Choice under risk
Prologue

• Uncertainty is a fact of life so people’s attitudes towards risk enter every realm of economic decision-making.

• We *must* study individual behavior with respect to choice involving uncertainty.

• Models of decision making under uncertainty play a key role in every field of economics.
Preferences

Let $X$ be some set of alternatives (consumption set). Formally, we represent the consumer’s preferences by a binary relation $\succeq$ defined on the consumption set $X$.

For any pair of baskets (or bundles) $x$ and $y$, if the consumer says that $x$ is at least as good as $y$, we write $x \succeq y$ and say that $x$ is weakly preferred to $y$.

Bear in mind: economic theory often seeks to convince you with simple examples and then gets you to extrapolate. This simple construction works in wider (and wilder circumstances).
The basic assumptions about preferences

The theory begins with two (not three!) assumptions about preferences

[1] Completeness

\[ x \succeq y \text{ or } y \succeq x \]

for any pair of bundles \( x \) and \( y \).

[2] Transitivity

if \( x \succeq y \) and \( y \succeq z \) then \( x \succeq z \)

or any three bundles \( x, y \) and \( z \).
Together, completeness and transitivity constitute the formal definition of *rationality* as the term is used in economics.

Rational economic agents are ones who [1] have the ability to make choices, and [2] whose choices display a logical consistency.

The preferences of a rational agent can be represented, or summarized, by a *utility function* (more later).
The paternity of decision theory and game theory (1944)
Preferences toward risk

The standard model of decisions under risk (known probabilities) is based on von Neumann and Morgenstern Expected Utility Theory.

Let $X$ be a set of lotteries, or gambles, (outcomes and probabilities). A fundamental assumption about preferences toward risk is independence:

For any lotteries $x, y, z$ and $0 < \alpha < 1$

\[ x \succ y \text{ implies } \alpha x + (1 - \alpha)r \succ \alpha y + (1 - \alpha)r. \]
Experiments à la Allais

Allais (1953) I

– Choose between the two gambles:

\[ A := 0.66 \rightarrow 24,000 \quad \quad B := 1 \rightarrow 24,000 \]

\[ 0.33 \leftrightarrow 25,000 \quad 0.01 \rightarrow 0 \]
Allais (1953) II

- Choose between the two gambles:

\[
\begin{align*}
C := & \quad \frac{.33}{\frac{.67}{\$0}} \\
D := & \quad \frac{.34}{\frac{.66}{\$0}}
\end{align*}
\]
The Marschak-Machina probability triangle

$H$, $M$, and $L$ are three degenerate gambles with certain outcomes $H>M>L$
EUT requires that indifference lines are parallel so one must choose either \( A \) and \( C \), or \( B \) and \( D \).
Contributions

Results have generated the most impressive dialogue between observation and theorizing (Camerer, 1995):

– Violations of EUT raise criticisms about the status of the Savage axioms as the touchstone of rationality.

– These criticisms have generated the development of various alternatives to EUT, such as Prospect Theory.
Limitations

Choice scenarios narrowly tailored to reveal “anomalies” limits the usefulness of data for other purposes:

- Subjects face “extreme” rather than “typical” decision problems designed to encourage violations of specific axioms.

- Small data sets force experimenters to pool data and to ignore individual heterogeneity.
Research questions

Consistency

– Is behavior under uncertainty consistent with the utility maximization model?

Structure

– Is behavior consistent with a utility function with some special structural properties?
Recoverability

– Can the underlying utility function be recovered from observed choices?

Extrapolation

– Given behavior in the laboratory, can we forecast behavior in other environments?
A new experimental design

An experimental design that has a couple of fundamental innovations over previous work:

– A selection of a bundle of contingent commodities from a budget set (a portfolio choice problem).

– A graphical experimental interface that allows for the collection of a rich individual-level data set.
The experimental computer program dialog windows
Rationality

Let \( \{(p^i, x^i)\}_{i=1}^{50} \) be some observed individual data (\( p^i \) denotes the \( i \)-th observation of the price vector and \( x^i \) denotes the associated portfolio).

A utility function \( u(x) \) rationalizes the observed behavior if it achieves the maximum on the budget set at the chosen portfolio

\[
u(x^i) \geq u(x) \text{ for all } x \text{ s.t. } p^i \cdot x^i \geq p^i \cdot x.
\]
Foundations of Economic Analysis (1947)

Paul A. Samuelson (1915-2009) – the first American Nobel laureate in economics and the foremost (academic) economist of the 20th century (and the uncle of Larry Summers...).
Revealed preference

A portfolio \( x^i \) is \textit{directly revealed preferred} to a portfolio \( x^j \) if \( p^i \cdot x^i \geq p^i \cdot x^j \), and \( x^i \) is \textit{strictly directly revealed preferred} to \( x^j \) if the inequality is strict.

The relation \textit{indirectly revealed preferred} is the transitive closure of the directly revealed preferred relation.
Generalized Axiom of Revealed Preference (GARP) If \( x^i \) is indirectly revealed preferred to \( x^j \), then \( x^j \) is not strictly directly revealed preferred (i.e. \( p^j \cdot x^j \leq p^i \cdot x^i \)) to \( x^i \).

GARP is tied to utility representation through a theorem, which was first proved by Afriat (1967).
**Afriat’s Theorem**  The following conditions are equivalent:

- The data satisfy GARP.

- There exists a non-satiated utility function that rationalizes the data.

- There exists a concave, monotonic, continuous, non-satiated utility function that rationalizes the data.
Afriat’s critical cost efficiency index (CCEI) The amount by which each budget constraint must be relaxed in order to remove all violations of GARP.

The CCEI is bounded between zero and one. The closer it is to one, the smaller the perturbation required to remove all violations and thus the closer the data are to satisfying GARP.
The construction of the CCEI for a simple violation of GARP

The agent is 'wasting' as much as \( A/B < C/D \) of his income by making inefficient choices.
The distributions of CCEI scores – two assets

![The distributions of CCEI scores – two assets](image)
The distributions of CCEI scores – three assets
Recoverability

- Revealed preference relations in the data contain the information that is necessary for recovering preferences.

- Varian (ECMA, 1982) uses GARP to generate an algorithm that can recover preferences from choices.

- This approach is purely nonparametric making no assumptions about the parametric form of the underlying utility function.
Risk neutrality
Infinite risk aversion
Loss / disappointment aversion
Decision-making quality

- In economics, heterogeneity in choices is attributed to heterogeneity in preferences, information, beliefs, or constraints.

- Several strands of empirical research consider heterogeneity in choices driven, instead, by differences in the quality of decision-making.

- The revealed preference criterion: if decisions are high quality then there exists a utility function that the choices maximize.
The CentERpanel

- A representative sample of over 2,000 Dutch-speaking households (5,000 individual members) in the Netherlands.

- A wide range of individual socio-demographic and economic information for the panel members.

- The subjects in the experiment were randomly recruited from the entire CentERpanel body.
Types of analysis


[3] The correspondence between wealth accumulation and behavior in the laboratory.
Risk aversion – the fraction of tokens allocated to the cheaper asset
Wealth differentials

The heterogeneity in wealth is not well-explained either by standard observables (income, education, family structure) or by standard unobservables (intertemporal substitution, risk tolerance).

If consistency with utility maximization in the experiment were a good proxy for (financial) decision-making quality then the degree to which consistency differ across subjects should help explain wealth differentials.
<table>
<thead>
<tr>
<th></th>
<th>Ln(hhld wealth)</th>
<th>Ln(hhld income '08)</th>
<th></th>
<th></th>
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<td>1.425** 1.348* 1.781** 1.728**</td>
<td>0.601*** 0.602*** 0.520*** 0.514***</td>
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<td>(0.565) (0.714) (0.746) (0.750)</td>
<td>(0.127) (0.127) (0.121) (0.121)</td>
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<td>-1.361 -1.366</td>
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<td>(0.838) (0.840)</td>
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<td>-1.361 -1.366</td>
<td>0.103</td>
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<td></td>
<td>(0.838) (0.840)</td>
<td>(0.072)</td>
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<td>Conscientiousness</td>
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<tr>
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<td>(0.072)</td>
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<td>Female</td>
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<td>0.000 0.000 0.000 0.000</td>
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<td>(0.000) (0.000) (0.000) (0.000)</td>
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<td>(0.000) (0.000) (0.000) (0.000)</td>
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<tr>
<td>Partner</td>
<td>0.682*** 0.682*** 0.733*** 0.714***</td>
<td>0.183 0.183 0.191 0.191</td>
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<td>0.092 0.092 0.095 0.095</td>
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<td>(0.103) (0.103) (0.095) (0.095)</td>
<td>(0.093) (0.093) (0.095) (0.095)</td>
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<td>Y Y Y Y</td>
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<td>5.862 5.879 5.880 5.841</td>
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<td></td>
<td>(5.862) (5.879) (5.880) (5.841)</td>
<td>(5.862) (5.879) (5.880) (5.841)</td>
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<td>$R^2$</td>
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<td>0.1794 0.1778 0.1801 0.1819</td>
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<td>517 517 494 494</td>
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<tr>
<td></td>
<td>Ln(hhld wealth)</td>
<td>Fraction of hhld wealth</td>
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<tr>
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<tr>
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<tr>
<td>CCEI</td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>1.907**</td>
<td>-0.100*</td>
<td>-0.179*</td>
<td>-0.003</td>
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<td></td>
<td>(0.751)</td>
<td>(0.057)</td>
<td>(0.096)</td>
<td>(0.050)</td>
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<td>0.686***</td>
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<td>-0.062***</td>
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<td>(0.133)</td>
<td>(0.013)</td>
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<td>0.312*</td>
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<td>0.019</td>
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<td>(0.018)</td>
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<tr>
<td>partner</td>
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<td>-0.058*</td>
<td>-0.008</td>
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<tr>
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<td>(0.021)</td>
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<td>(0.014)</td>
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<tr>
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<td>-0.043***</td>
<td>0</td>
</tr>
<tr>
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<td>(0.107)</td>
<td>(0.009)</td>
<td>(0.013)</td>
<td>(0.007)</td>
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<td>Constant</td>
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<td>1.42</td>
<td>-0.287</td>
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<td>(6.032)</td>
<td>(1.197)</td>
<td>(1.388)</td>
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<tr>
<td>$R^2$</td>
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<td>0.104</td>
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<td># of obs.</td>
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</table>
Conclusion

Judgment about the quality of decision-making is generally made difficult by twin problems of *identification* and *measurement*:

- **The identification problem** Distinguishing differences in decision-making quality from unobserved differences in preferences, information, beliefs or constraints.

Identification is important because welfare conclusions and thus (constrained) optimal policy will depend on the sources of any systematic differences in choices.

- **The measurement problem** Defining (and implementing) a *portable, practical, autonomous, quantifiable, and economically interpretable* measure of decision-making quality.
Individual-level data

- A review of the full data set reveals striking regularities within and marked heterogeneity across subjects.

- A quick look at the individual-level data indicates that whatever is going on is not consistent with the standard interpretation of EUT.

- Examples that reveal some of the unexpected features of the data and illustrate the role of heuristics.
Diagonal

Scatter plot of \( x_1 \) and \( x_2 \) when \( \text{Prob}(x_1) = 1/2 \) for ID 304

Scatter plot of \( \log(p_1/p_2) \) and \( x_1 \) when \( \text{Prob}(x_1) = 1/2 \) for ID 304

Scatter plot of \( \log(p_1/p_2) \) and \( x_1 \) when \( \text{Prob}(x_1) = 1/2 \) for ID 304
Diagonal and boundary

Scatter plot between x1 and x2 when Prob(x1) = 1/2 for ID 307
Diagonal, boundary and smooth
Diagonal, boundary and smooth

Scatter plot between x1 and x2 when Prob(x1) = 1/2 for ID 327

Scatter plot of log(p1/p2) and x1/x1+x2 when Prob(x1) = 1/2 for ID 327

Scatter plot of log(p1/p2) and p1/x1/(p1/x1+p2/x2) when Prob(x1) = 1/2 for ID 327
Center
The Token Shares in the Risk Treatment for Subject ID: 922

Vertex
The Token Shares in the Risk Treatment for Subject ID: 913
Centroid (budget shares)
The Budget Shares in the Risk Treatment for Subject ID 1001

Edge
The Token Shares in the Risk Treatment for Subject ID 906
**Bisector**

The Token Shares in the Risk Treatment for Subject ID 1003

**Center and bisector**

The Token Shares in the Risk Treatment for Subject ID 1007
Edge and bisector

The Token Shares in the Risk Treatment for Subject ID: 1016

Center, vertex, and edge

The Token Shares in the Risk Treatment for Subject ID: 1023
Vertex and edge

Center and bisector