Comments on "Estimation of a Stochastic Model of Reproduction: An Econometric Approach" *

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JAMES HECKMAN and Robert Willis have embarked on a program of research that extends economic rationality to the bedroom and promises to make them the Masters and Johnson of economics. Their paper contains some ingenious ideas for modeling and estimation; ideas that are potentially useful in other areas of economics where discrete choices or outcomes occur. On the other hand, the authors' empirical results are not an unmixed success, and fail to establish that the technology of childbearing is a fertile area for application of the theory of rational behavior. The lack of significance of economic factors in explaining fecundability suggests that this paper may be approaching the outer limits of the universe of the new home economics.

My discussion will be divided into three parts. First, I shall comment on the modeling of the childbearing technology. Second, I shall make a few observations on the statistical methods developed in the paper. Third, I shall comment on the empirical results.

A. MODELING FECUNDABILITY

The authors stress the sequential, uncertain nature of contraception and childbearing decisions and suggest that a realistic model of the reproductive history of the household must take into account the stream of incoming information on child quality, income, and occupational opportunities. Second, the authors emphasize the imperfection of contraceptive techniques and their cost, which suggests that particular stress must be placed empirically on contraceptive method and regularity of use. Third, the authors recognize the importance of

* This comment was prepared after extensive discussions with Donald Sant of the University of California, Berkeley, who is responsible for the empirical results reported here.

variation across the population in "natural" fertility and the role such variability plays in determining the way to model behavior and what statistical methods to adopt.

It is only with respect to the last point that the authors have fully succeeded in achieving the theoretical desiderata in their empirical analysis. Neither the consumer's optimization model written down by the authors to motivate the estimated equations nor the choice of independent variables in the empirical analysis conforms well to a model of sequential information gathering. For example, predicted income at age 40 is used as an independent variable in the empirical analysis; the possibility is precluded that information on income evolves over the first ten years of marriage. Second, the authors make no use of the choice-of-technique data in the National Fertility Study, distinguishing only contraceptors and noncontraceptors. The decision to contracept and, if so, what technique to use should be thought of as behavior jointly determined with the monthly probability of conception. There are no conceptual problems, although there may be statistical ones, involved in looking at monthly probabilities of conception conditioned on the decision to contracept, as the authors do now. One could, in fact, go further and look at these probabilities conditioned on choice of technique. However, this analysis leaves the decision to contracept unexplained.

One can formulate a model of the joint events of choice of contraceptive technique j (with no contraception being one alternative, $j = 0$) and conception, with the schematic form

$$(1)\quad \left[\begin{array}{l}\text{Probability}\\ \text{of choosing}\\ \text{contraceptive}\\ \text{technique } j\end{array}\right] = f\left(\left[\begin{array}{l}\text{Socio-}\\ \text{economic}\\ \text{variables}\end{array}\right], \left[\begin{array}{l}\text{Costs of each}\\ \text{contraceptive}\\ \text{technique}\end{array}\right], \left[\begin{array}{l}\text{Natural}\\ \text{fecund-}\\ \text{ability}\end{array}\right],\right.$$

$$\left.\left[\begin{array}{l}\text{Reliability}\\ \text{of each}\\ \text{contraceptive}\\ \text{technique}\end{array}\right], \left[\begin{array}{l}\text{Safety of each}\\ \text{contraceptive}\\ \text{technique}\end{array}\right], \left[\begin{array}{l}\text{Nuisance}\\ \text{cost of each}\\ \text{contraceptive}\\ \text{technique}\end{array}\right], \ldots\right)$$

$$(2)\quad \left[\begin{array}{l}\text{Monthly}\\ \text{probability}\\ \text{of conception}\\ \text{conditioned on}\\ \text{contraceptive}\\ \text{technique}\end{array}\right] = g\left(\left[\begin{array}{l}\text{Socio-}\\ \text{economic}\\ \text{variables}\end{array}\right], \left[\begin{array}{l}\text{Choice of}\\ \text{contra-}\\ \text{ceptive}\\ \text{technique}\end{array}\right], \left[\begin{array}{l}\text{Natural}\\ \text{fecund-}\\ \text{ability}\end{array}\right], \ldots\right)$$

Equation 1 gives the frequencies of choosing alternative contraceptive techniques in the subpopulation facing specified values of the right-hand-side variables. Equation 2, the one considered by the authors, gives the monthly probability of conception for the subpopulation with a specified natural fecundability who choose specific contraceptive techniques. Natural fecundability is unobserved. One can think of generating a sample by first specifying the observed socioeconomic variables and characteristics of contraceptive techniques; second, drawing a natural fecundability level from the distribution of this variable in the subpopulation with the observed independent variables; third, drawing a contraceptive technique from the multinomial distribution with probabilities given by equation 1, and fourth, drawing sequentially from the negative binomial distribution with probabilities given by equation 2 until pregnancy occurs, or the experiment period ends. This description could be formalized to yield a likelihood function from which maximum likelihood estimators of the parameters of the functions in equations 1 and 2 could be obtained. It should be noted that consideration of equation 2 above may lead to statistical difficulties unless the event of the choice of contraceptive technique is independent of natural fecundability.

B. STATISTICAL METHODS

The authors' treatment of variation of natural fertility in the population and their development of statistical methods to fit this structure deserve special commendation. There has been a tendency in the new home economics to emulate the traditional practice in consumer theory: treating the choices of a population as if they were generated by a single "representative" consumer. Thus, for example, a model of choice of number of children is postulated with a representative consumer demanding, say, 2.2 children, with price and income elasticities determined by the usual marginal calculations. In fact, the population is made up of some families with two children and some with three, depending on tastes, and the effect of price and income changes is at the *extensive* margin, where families switch from two to three. Heckman and Willis do a great service by abandoning this tradition of the representative consumer and modeling explicitly the binary nature of the observed outcome.

An important aspect of this work is the recognition of variability of natural fecundability across women and (to a lesser extent) across time for the same woman. The authors introduce a "components of variance" structure to account for this unobservable effect; one of the most interesting of the empirical findings is the confirmation of the existence of these differences. It should be pointed out that this topic is one of considerable current interest. Recent related papers have been circulated by Gary Chamberlain and Zvi Griliches [2], Arthur Goldberger [3], and Robert Hall [4].

The normally distributed error components assumed by the authors lead to their forbidding equation 22, giving the monthly probability of conception. Alternative distributional assumptions are equally plausible and lead to forms which are easier to analyze and utilize in iterative statistical procedures. Let $P(\alpha,\epsilon)$ denote the monthly probability of *not* conceiving, where α is a term summarizing the observed socioeconomic factors and ϵ is unobserved natural fecundability. Let $h(\epsilon)$ denote the frequency distribution of ϵ in the population. Then, the probability of *not* conceiving for j months is given by

$$Q_j = \int_0^\infty P(\alpha, \epsilon)^j h(\epsilon) d\epsilon$$

Now suppose that $P(\alpha, \epsilon) = e^{-\alpha-\epsilon}$, where $\alpha > 0$, $\epsilon > 0$, and suppose that ϵ has a gamma distribution with mean one and variance v,

$$h(\epsilon) = \frac{v^{-1/v}\epsilon^{(1/v)-1}e^{-\epsilon/v}}{\Gamma(1/v)}$$

Then,

$$Q_j = \frac{\displaystyle\int_0^\infty e^{-j\alpha-j\epsilon}v^{-1/v}\epsilon^{(1/v)-1}e^{-\epsilon/v}d\epsilon}{\Gamma(1/v)} = \frac{e^{-j\alpha}}{(1+jv)^{1/v}}$$

The monthly probability of conception is then

$$P_j = Q_j - Q_{j+1} = \frac{e^{-j\alpha}}{(1+jv)^{1/v}} - \frac{e^{-(j+1)\alpha}}{(1+(j+1)v)^{1/v}}$$

While this expression is quite nonlinear in v and α, it avoids the difficulties of manipulating integrals and performing numerical integration. For example, one easily sees that the conditional probability of *not* becoming pregnant in period j,

$$Q_{j+1}/Q_j = \frac{e^{-\alpha}}{\left(1 + \dfrac{v}{1+jv}\right)^{1/v}}$$

increases as j increases if, and only if, $v > 0$.

C. EMPIRICAL RESULTS

In the joint determination of the technique of contraception and the occurrence of pregnancy outlined earlier in these comments, it is clear that economic choice, conditioned on costs and socioeconomic characteristics, will influence primarily the choice of contraceptive technique. Except for socioeconomic influences on coital frequency, one would expect the monthly probability of conception to be biologically determined, and to display the "serial correlation" effect associated by the authors with variations in natural fecundability across the population. The authors' empirical results tend to confirm these expectations. Only age of wife is significant among socioeconomic variables, probably because it influences the distribution of natural fecundability. The "serial correlation" effect is highly significant.

The family choice of contraceptive technique, not investigated by the authors, promises to be a much more fruitful ground for investigating economic influences on fertility. If this relation failed to exhibit a dependence on socioeconomic factors, expected and completed family sizes would also be independent of these factors. But there is considerable evidence that socioeconomic factors *are* important in determining fertility (see T. Schultz [6]). I conclude these comments by reporting on some further estimates of the relation between socioeconomic factors and expected family size obtained by Donald Sant of the University of California. These estimates confirm the importance of education and income in influencing the expectation of having additional children, and provide indirect evidence that families form expectations on conception probabilities that depend on socioeconomic factors via the choice of contraceptive technique.

The sample consists of families drawn from the Survey of Economic Opportunity according to the following criteria: residence in twelve identifiable SMSA's, intact family, age of mother between 18 and 35, and family income $4,000 or more. Binary logit models were fitted to individual observations by the maximum likelihood method for the subsamples of families having two, three, or four children, with the dependent variable being a response from the mother that she expected to have one or more additional children. The results are given in Table 1. All independent variables are dichotomized, and standard errors are given in parentheses. The coefficients of each type of socioeconomic variable (sex ratio, education of wife, family income) are constrained to sum to zero. With respect to family income, the results show a consistent drop in the probability of expecting additional children at high incomes. presumably because of the opportunity cost

TABLE 1

Dependent Variable: Expect to Have an Additional Child; Model: Maximum Likelihood Estimation of Binary Logit Model

	Current Number of Children		
Independent Variable	2	3	4
Constant	−.761	−1.364	−1.131
	(.233)	(.327)	(.488)
Sex ratio of current family			
0 boys	.032	.438	.946
	(.191)	(.316)	(.716)
1 boy	−.214	−.213	.068
	-	-	-
2 boys	.182	−.630	−.899
	(.192)	(.282)	(.413)
3 boys	-	.405	.038
		(.352)	(.437)
4 boys	-	-	−.017
			(.767)
Education of wife			
Less than 12 years	−.469	−.748	.050
	(.257)	(.340)	(.386)
12 years	−.204	.086	.448
	-	-	-
13–15 years	.338	.056	−.498
	(.268)	(.440)	(.517)
More than 15 years	.335	.606	-
	(.352)	(.480)	
Family Income			
\$4,000 to \$8,000	.205	.393	−.672
	(.227)	(.353)	(.491)
\$8,000 to \$12,000	.288	.219	1.016
	-	-	-
Over \$12,000	−.493	−.612	−.344
	(.369)	(.569)	(.783)
Race			
Black	−.165	−.290	.400
	(.147)	(.185)	(.275)
Sample size	324	247	163

NOTE: Standard errors are in parentheses.

of having children and the income effect on the demand for expensive effective contraceptive techniques. A clear economic disincentive to large lower-income families also appears. Wife's education shows little consistent relation to the expectation of more children, although the results for families with two or three children suggest that the expectation may rise with education, contrary to the usual conclusion that opportunity cost of children rises with education. It would be interesting to isolate the employment opportunities of the wife from wife's education in assessing this effect. Sex ratio appears to be an important determinant of the expectation to have more children in a large family, with an imbalance of either sex tending to increase the expectation. In a family with three children, having at least one child of each sex is a significant disincentive to additional children. There is some proboy bias in three-child families, in that the sum of the coefficients in predominantly (two or three) boy families is negative. Similar conclusions hold for families with four children; there is a significant incentive to expect additional children in a no-boy family.

These empirical results tend to support the conclusion that fertility decisions are sequential, depending on cumulative information such as sex ratio; and are significantly influenced by income, and to a lesser degree, by education. Since contraceptive technique is the instrument by which families can control family size, these results suggest that the authors' methods applied to the choice of technique relation should yield significant results. The theoretical and statistical tools developed by the authors offer the possibility of fruitful and revealing glimpses into the economic determinants of these aspects of sexual behavior.

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