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INCOME TAX EVASION: A THEORETICAL ANALYSIS

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1. Introduction

Theoretical analysis of the connection between taxation and risk-taking has mainly been concerned with the effect of taxes on portfolio decisions of consumers, Mossin (1968b) and Stiglitz (1969). However, there are some problems which are not naturally classified under this heading and which, although of considerable practical interest, have been left out of the theoretical discussions. One such problem is tax evasion. This takes many forms, and one can hardly hope to give a completely general analysis of all these. Our objective in this paper is therefore the more limited one of analyzing the individual taxpayer's decision on whether and to what extent to avoid taxes by deliberate underreporting. On the one hand our approach is related to the studies of economics of criminal activity, as e.g. in the papers by Becker (1968) and by Tulkens and Jacquemin (1971). On the other hand it is related to the analysis of optimal portfolio and insurance policies in the economics of uncertainty, as in the work by Arrow (1970), Mossin (1968a) and several others.

We shall start by considering a simple static model where this decision is the only one with which the individual is concerned, so that we ignore the interrelationships that probably exist with other types of economic choices. After a detailed study of this simple case (sections

* Tax evasion as a topic for theoretical investigation was suggested by J.A. Mirrlees (1971) in a paper prepared for the International Economic Association's Workshop in Economic Theory, which was held in Bergen in the summer of 1971. This provided us with the initial stimulus to write the present article. We have received valuable comments and suggestions from A.B. Atkinson, Karl Borch, Jacques Drèze, Leif Johansen and a referee.

2–4) we proceed with an analysis of the dynamic case where the individual has to make a sequence of tax declaration decisions (section 5). We conclude (section 6) with an informal discussion of some further problems in this field, including the optimal design of tax systems.

2. The nature of the optimum

The tax declaration decision is a decision under uncertainty. The reason for this is that failure to report one's full income to the tax authorities does not automatically provoke a reaction in the form of a penalty. The taxpayer has the choice between two main strategies: (1) He may declare his actual income. (2) He may declare less than his actual income. If he chooses the latter strategy his payoff will depend on whether or not he is investigated by the tax authorities. If he is not, he is clearly better off than under strategy (1). If he is, he is worse off. The choice of a strategy is therefore a non-trivial one.

We shall assume that the tax-payer's behaviour conforms to the Von Neumann–Morgenstern axioms for behaviour under uncertainty. His cardinal utility function has income as its only argument; this must be understood as the indirect utility function with constant prices. Marginal utility will be assumed to be everywhere positive and strictly decreasing, so that the individual is risk averse.

Actual income, W, is exogenously given and is known by the taxpayer but not by the government's tax collector.[1] Tax is levied at a constant rate, θ, on declared income, X, which is the taxpayer's decision variable. However, with some probability p the taxpayer will be subjected to investigation by the tax authorities, who will then get to know the exact amount of his actual income. It this happens the taxpayer will have to pay tax on the undeclared amount, W-X, at a penalty rate π which is higher than θ.

This formal representation of the taxpayer's choice situation is in some ways a significant simplification of his real world situation; in particular, the present formulation ignores some of the uncertainty elements. First, it abstracts from the fact that the tax laws to some

[1] The analysis would be essentially unchanged if we were to assume (more realistically) that a part of the actual income were known by the government. Clearly, it would never pay to try to avoid taxes on that part, so the analysis would then be valid for that part of actual income which is unknown by the government.

extent leave it to the discretion of the courts to determine whether the penalty will be of the type discussed here or take the form of a jail sentence; it may also be a combination of both. Second, even if jail is not an alternative, the penalty rate θ may itself be uncertain from the point of view of the taxpayer. Even though we ignore these points, we hope to have retained enough of the structure of the problem to make the theoretical analysis worthwhile.

The taxpayer will now choose X so as to maximize

$$E[U] = (1-p)\,U(W-\theta X) + pU(W-\theta X-\pi(W-X))\,. \tag{1}$$

For notational convenience we define

$$\begin{aligned} Y &= W - \theta X\,, \\ Z &= W - \theta X - \pi(W-X)\,. \end{aligned} \tag{2}$$

The first-order condition for an interior maximum of (1) can then be written as

$$-\theta(1-p)\,U'(Y) - (\theta-\pi)\,pU'(Z) = 0\,. \tag{3}$$

The second-order condition

$$D = \theta^2(1-p)\,U''(Y) + (\theta-\pi)^2 pU''(Z)\,, \tag{4}$$

is satisfied by the assumption of concavity of the utility function.

In this analysis the conditions for an interior maximum to exist are of particular importance. Clearly, it cannot be assumed a priori that $0 < X < W$, because whether or not this will be true, should depend on the values of the parameters. To see what conditions on parameter values are required for an interior solution we evaluate expected utility at $X = 0$ and $X = W$. Since expected marginal utility is decreasing with X, we must have that

$$\left.\frac{\partial E[U]}{\partial X}\right|_{X=0} = -\theta(1-p)\,U'(W) - (\theta-\pi)\,pU'(W(1-\pi)) > 0 \tag{5}$$

and

$$\left.\frac{\partial E[U]}{\partial X}\right|_{X=W} = -\theta(1-p)\,U'(W(1-\theta)) - (\theta-\pi)pU'(W(1-\theta)) < 0\,. \tag{6}$$

These conditions can be rewritten as

$$p\pi > \theta\left[p + (1-p)\,\frac{U'(W)}{U'(W(1-\theta))}\right], \tag{5'}$$

$$p\pi < \theta\,. \tag{6'}$$

(6′) implies that the taxpayer will declare less than his actual income if the expected tax payment on undeclared income is less than the regular rate. Since the bracketed factor in (5′) is obviously positive and less than one, the two conditions do give us a set of positive parameter values which will guarantee an interior solution. It is with such solutions that we shall be concerned in later selections.

This is a very simple theory, and it may perhaps be criticized for giving too little attention to nonpecuniary factors in the taxpayer's decision on whether or not to evade taxes. It need hardly be stressed that in addition to the income loss there may be other factors affecting utility if one's attempt at tax evasion is detected. These factors may perhaps be summarily characterized as affecting adversely one's reputation as a citizen of the community; we may represent this by an additional variable, s, in the utility function. We now write expected utility as

$$E[U] = (1-p)\,U(Y,s_0) + pU(Z,s_1)\,. \tag{7}$$

Thus, the variable s takes on different values according to what state of the world obtains (whether or not the evasion is detected). As a convention we assume $U(Y, s_0) > U(Y, s_1)$. The first-order condition is then

$$-\theta(1-p)\,U_1(Y,s_0) - (\theta-\pi)pU_1(Z,s_1) = 0\,, \tag{8}$$

where U_1 now denotes the derivative of U with respect to the income variable. Of special interest is now the condition on parameter values which must hold for $X < W$. Proceeding as in the cases studied above we obtain this condition as

$$p\pi < \theta\left[p + (1-p)\frac{U_1(W(1-\theta), s_0)}{U_1(W(1-\theta), s_1)}\right]. \tag{9}$$

Observe first that (9) reduces to (6′) if $U_1(W(1 - \theta), s_0) = U_1(W(1 - \theta), s_1)$, so that a change in the state variable leaves the *marginal* utility of income unaffected. The most natural assumption is perhaps $U_1(W(1 - \theta), s_0) < U_1(W(1 - \theta), s_1)$; a better reputation decreases the marginal utility of income so that "reputation" and income are substitutes in the cardinal sense. This would make the expression in brackets in (9) less than one and the right-hand side of the inequality less than θ, so that the condition for "profitable" tax evasion would become stricter. Depending on the value of $U_1(W(1 - \theta), s_0)/U_1(W(1 - \theta), s_1)$, one might observe different "break-even" values of the parameters for different taxpayers.[2]

3. Comparative static results

We shall now examine the way in which reported income depends on the parameters of the model, W, θ, π and p. We shall do this using the simpler of the two models above, in which the only argument in the taxpayer's utility function is his net income. This does represent some simplification of the argument compared to the alternative model, in so far as the various derivatives with respect to income will depend upon the value of s. The reader will notice that some but not all of our results are affected by this simplification. Moreover, if the reader is prepared to accept the view that the influence on e.g. the relative risk aversion function of a change in s is insignificant compared to the effect of a change in income, then the results reported here can be seen as approximative results for the more complicated model.

We shall make use of the well-known Arrow–Pratt risk aversion measures to evaluate our results. These are the absolute and the relative risk aversion functions, defined as

$$R_A(Y) = -\frac{U''(Y)}{U'(Y)}, \qquad R_R(Y) = -\frac{U''(Y)Y}{U'(Y)}, \tag{10}$$

[2] One should be aware that in a cross-section of taxpayers θ and π might also vary considerably if they are interpreted as *marginal* tax rates. One might also expect the subjective assessment of the probability of detection to differ a lot between taxpayers.

respectively. (The functions could of course equally well have been written with Z or any income variable as the argument.)

There seems to be a general presumption that absolute risk aversion is decreasing with income; the case of relative risk aversion is more complicated, and we shall not commit ourselves to any specific hypothesis as to its shape.[3] Differentiating (3) with respect to W and solving for $\partial X/\partial W$, we obtain

$$\frac{\partial X}{\partial W} = \frac{1}{D}\left[\theta(1-p)\,U''(Y) + (\theta-\pi)(1-\pi)\,pU''(Z)\right]. \qquad (11)$$

Substituting from (3) we can rewrite this as

$$\frac{\partial X}{\partial W} = -\frac{1}{D}\,\theta(1-p)\,U'(Y)\left[-\frac{U''(Y)}{U'(Y)} + (1-\pi)\frac{U''(Z)}{U'(Z)}\right]$$

or, using (10),

$$\frac{\partial X}{\partial W} = -\frac{1}{D}\,\theta(1-p)\,U'(Y)[R_A(Y) - (1-\pi)R_A(Z)]. \qquad (12)$$

On the assumption of decreasing absolute risk aversion $R_A(Y) < R_A(Z)$. However, the sign of the bracketed expression depends on the value of π. Only in the case of $\pi \geqslant 1$ can we conclude that the derivative is unambigously positive.

It is perhaps of somewhat greater interest to study the sign of the derivative $\partial(X/W)/\partial W$; i.e. how does the fraction of actual income declared vary as actual income changes? Since we have that

$$\frac{\partial(X/W)}{\partial W} = \frac{1}{W^2}\left(\frac{\partial X}{\partial W}W - X\right),$$

we can substitute from (11) and (4) to obtain

[3] For a lucid discussion of these measures see Arrow (1970). They have been used in the analysis of taxation and risk-taking by Mossin (1968b) and Stiglitz (1969).

$$\frac{\partial(X/W)}{\partial W} = \frac{1}{W^2}\frac{1}{D}\Big[\theta(1-p)\,U''(Y)W + (\theta-\pi)(1-\pi)pU''(Z)W$$

$$- \theta^2(1-p)\,U''(Y)X - (\theta-\pi)^2 pU''(Z)X\Big] .$$

Collecting terms and substituting from (2) we can write

$$\frac{\partial(X/W)}{\partial W} = \frac{1}{W^2}\frac{1}{D}\,[\theta(1-p)\,U''(Y)Y + (\theta-\pi)pU''(Z)Z] .$$

We can now substitute in this expression from the first-order condition (3). This yields

$$\frac{\partial(X/W)}{\partial W} = -\frac{1}{W^2}\frac{1}{D}\,\theta(1-p)\,U'(Y)[R_R(Y) - R_R(Z)] . \tag{13}$$

We can then conclude that when actual income varies, the fraction declared increases, stays constant or decreases according as relative risk aversion is an increasing, constant or decreasing function of income.

It is not easy to select one of these hypotheses about the relative risk aversion function as the most realistic one. We shall therefore be content with adding this result to those of a similar nature that already exist in the economics of uncertainty. However, it is of some interest in itself to observe that even a model a simple as the present one does not generate any simple result concerning the relationship between income and tax evasion.

We now differentiate (3) with respect to θ. This yields

$$\frac{\partial X}{\partial \theta} = -\frac{1}{D}X[\theta(1-p)\,U''(Y) + (\theta-\pi)pU''(Z)]$$

$$+\frac{1}{D}\,[(1-p)\,U'(Y) + pU'(Z)] .$$

Substituting from (3) we can rewrite this as

$$\frac{\partial X}{\partial \theta} = \frac{1}{D}\,X\theta(1-p)\,U'(Y)[R_A(Y) - R_A(Z)]$$

$$+\frac{1}{D}\,[(1-p)\,U'(Y) + pU'(Z)] . \tag{14}$$

The second of the two terms on the right is unambiguously negative. The first term is positive, zero or negative according as absolute risk aversion is decreasing, constant or increasing. Of these decreasing absolute risk aversion seems to be the most attractive assumption, but we must then conclude that no clearcut hypothesis emerges as to the connection between the regular tax rate and reported income.

The economic meaning of this result is best seen if we regard the two terms in (14) as the income effect and the substitution effect, respectively. The latter is negative because an increase in the tax rate makes it more profitable to evade taxes on the margin. The former is positive because an increased tax rate makes the tax payer less wealthy, reducing both Y and Z for any level of X, and this, under decreasing absolute risk aversion, tends to reduce evasion.

The next question we investigate is how reported income depends on the penalty rate. From (3) we get

$$\frac{\partial X}{\partial \pi} = -\frac{1}{D}(W-X)(\theta-\pi)pU''(Z) - \frac{1}{D}pU'(Z)\,. \tag{15}$$

These terms are both positive, so that an increase in the penalty rate will always increase the fraction of actual income declared.

Finally, we differentiate (3) with respect to p to obtain

$$\frac{\partial X}{\partial p} = \frac{1}{D}\left[-\theta U'(Y) + (\theta-\pi)U'(Z)\right]\,. \tag{16}$$

This derivative is positive; an increase in the probability of detection will always lead to a larger income being declared.

Summing up the comparative static analysis of our model, we may note that although it does not yield any clear-cut results in the analysis of changes in actual income and in the tax rate, unambiguous results can be derived for the two parameters of the model which are of particular interest for policy purposes in this field, viz. the penalty rate and the probability of detection. The former is a parameter over which the tax authority exercises direct control; the latter it may be assumed to control indirectly through the amount and efficiency of resources spent on detecting tax evasion. The model implies that these two policy tools are substitutes for each other. While the expected tax yield would fall with a decrease of p, the loss of tax revenue could be compensated by an increase of π.

4. Variable probability of detection

We have assumed the probability of detection to be exogenously given to the individual taxpayer; consequently it is independent of the amount of income he reports. This may not be entirely satisfactory, but a natural hypothesis on the nature of the dependence does not immediately suggest itself. If we write $p = p(X)$, should $p'(X)$ be positive or negative? On the one hand the tax authorities might believe that the rich are most likely to evade taxes, thus making $p'(X) > 0$. On the other hand they might base their policy on the statistical hypothesis that in the absence of any knowledge about actual income, a person with a low reported income is more likely to be an evader; the tax authorities would then formulate a rule according to which $p'(X) < 0$.

It seems difficult to choose between these two hypotheses unless we introduce the further assumption that although the tax authorities do not know the taxpayer's actual income they do know his profession, and they have some ideas about normal incomes in the various professions. They would then formulate a $p(X)$ function for each profession, and each such function would have $p'(X) < 0$; a person reporting an income below the average of his profession is more likely to be investigated than one reporting an income above the average. This might well be consistent with the first of the two hypotheses mentioned above, since the $p(X)$ functions might shift upward with increasing average professional income. Within our framework of individual choice $p'(X) < 0$ seems the more natural hypothesis and will be adopted in the following.

It is interesting to see how this added complication affects our comparative static results. Expected utility must now be written as

$$E[U] = [1 - p(X)]\, U(Y) + p(X)\, U(Z)\,, \tag{17}$$

and the first-order condition becomes

$$\begin{aligned} &-p'(X)\, U(Y) - \theta[1 - p(X)]\, U'(Y) \\ &+p'(X)\, U(Z) - (\theta - \pi)\, p(X)\, U'(Z) = 0\,. \end{aligned} \tag{18}$$

One small problem arises now because the dependence of p on X might create non-concavities in $E[U]$. Although we shall only be concerned with local properties of $E[U]$ we may as well eliminate this problem by assuming very naturally that $p''(X) \geqslant 0$. Then all terms in the second-order derivative, which we shall now write as D^*, will be negative.

We now limit ourselves to an investigation of the effect of changes in the two policy parameters which are presumably most relevant for the control of tax evasion, viz. the penalty rate and the probability of detection. Differentiating (18) with respect to π yields

$$\frac{\partial X}{\partial \pi} = -\frac{1}{D^*}(W-X)(\theta-\pi)p(X)U''(Z) - \frac{1}{D^*}p(X)U'(Z) + \frac{1}{D^*}(W-X)p'(X)U'(Z) \,. \tag{19}$$

The first two terms on the right correspond to the two terms in (15) and are both positive. The dependence of p on X adds a third term which is also unambiguously positive. The conclusion from the simpler model therefore carries over; a rise in the penalty rate will lead to an increase in declared income.

Our previous derivative $\partial X/\partial p$ has no direct counterpart in the present model, since p is now endogenously determined. However, it is possible to study a shift in the $p(X)$ function, e.g. by writing it as $p(X) + \epsilon$, differentiating with respect to ϵ and evaluating the derivative at $\epsilon = 0$. The result is then

$$\frac{\partial X}{\partial \epsilon} = \frac{1}{D^*}[-\theta U'(Y) + (\theta-\pi)U'(Z)] \,, \tag{20}$$

which is an expression of exactly the same form as the previous one in (16) and therefore positive. A positive shift in the $p(X)$ function will increase declared income and reduce tax evasion.

5. The dynamic case

We now leave the problem where the individual has only to make one declaration, or where his problems in different time periods are independent, and consider the more general case where the individual must make a sequence of (interrelated) decisions. Essentially, the problem arises because it is plausible that if the individual is discovered cheating today he will be investigated, and thus if he was cheating discovered, for yesterday.

The purpose of this section is to investigate the dynamic rather than the comparative static aspects of his declarations: for example whether for fixed parameters (tax rates, etc.) his declarations will increase or decrase over time, rather than whether in a fixed period the declaration will increase or decrase if a parameter is changed. The latter question is still of interest, but as it may be investigated using the methods of the previous sections is not discussed here.

We work in discrete time, and to simplify the analysis we assume that the individual has an infinite life expectancy; as we shall see the individual breaks his planning period down into a number of finite length subperiods, so this involves no serious loss of generality – except for the individual who would be near the end of his life. To abstract from other problems we will also assume that the individual has no time preference, and does not anticipate or postpone income by borrowing or saving.

To formalize our rationale for examining the dynamic problem we continue to assume that there is a fixed probability p of the individual's being discovered in period t evading tax in that period, if he does evade. However, if he is discovered in period t he is now investigated, and therefore discovered, for all preceding periods back to the time when he last paid the full amount – either voluntarily or because he was discovered. The individual has a fixed income in all periods, which, as we no longer change, we shall normalize to unity; he may, however, vary the amount he declares X_t in each period t (measured from some time when a full declaration was last made), provided that he neither declares a negative income nor more than he receives, that is $0 \leqslant X_t \leqslant 1$. Now if the individual is not discovered in period t his post tax income is simply

$$Y_t = 1 - \theta X_t \,, \tag{21}$$

while if he is discovered he must pay a penalty on all he has evaded since the time when he last paid the full amount, so his post-tax income is

$$Z_t = 1 - \theta X_t - \Pi \sum_{\tau=1}^{t} (1 - X_\tau) \,. \tag{22}$$

Such a penalty rule is clearly arbitrary, but is not without interest. Investigation is obviously costly for the authorities (or otherwise $p = 1$), and a plausible rule-of-thumb for rationing these investigations would be to make a preliminary random investigation in each period, and then continue investigating backwards as long as this yielded some revenue, stopping when it did not. Our rule would be consistent with such behaviour. We latter comment on the alternative of investigating the entire past of a discovered evader, but ignore the possibility of his being discovered today affecting his future p.

The problem involves interrelationships between declarations at different times in two ways: firstly today's decision must be influenced by past declarations, since these determine the penalty if caught; and secondly, a decision to cheat today involves mortgaging the future, since the stochastic penalty is in effect delayed. Before considering the consistent individual who appreciates both of these interdepencies it is constructive to consider the simpler case of the myopic individual who appreciates only the first.

The myopic individual then ignores the effect of his actions today on his future, and as he must take the past as given he is essentially in a static framework. This case is, however, worth a brief examination here as the form of the problem is slightly different to that of the static problem (specifically the penalty is no longer proportional to the underdeclaration), and also as it simply illustrates the concepts we shall be interested in.

The questions we shall consider are the following: (1) whether the individual will initially make a partial underdeclaration, that is whether $0 < X < 1$; (2) whether in some period, say T, he will declare his full income, that is whether there exists a T such that $X_T = 1$; and (3) whether his declarations will increase (or decrease) over time, that is whether $X_t > X_s$ for $t > s$. These three properties essentially define the qualitative nature of his declaration path over time.

Since the myopic individual ignores the future and must take the past as given he maximizes, in each period t, the expected value of his utility level in that period, that is

$$E[U_t] = (1-p)\,U(Y_t) + pU(Z_t)\,, \tag{23}$$

where Y_t (post-tax income if not discovered) and Z_t (post-tax income if discovered) are as defined above.

We first note that for $t = 1$ we have $Y_1 = 1 - \theta X_1$ and $Z_1 = 1 - \theta X_1 - \Pi(1 - X_1)$, so the problem is identical to the static case. It follows that there will be situations where an initial partial underdeclaration is made, which answers our first question; these are the interesting cases which we shall consider.

To answer our second question we first show that X_t does not tend asymptotically to 1 (for we later require inf $\{1 - X_t\} > 0$). If $X_t < 1$ for all t it is clear that

$$(1-p)\,U(Y_t) + pU(Z_t) > U(1-\theta)\,. \tag{24}$$

Then taking $t \to \infty$ we would have, if $X_t \to 1$,

$$pU\left(1 - \theta - \Pi \sum_1^\infty (1-X_\tau)\right) > pU(1-\theta)\,,$$

which would imply $\Sigma_1^\infty\ (1 - X_\tau) < 0$, which is impossible as $X_t < 1$. Having cleared up this minor point we now show that the left side of (24) tends to minus infinity; since the right side is constant this means that (24) cannot hold for all t, so that at some t, $X_t = 1$. The first term $(1 - p)\ U\ (Y_t) \leqslant (1 - p)\ U\ (1)$ is clearly bounded. In the second we have

$$Z_t = 1 - \theta X_t - \Pi \sum_1^t (1-X_\tau) \leqslant 1 - \theta X_t - \Pi t K \to -\infty\,,$$

where $K = \inf \{1 - X_\tau\} > 0$; it follows that the second term itself, $U(Z_t)$, and thus the left side, tend to minus infinity.

The answer to our third question is in the affirmative, that is declarations increase over time. To show this we first digress to consider our simple static model with a fixed penalty C, so we have $Z = 1 - \theta X - \Pi(1 - X) - C$, rather than $Z = 1 - \theta X - \Pi(1-X)$. Proceeding as in section 3 we may obtain the effect of a change in this fixed penalty as

$$\frac{\partial X}{\partial C} = -\frac{1}{D}(\theta - \Pi)\,pU''(Z) > 0\,.$$

By integrating it follows that if $C' > C$ then $X' > X$ (providing X and X' are both interior solutions). The relevance of this to our problem is immediate, for the passage of time is equivalent to the increase of a fixed penalty; this is because we may always write Z_t as $1 - \theta X_t - \Pi(1-X_t) - C_t$ where $C_t = \Pi \sum_1^{t-1} (1-X_\tau)$. Since $C_{t+1} = C_t + \Pi(1-X_t) > C_t$ it follows that $X_{t+1} > X_t$.

We now turn to the consistent individual, and recall that the essential difference between this and the myopic is that the consistent appreciates that by cheating today he is placing himself in a worse position tomorrow. Because of this we may obtain at least intuitive ideas on his declaration path from that of the myopic individual.

Since the individual considers the whole of the future he maximizes lifetime utility, which we specify to take the simple form $\sum_1^\infty E[U_t]$. When this infinite sum does not converge, which will typically be the case, we specify that he maximizes $1/T \sum_1^T E[U_t]$, where T is some time which divides the future into independent periods; specifically, $E[U_s]$ is independent of X_t if $s \leq T < t$. Clearly T will be the period when the individual first plans to declare his full income, or alternatively is discovered. For this to be well-defined we must of course ensure that $T < \infty$, but this is simply our second question, which we consider below.

First, however, we note that an initial partial underdeclaration is possible, so $0 < X_1 < 1$, or equivalently, $T > 1$. This may be shown in exactly the same way as in the static or myopic cases.

To answer our second question, we use the analysis for the myopic individual. If in some period t the taxpayer declares his full income the sum of his future expected utilities is

$$[U(1-\theta)] + [(1-p)\,U(1-\theta X_{t+1}) + pU(1-\theta X_{t+1} - \Pi(1-X_{t+1}))] + \dots, \tag{25}$$

while if $X_t < 1$ the corresponding sum, if he is not discovered in period t, is

$$[(1-p)\,U(Y_t) + pU(Z_t)] + [(1-p)\,U(1-\theta X'_{t+1}) + pU(1-\theta X'_{t+1} - \Pi(1-X'_{t+1}) - \Pi \sum_1^t (1-X_\tau))] + \dots \tag{26}$$

though if he is discovered it is

$$[(1-p)\,U(Y_t)+pU(Z_t)] \\ +\,[(1-p)\,U(1-\theta X''_{t+1})+pU(1-\theta X''_{t+1}-\Pi(1-X''_{t+1}))]\,+\ldots\,. \tag{27}$$

In these expressions X_{t+1}, X'_{t+1}, and X''_{t+1} are the respective optimal declarations in period $t+1$; of course he only knows if he is discovered in period t at the end of that period, so the probabilistic nature of the first terms in (26) and (27) make sense. In parallel with the myopic individual it is clear that T is finite if the individual terms of (25) are not less than the corresponding terms of both (26) and (27), with strict inequality from some term. Now it is clear that the second term in (26) is less than the second term in (25), for it is the highest expected utility level achievable with the positive fixed penalty $\Pi\Sigma_1^t(1-X_\tau)$ as opposed to that achievable with zero fixed penalty; the second term in (27) is of course equal to that in (25). This argument may be repeated for all subsequent terms, so the (weak inequality) condition is fulfilled for all terms beyond the first. For the first term, however, the myopic argument immediately tells us that the (strong inequality) condition is fulfilled for some t. It follows that T is finite.

An interesting corollary to this, apart from the choice process being well-defined, is that the consistent individual will always declare more than the myopic: it is then indeed "short-sighted to evade taxes". Finally, if the individual knows that once he is discovered his whole past will be investigated, his behaviour is straightforward: he will act exactly as he would in the case we have considered until period T, and thereafter declare everything.

6. Concluding remarks

We have examined some static and dynamic aspects of the decision to evade income taxes. The model we have used is clearly rather special, and we can claim no more for it than that it seems to yield some insight into the structure of the problem. We also hope that the approach will suggest other topics for research in the field, both theoretical and empirical.

Of theoretical topics the ones which immediately suggest themselves are perhaps various generalizations of the present model. One possibility is to extend the model to take account of labour supply decisions;

one might hope to discover some interesting connections between incentives to avoid taxes and to supply work effort. However, although we have studied this case, we have not been able to come up with any interesting and reasonably simple results. Another possible extension would be to incorporate saving and portfolio decisions. It might also be worthwhile to analyze more complicated income tax schemes than the simple proportional case which we have examined.

It would also be of interest to see a discussion of tax evasion within the framework of optimal taxation theory. This theory assumes of course that there is no evasion whatever. One conclusion which is classic is that to promote an efficient allocation of resources taxes should be levied primarily on commodities that are inelastic in demand or supply. In particular, it seems to be widely agreed that an income tax is the best means by which to effect a redistribution of incomes if labour is perfectly inelastic in supply. This conclusion stands in obvious need of modification if it is realized that an income tax probably offers much larger opportunities for tax evasion than commodity taxes do. The policy tools available to the government for the purpose of counteracting the tendency to evasion are the tax rates themselves, the penalty rates and the expenditure on investigation, which determines the probability of being detected. To assess the efficiency of these tools one would need empirical estimates of the effects discussed in this paper.

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