Notice that ρV_n will be higher when b is higher, when α is higher, and when λ is higher. So in particular the value of being unemployed is higher when when the arrival rate is higher.

Adding a minimum wage

With a minimum wage m the worker's value of search is $V_n(m)$, which we will have to solve for. As before assume wages are determined by a rent-splitting wage process. Then ignoring the minimum wage, the wage when the value of the match is θ would be:

$$w = \alpha \theta + (1 - \alpha)\rho V_n(m).$$

Define $\hat{\theta}$ as the value such that

$$m = \alpha \widehat{\theta} + (1 - \alpha) \rho V_n(m)$$

For $\theta > \hat{\theta}$ the minimum wage is not a problem. But for a range of lower values the minimum is binding. Assuming $\hat{\theta} > m$ there is a range of $\theta's$ that are efficient (i.e., have match value at least as big as the minimum) but under the ordinary wage model would be paid less than the minimum. Flinn assumes these matches are consumated and the wage is set to m, generating a spike at the minimum wage. This is a nice idea and gives a simple explanation for the spike in wages at the minimum wage. (See figure at end of lectures). Notice that the fraction of workers at the spike is the fraction of matches with $\theta \in [m, \hat{\theta}]$. The width of this interval is:

$$\widehat{\theta} - m = \frac{(1-\alpha)}{\alpha}(m - \rho V_n(m))$$

which is bigger, the larger is firm's bargaining power and larger is the gap $m - \rho V_n(m)$.

The value of unemployment is now:

$$\rho V_n(m) = b + \frac{\lambda}{\rho + \eta} \int_m^{\widehat{\theta}} (m - \rho V_n(m)) dG(\theta) + \frac{\lambda \alpha}{\rho + \eta} \int_{\widehat{\theta}} (\theta - \rho V_n(m)) dG(\theta)$$
(1)

Notice that given a value for λ and m (and the other parameters α, ρ, η and the d.f. G) this equation can be solved for $\rho V_n(m)$, and as in the case with no min. wage, $\rho V_n(m)$ will be higher when λ is higher.

The presence of the minimum wage creates a wedge between $V_n(m)$ and $V_e(m)$. The lowest-wage job is now more valuable than unemployment (whereas in a standard model the job that is just acceptable has the same value as continuing to search). This is an interesting feature of the model to think about.

Equilibrium

The final step it to endogenize λ . This will be done, as in the standard S-M model, by looking at firm's vacancy creation decisions, and combining this with a Beveridge type relation.

We have some additional notation and assumptions:

- participation equation: $Q(\rho V_n)$ – people decide to enter labor force based on ρV_n

 $-\ell =$ fraction of workers who participate (either work or search)

 $\ell = Q(\rho V_n(m))$

The participation margin makes the minimum wage "interesting" because a higher value of m will cause more workers to enter the labor market, and can actually lead to more employment in the economy.

 $-\widetilde{u} = \text{total number (mass) of unemployed}$

-v = total number (mass) of vacancies

 $-k = \tilde{u}/v$ = ratio of searchers to job openings (note that $k = 1/\theta$ using the standard notation)

 $-m(\widetilde{u}, v) =$ matching function = flow rate of matches. Standard c.r.s. assumption on m(.):

$$m(\widetilde{u}, v) = vq(\widetilde{u}/v) = vq(k)$$

for some increasing function q(.). With this assumption we get the arrival rate of offers (to workers) is:

$$\lambda = \frac{m(\widetilde{u}, v)}{\widetilde{u}} = \frac{q(k)}{k} \ (= \theta \boldsymbol{q}(\theta) \text{ in usual notation})$$

and the job filling rate is:

$$\frac{m(\widetilde{u},v)}{v} = q(k).$$

What determines v? As in the simple S-M model this comes from the vacancy creation decision. Assume firms can create a vacancy for cost c. The expected value of a vacancy is

$$\rho V_v = -c + q(k)(1 - G(m))(J - V_v)$$

where J = the expected profits of a consumated match. If we assume $V_v = 0$ (vacancies are created until the net profit is 0), we get

$$c = q(k)(1 - G(m))J$$

$$\Rightarrow \frac{1}{v} = \widetilde{u}q^{-1}\left(\frac{c}{J(1 - G(m))}\right).$$
(2)

Note that equation (3) is the same as we had in the basic S-M model, except we have to allow for truncation of low-productivity offers. For a given amount of unemployment and values for c and J this gives the amount of vacancies created.

What is J? For any given match the firm's expected discounted profit is $J(\theta)$, where

$$(\rho + \eta)J(\theta) = (\theta - w(\theta)) + \eta V_v.$$

With $V_v = 0$, we get

$$\begin{split} I(\theta) &= \frac{\theta - w(\theta)}{\rho + \eta} \\ &= \frac{\theta - m}{\rho + \eta} \text{ if } \theta \le \widehat{\theta} \\ &= (1 - \alpha) \frac{(\theta - \rho V_n(m))}{\rho + \eta} \quad \text{if } \theta > \widehat{\theta} \end{split}$$

which generalizes the expression from the basic S-M model. Thus

$$J = E[J(\theta)|\theta \ge m]. \tag{3}$$

Given m and $\rho V_n(m)$ we can find J. This will be lower when $V_n(m)$ is higher because workers get a higher wage when their fallback option is higher.

Finally, what is \tilde{u} ? Recall that in a model where U unemployed people have a job finding rate of f and E = L - U employed people have a job-losing rate of s that in steady state s(L - U) = Uf, implying that the steady state unemployment rate is u = U/L = f/(f+s). In this model the job loss rate is η and the job finding rate is $\lambda(1-G(m)) = \frac{q(k)}{k}(1-G(m))$. So the unemployment rate is

$$u = \frac{\eta}{\eta + \frac{q(k)}{k}(1 - G(m))}.$$

Again, this generalizes the expression in the standard S-M model to allow for low-productivity matches. If the size of the labor force is ℓ then

$$\widetilde{u} = u\ell = \frac{\eta}{\eta + \frac{q(k)}{k}(1 - G(m))}Q(\rho V_n(m))$$
(4)

This equation is the "generalized Beveridge curve".

So now we are ready to discuss the equilibrium. The primitives are

$$\rho, b, \eta, \alpha, G(.), Q(.), q(.), c, m$$

The endogenous variables are

$$\ell, u, v, \rho V_n(m)$$

Flinn notes that there is a simple recursive algorithm:

- 1. choose a value for λ
- 2. using equation 1 (above) solve for $\rho V_n(m)$
- 3. given $\rho V_n(m)$ find $\ell = Q(x)$, and also solve for J using (3).
- 4. using equation 4 (above) solve for \widetilde{u}

- 5. using equation 2 (above) solve for v
- 6. this generates a new value of $\lambda = q(\frac{\tilde{u}}{v})/(\frac{\tilde{u}}{v})$.

III. KLNK

KLNK is an example of a recent paper that uses matching function ideas to think about observed patterns of unemployment. The motivating question is how to explain the apparent outward shift in the US Beveridge curve after 2008 (see figure at end of notes), and the rise in long term unemployment.

Their basic hypothesis is that people who have been unemployed for longer are less likely to be hired: so they introduce a term that multiplies the "matches per searcher" function coming from a standard S-M model, reflecting the hiring probability for a searcher who has been unemployed with duration d. They also allow non-participants to search too, effectively "clogging up" the labor market for the unemployed.

Notation and Setup

u.

- $P_t, E_t, U_t, N_t = \#$ of population, employed, unemployed, non-participants - $e_t = E_t/P_t$, $n_t = N_t/P_t$, $u_t = U_t/P_t$ note non-standard but useful defn of

- λ_t^{ij} = monthly transition rate from state i to state j e.g, $\lambda_t^{EU}, \lambda_t^{EN}, \ldots$
- $V_t = \#$ vacancies (from JOLTS)
- matching function $M(U + sN, V) = m_0(U + sN)^{\alpha}V^{1-\alpha}$ $x_t = \frac{V_t}{U_t + sN_t}$ = tightness in period t

- baseline
$$\lambda_t^{UE} = \frac{U_t}{U_t + sN_t} \times \frac{m_0(U_t + sN_t)^{\alpha}V_t^{1-\alpha}}{U_t} = m_0 x_t^{1-\alpha}$$

$$-\lambda_t^{NE} = \frac{sN_t}{U_t + sN_t} \times \frac{m_0(U_t + sN_t)^{\alpha} V_t^{1-\alpha}}{N_t} = sm_0 x_t^{1-\alpha}$$

The key assumption generating duration dependence is that the rate of moving from U to E for a searcher who has been unemployed d periods is:

$$\lambda_t^{UE}(x_t, d) = A(d)m_0 x_t^{1-\alpha}$$

where $A(d) = (1 - a_1 - a_2) + a_1 exp(-b_1d) + a_2 exp(-b_2d)$. This functional form is consistent with 2 groups of people in the pool with different rates of decay of their exit rates. KLNK use CPS data which do not ask N's how long they have been out of work, so they do not allow duration dependence in the job-finding rate of N's.

Calibration and Simultion (see paper for more details) 1. They estimate $\lambda_t^{UN}, \lambda_t^{UE}, \lambda_t^{EN}, \lambda_t^{EU}, \lambda_t^{NE}, \lambda_t^{NU}, V_t, U_t, N_t$ and the distribution of d at each t, which they call Θ_t . These are all estimated from CPS data.

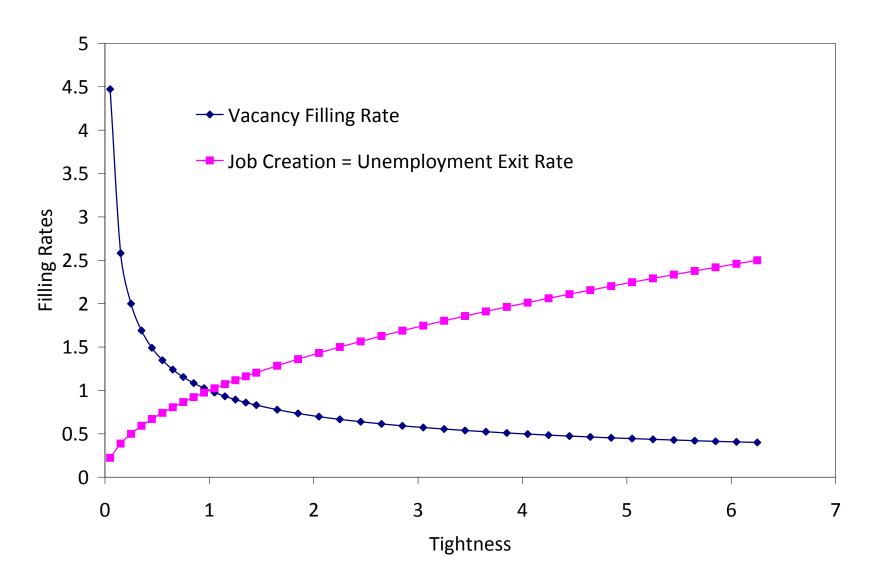
2. They estimate the function A(d) using data on relative rates of job-finding for people with different d's using 2002-2007 data. See figure 7 at end of notes. 3. They estimate m_0, s, α using data on UE and NE transition rates from 2002-2007 (and the distributions of d in each month, together with $\widehat{A}(d)$. They get:

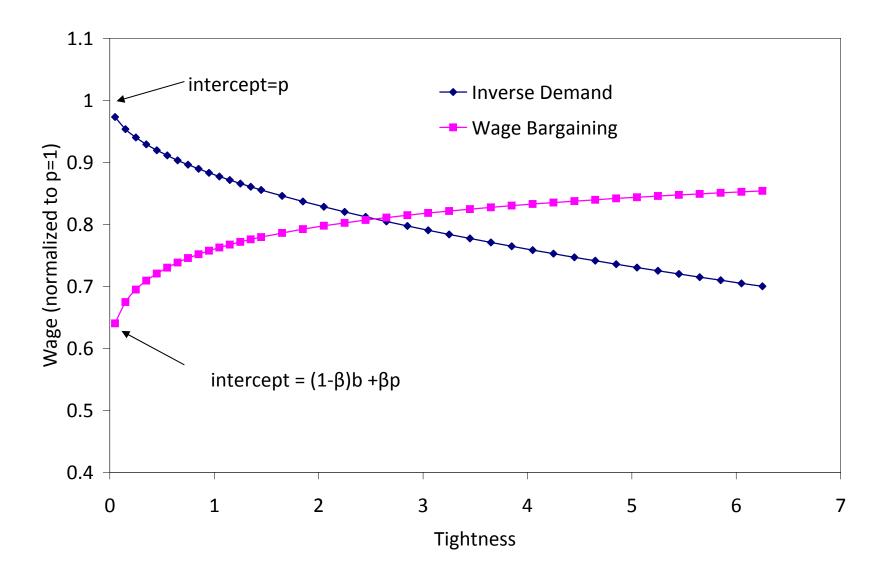
$$\alpha = 0.753$$
$$m_0 = 0.435$$
$$s = 0.218$$

Notice that α is a lot bigger than 0.5. Also s is small but non-zero, confirming that the N's exert some pressure on the labor market.

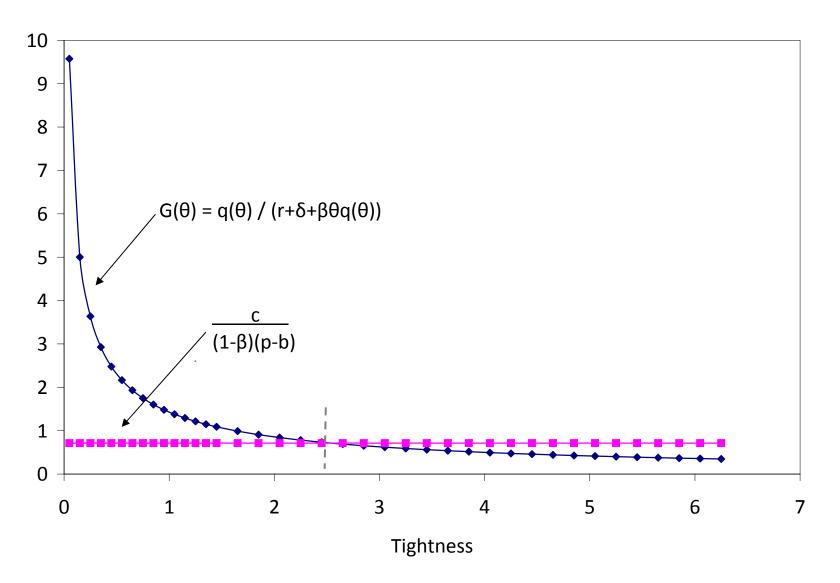
4. Then using the *actual* vacancies V_t and *actual* transitions $\lambda_t^{UN}, \lambda_t^{NU}, \lambda_t^{EN}, \lambda_t^{EU}$ but model-based (i.e., estimated) transitions $\lambda_t^{UE}, \lambda_t^{NE}$ they simulate the evolution of the labor market from 2008 to 2013. The goal here is to see how much of the fall in "job creation" rates can be explained by the rise in the mean duration of joblessness among searchers following the huge shocks in $\lambda_t^{EN}, \lambda_t^{EU}$ that occured at the start of the Great Recession.

Vacancy Filling and Job Creation Rates

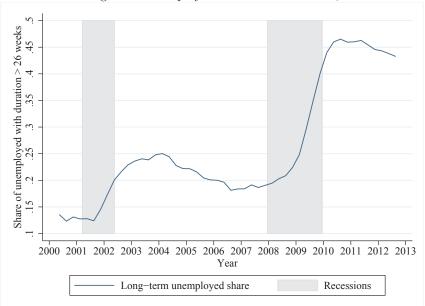




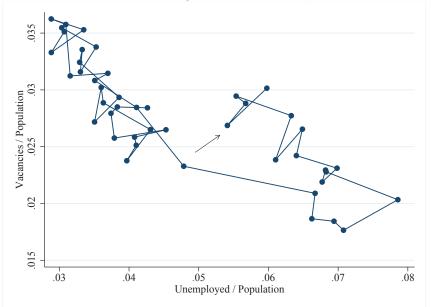
Determination of Tightness from Vacancy Creation Side



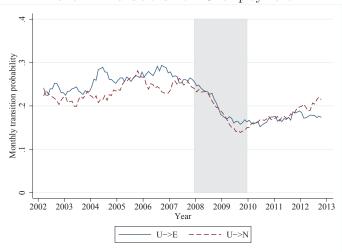
Panel A: Long-term Unemployment Share in the U.S., 2000-2013



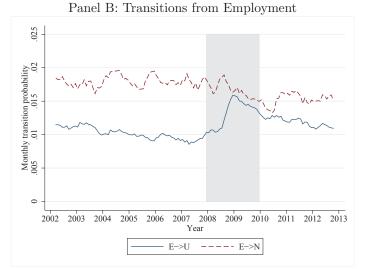
Panel B: The Beveridge Curve in the U.S., 2000-2013



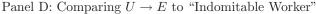
<u>Notes</u>: This figure uses data from the CPS and from JOLTS. Panel A shows the share of unemployed workers aged 25-55 that have unemployment durations of more than 26 weeks. The pooled, cross-sectional data come from monthly CPS surveys. In this panel and in Figures 3 through 5, month fixed effects have been residualized out of the data to account for seasonality, and the data are smoothed by taking a three-month average around each observation. Panel B shows the Beveridge curve, the relationship between unemployment and vacancies, with both series normalized by the total population (i.e., labor force plus non-participants). The arrow in panel B indicates the apparently outward movement of the Beveridge curve after 2008.

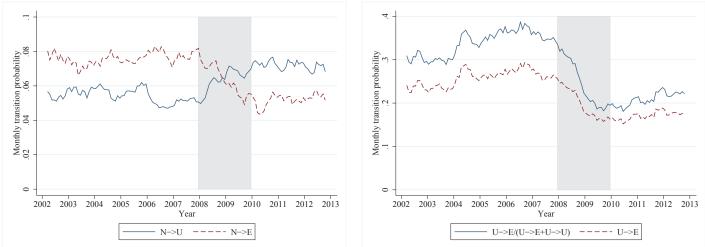


Panel A: Transitions from Unemployment



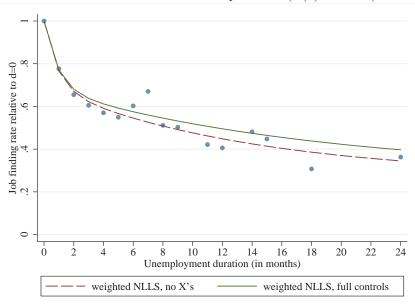
Panel C: Transitions from Non-Participation





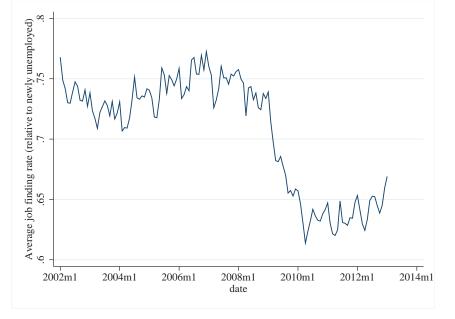
Notes: These figures use data from the CPS. See notes to Figure 2 for more information on the sample construction.



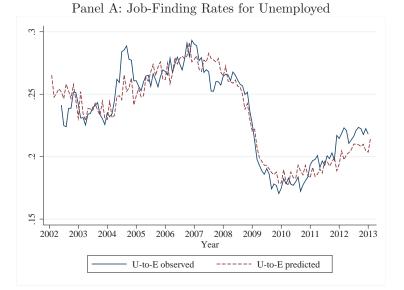


Panel A: Estimated Duration Dependence (A(d)function)

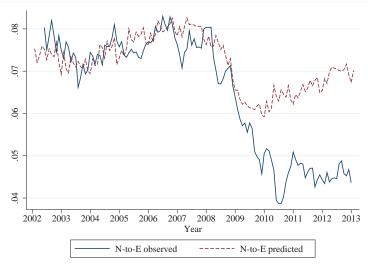
Panel B: Predicted Job Finding Probability, \overline{A} , Based on Distribution of Unemployment Durations and A(d)



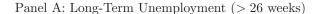
<u>Notes</u>: In Panel A, the figure uses data from the CPS and estimates (via NLLS) the negative exponential relationship between monthly job finding probability and unemployment duration. The NLLS uses CPS sample weights. The following functional form is used to estimate duration dependence: $A(d) = (1 - a_1 - a_2) + a_1 \exp(-b_1 \times d) + a_2 \exp(-b_2 \times d)$. The fitted values from the estimates with controls (solid line) are used to construct the counterfactuals shown in Figures 7 through 10. The controls used are the following: gender, fifth-degree polynomial in age, three race dummies (white/black/other), five education category dummies (high school dropout, high school graduate, some college, college graduate, and other), and gender interactions for all of the age, race, and education variables. Only monthly cell means with at least 30 observations are shown. In Panel B, the figure is generated by using estimates of how job finding probability varies with unemployment duration interacted with observed distribution of unemployment durations. Thus, the line in this figure shows the extent to which we would predict changes in job finding probability based solely on observed changes in distribution of unemployment duration. The y-axis scale is normalized so that a value of 1 indicates average job finding probability for a newly unemployed worker.



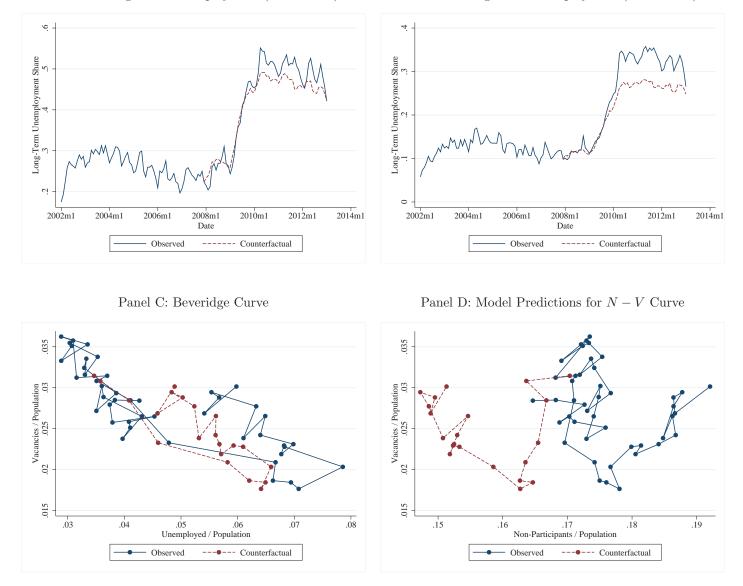




<u>Notes</u>: These figures report the model-generated predicted job-finding rates for unemployed workers and non-participants, where the predictions are based on model estimates calibrated to match 1/2002-12/2007 time period. See main text for more details.



Panel B: Long-Term Unemployment (> 52 weeks)



Notes: These figures use data from the CPS and JOLTS. See main text for more details on model calibration.