Professor Kauffman and M. Cruon have written a text on dynamic programming intended for engineers or those of equivalent mathematical training. They have chosen to stress the conceptual nature and general theoretical structure of dynamic programming rather than the rich but miscellaneous collection of results on specific problems. The expository style is in the best French tradition, elegant and limpid. The first four chapters deal with dynamic programs under certainty and probabilistic risk and for finite and infinite horizons. The concept of a dynamic program is introduced by carefully worked examples and restated in fairly abstract form. A noteworthy feature is the interpretation of dynamic programs in terms of graph theory. The indicated computational methods are those based directly on the principle of optimality. A similar discussion holds for the case where the result of a decision is a random variable.

The mathematically deeper developments are concerned with the stationary infinite-horizon case. The authors follow Bellman, with some minor improvements and greater elegance of notation; essentially they are concerned to show that the functional equation resulting from the application of the principle of optimality has a unique solution. The hypotheses invoked are, however, very restrictive; essentially only the case of a single state variable is considered, and then the value of the state variable must be shrinking (or at least not increasing, depending on other hypotheses) at each step. Under the same hypotheses the optimal strategies can be shown to be stationary.

The authors distinguish clearly the various alternative criteria that may be employed in infinite-horizon models—sum of returns, sum of discounted returns, and limit of the average return—and establish some connections among them. In particular, they present conditions under which policies which maximize the sum of discounted returns tend to a policy which maximizes average return as the discount rate approaches one. The sufficient conditions are again very restrictive.

It is unfortunate that the authors were unable to take advantage of the recent general theoretical results of Blackwell and Derman or the more specialized results of von Weizsäcker, Koopmans and others on economic growth models; as a result, the range of applicability of the theorems proved is very narrow.

The fifth chapter is an exposition of Howard's algorithm for finite-state finite-decision dynamic programming. The clarity of the presentation is outstanding, but its usefulness for text or references purposes is somewhat impaired by incompleteness; on several occasions the authors simply refer to Howard for a result instead of proving it themselves.

The sixth chapter presents a few generalizations.

In sum, this book is distinguished by elegance and clarity. But it lacks the variety of application which makes dynamic programming such a rich, open field; and its theoretical material has already been supplanted.

Manufacturing Production Functions in the U. S., 1957: An Interindustry and Interstate Comparison of Productivity. *George H. Hildebrand and Ta-chung Liu* (Cornell Studies in Industrial and Labor Relations, IV). Ithaca, N. Y.: Cornell University Press, 1965. Pp. xi+224. $6.00.

DANIEL MCFADDEN, *University of California, Berkeley, and University of Chicago*

THIS applied econometrics monograph presents estimates of production functions and labor demand functions for fifteen two-digit industries in American manufacturing. The study is based on cross-section analysis of state-by-state census data for 1957. Several theoretical innovations, a readable synthesis of a large volume of data, and an excellent review and summary of empirical findings make this an essential handbook for anyone interested in econometric studies of production.

For each industry, the authors fit variants of an econometric model in which three endogenous variables—the actual demands for production and non-production labor (L_p and L_n) and value added (V)—are determined as (essentially) log linear functions of the wage rates of production and non-production employees (w_p and w_n), the lagged capital stock (K_{-1}), and lagged labor demands. The non-linearity in the system is the consequence of an adjustment of measured production labor and capital stock to reflect differences in "quality" or "technological level" over the sample. The state educational level of the labor force (q) is used to index labor quality, while the ratio of depreciated to gross asset value (R) used to index the vintage of capital. While these variables are found to have good explanatory power for most industries, there is no independent evidence on their accuracy as quality indices. The authors make the adjustments $\overline{L}_p=(L_p)^{b_0 \log q}$ and $\overline{K}_{-1}=(K_{-1})^{e_0 \log R}$, where $\overline{L}_p$ and $\overline{K}_{-1}$ are unmeasured "true" inputs, and b_0, e_0 are parameters (q and R are scaled so they always exceed one). A disadvantage of this form of adjustment is that the "true" input level is not always monotone increasing in the quality index: if $L_p<1$, say, then a rise in q lowers $\overline{L}_p$. Further, the parameters b_0 and e_0 are not identified in the econometric model, leading to a confounding of "quality" and "scale" effects. Our further discussion of the model will be simplified without loss of generality by suppressing the quality adjustment terms and treating L_p and K_{-1} as already adjusted input variables.

The fundamental production relation for a given industry is assumed to be of the Cobb-Douglas form

$$Q_i = a' L_{pi}^{b'} L_{ni}^{c'} K_{-1i}^{e'} M_i^{\delta} u_i \tag{1}$$

where M_i and Q_i denote intermediate good input and output, respectively, u_i is an unobserved stochastic term, and i indexes the observations. A demand function

$$Q_i = d_i P_i^{-h} \tag{2}$$

is assumed to determine the output price P_i of the establishments included in observation i. Here P_i is interpreted as the price level of the observed unit *relative* to that of the industry, so that h is a cross-elasticity of demand. The authors do not postulate a functional form for the demand scale parameter d_i. However, they assume that output price is unrelated to the exogenous variables of the system, which implies the relation

$$d_i = \epsilon_i Q_i^{\alpha} \tag{3}$$

with $\alpha=1$, where ϵ_i is a stochastic term. Some indirect evidence from international cross-section data supports this specification (see Arrow [1]). A reasonable alternative specification of demand scale is that states in which average establishments are largest are net exporters, implying in (3), say, that $0\leqq\alpha<1$. If this alternative is correct, then the authors underestimate the input-output elasticities b', c', e'.

The intermediate good input is assumed to be determined by instantaneous short-run profit maximization, giving the maximization condition

$$\left(1-\frac{1}{h}\right)\delta Q_i/M_i = m_i/P_i, \tag{4}$$

where m_i is the intermediate good price. Since value added is defined as $V_i=P_iQ_i-m_iM_i$, (4) yields the proportionality

$$V_i = vP_iQ_i, \tag{5}$$

TABLE 1

SIC Number	Industry	Number Obs.	v[1]	Mean[5] γ_p	Authors' Estimates ϕ_p	Authors' Estimates h	Authors' Estimates s	h_{min}[2]	Corrected Estimates for $\alpha=1$, $\phi_p=1.0$: h	β	Mean MPP_p	s	Corrected Estimates for $\alpha=1$, $\phi_p=0.7$: h	β	Mean MPP_p	s
20	Food Products	41	.26	1.11	.85	4.2	1.24	3.9	39	.24	1.02	1.06	10.6	.18	.77	1.04
22	Textiles	21	.25	—	—	—	—	4.0	—	—	—	—	—	—	—	—
23	Apparel	24	.36	1.14	.90	4.8	1.04	2.8	23	.33	1.05	1.01	7.3	.26	.82	1.01
24	Lumber[3]	23	.32	1.18	.80	3.1	1.10	3.1	20	.28	1.03	1.03	7.6	.22	.81	1.03
25	Furniture	21	.43	—	—	—	—	2.3	—	—	—	—	—	—	—	—
26	Pulp & Paper	30	.36	1.48	.90	2.6	1.10	2.8	8.6	.28	1.15	1.03	5.3	.21	.86	1.02
28	Chemicals	32	.40	1.55	.70	1.8	1.18	2.5	7.0	.30	1.16	1.05	4.6	.23	.89	1.04
29	Petroleum & Coal	18	.20	1.00	.70	3.3	1.00	5.0	+00	.20	1.00	1.00	16.7	.15	.75	1.00
30	Rubber[3]	16	.46	1.48	.95	2.8	1.08	2.2	6.8	.36	1.16	1.02	4.2	.29	.93	1.02
31	Leather[3]	17	.42	1.32	.95	3.6	.92	2.4	9.9	.35	1.10	.97	5.1	.28	.88	.98
32	Stone, Clay, & Glass	27	.49	1.70	.80	1.9	1.09	2.1	5.1	.37	1.28	1.04	3.6	.29	1.01	1.03
33	Primary Metals[3]	29	.36	1.70	.80	1.9	1.12	2.8	6.8	.25	1.18	1.03	4.8	.19	.90	1.02
34	Fabricated Metals	33	.40	1.27	.90	3.5	1.02	2.5	12	.35	1.11	1.01	5.6	.27	.86	1.01
35	Non-electrical Machinery	30	.46	1.17	.90	4.4	1.07	2.2	15	.42	1.07	1.03	5.5	.34	.86	1.02
36	Electrical Machinery	25	.47	1.13	.90	4.9	.95	2.1	18	.44	1.06	.98	5.5	.35	.84	.98
37	Transportation Equipment	29	.38	.88	.70	5.0	1.01	2.6	[4]	—	—	—	12.7	.33	.76	1.00
38	Instruments[3]	14	.47	1.06	.85	5.0	1.11	2.1	37	.46	1.04	1.06	6.2	.37	.84	1.04

[1] v = Value Added/Value of Shipments. From *1958 Census of Manufactures* and "The Interindustry Structure of the U.S.," *Survey of Current Business*, Nov. 1964.

[2] The minimum value of h which is consistent with profit maximization in intermediate good input, $h_{min}=1/v$.

[3] The authors have fitted a production function with total labor, rather than production and non-production labor separately, for these industries. Hence, γ_p, ϕ_p, and MPP_p denote marginals for *total* labor in these industries.

[4] The observed γ_p is inconsistent with profit maximization for $\phi_p > .88$.

[5] The authors have taken γ_p to be the MPP_p. When their specification $h < +\infty$ and $\alpha=1$ is correct, this estimate is too high.

TABLE 2

SIC Number	Industry	Correlation of Relative Shares and Relative Input Prices[1]	Mean Capital's Share of Value Added	Elasticity of Substitution Estimates[2]		
				Method A	Method F-R	Method H-L
20	Food Products	−.55 (1%)	.52	.78 (.15)	.51 (.12)	29.1
22	Textiles	−.59 (1%)	.37	.84 (.16)	.48 (.16)	1.71
23	Apparel	−.50 (2%)	.37	1.16 (.20)	.66 (.12)	1.45
24	Lumber	−.52 (2%)	.33	.99 (.07)	.79 (.08)	1.00
25	Furniture	−.40 (.10%)	.39	1.17 (.12)	.58 (.07)	.90
26	Pulp & Paper	+.08	.49	.65 (.41)	1.10 (.22)	1.19
28	Chemicals	−.50 (1%)	.63	.62 (.21)	.63 (.12)	1.27
29	Petroleum & Coal[3]	−.76 (1%)	.60	.52 (.48)	.33 (.13)	.34
30	Rubber	−.30	.43	1.31 (.34)	.68 (.19)	1.34
31	Leather	−.55 (5%)	.35	.77 (.27)	.72 (.10)	.78
32	Stone, Clay & Glass	−.15	.49	.85 (.19)	.92 (.11)	1.35
33	Primary Metals	−.56 (1%)	.42	.85 (.68)	.46 (.15)	1.24

[1] If the Cobb-Douglas specification is correct, this correlation should be zero. Significance levels (two-sided tests) are given in parentheses. These estimates contain some bias due to the measurement of capital price as a residual from capital share data.

[2] If the production function is Cobb-Douglas, then the elasticity of substitution σ should equal one. The production functions (8) have $\sigma = b[1 - g/s_k]^{-1}$, where s_k is the share of capital in value added. Method A estimates are obtained from the regression (7) under the specification $g = 0$, $\sigma = b$ (See Arrow [1]). Method H-L estimates are obtained from the Hildebrand-Liu parameter values for g and b in (7) and the mean value of s_k over the sample. Method F-R estimates are obtained from a regression equation for the factor ratio, $K_i/L_i = A(w_i/r_i)^{\sigma} u_i''$, where r_i is capital price measured as a residual from capital share data, and u'' is a stochastic term. These estimates contain some bias due to measurement error. Standard errors are in parentheses.

[3] The volume of output is found to be a significant explanatory variable in the Method FR estimating relation for σ in the Petroleum and Coal and Non-electrical machinery industries, indicating a bias toward higher capital intensities at higher scale levels when relative input prices are fixed. The Method FR estimates are corrected for this bias; the Method A and H-L estimates are not.

TABLE 2 (*continued*)

SIC Number	Industry	Correlation of Relative Shares and Relative Input Prices[1]	Mean Capital's Share of Value Added	Elasticity of Substitution Estimates[2]		
				Method A	Method F-R	Method H-L
34	Fabricated Metals	−.37 (5%)	.40	.62 (.21)	.50 (.13)	.72
35	Non-electrical Machinery[4]	−.91 (1%)	.40	.93 (.47)	−.03 (.09)	.63
36	Electrical Machinery[3]	−.75 (1%)	.44	.61 (.36)	.50 (.09)	.81
37	Transportation Equipment	−.33 (10%)	.35	1.19 (.51)	.73 (.15)	2.58
38	Instruments	−.69 (1%)	.37	.74 (.27)	.45 (.17)	1.47

where $v=1-\delta(1-1/h)$. Equations (1)–(5) can be solved to yield a partially reduced form

$$V_i = aL_{pi}^{b}L_{ni}^{c}K_{-1i}^{e}u_i, \tag{6}$$

where, defining $\beta=h[v(h-1)-\alpha(1-v)]/(h-1)(h+\alpha-1)$, the parameters satisfy $b'=\beta b$, $c'=\beta c$, $e'=\beta e$; a is a constant term; and $u_i^{\beta}=m_i^{-\delta}\epsilon_i^{1/(h+\alpha-1)}u_1'$. The unobserved price m_i and the stochastic terms ϵ_i and u_i' are tacitly assumed to be such that the u_i are independently distributed log normally, and ordinary and two-stage least squares estimates of the parameters are obtained.

The authors form equations (5) and (6) without considering explicitly the impact of the intermediate good input and the output demand structure on the parameters b, c, e. Consequently, their empirical results in several cases require re-interpretation or correction. Letting $\gamma_{pi}=bV_i/w_{pi}L_{pi}$, the marginal physical product of production labor (in dollars of output per dollar of labor input) is $MPP_{pi}=(\beta/v)\gamma_{pi}$. If the authors' specification $\alpha=1$ and $h<+\infty$ is correct, then their estimates of the MPP_{pi} and the corresponding marginal revenue products, $\phi_{pi}=(1-1/h)\ MPP_{pi}$, are too high. This correction leads to quantitatively important differences in most of the monograph's parameter estimates, since the authors use "reasonable" values of ϕ_{pi} to determine the non-identified demand elasticity h. In Table 1, values of v and estimates of h, β, mean MPP_p, and returns to scale $s(=b'+c'+e'+\delta)$ in the production function (1) are determined for the values $\phi_{pi}=1.0$ and $\phi_{pi}=0.7$. The table shows that the authors have underestimated the cross-elasticity h in this ϕ_{pi} range by a factor of 2 to 3, and have significantly over-estimated deviations from constant returns to scale.

The first two chapters of the monograph review the existing empirical literature on production functions, and provide tests of the authors' Cobb-Douglas specification vs. other functional forms of the production function, particularly the now-popular C.E.S. production function. For C.E.S. production functions, the regression formula

$$\frac{V_i}{L_i} = Aw_i^b(K_{-1i}/L_i)^g u_i'' \tag{7}$$

must hold with $g=0$ (where L_i is total labor input, w_i is the overall wage rate, and u_i'' is a stochastic term). When labor is paid its marginal physical product, (7) gives a differential specification of a family of value-added production functions. For $b\neq 1$ and $b+g\neq 1$, the possible solutions are a form discovered by Bruno (see Nerlove [2]),

$$V_i = [\beta K_{-1i}^{1-1/b} + \alpha K_{-1i}^{-g/b} L_i^{g/b+1-1/b}]^{b/(b-1)} \tag{8-a}$$

and the Cobb-Douglas form

$$V_i = aK_{-1i}^{g/(1-b)} L_i^{1-g/(1-b)} \tag{8-b}$$

where α, β, a are constants chosen to satisfy (7). When $g=0$, $(8-a)$ is the C.E.S. form. For (8) to have the usual properties of production functions, the parameters must satisfy $b>0$ and $0\leqq g/(1-b)<1$.

Finding g significantly non-zero in the regression (7) for a majority of the industries studied, the authors reject the C.E.S. specification in favor of the Cobb-Douglas form. However, a direct test of the invariance of relative shares under changes in the input price ratio implies that the Cobb-Douglas specification is not a particularly good one for most industries. Further, most industries are found to have an elasticity of substitution (an index of the sensitivity of cost-minimizing input proportions to changes in relative input prices) which deviates significantly from the unitary value which must hold under the Cobb-Douglas specification. These results are summarized in Table 2; other C.E.S. estimates are reviewed in Nerlove [2].

While the possible specification errors detailed in this review suggest that the authors' quantitative results be viewed with scepticism, the high quality and originality of the volume is impressive. The book is warmly recommended to economists and econometricians.

REFERENCES

[1] Arrow, K. J., H. Chenery, B. Minhas, R. Solow, "Capital-Labor Substitution and Economic Efficiency," *Review of Economics and Statistics*, 1961.

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Management Goals and Accounting for Control, vol. 3 in the Series: *Yuji Ijiri*. Studies in Mathematical and Managerial Economics, Amsterdam: North-Holland Publishing Company, 1965. $6.50.

NICHOLAS DOPUCH, *Carnegie Institute of Technology*

This third volume in the series, "Studies in Mathematical and Managerial Economics" (edited by Henri Theil), differs from its predecessors in one significant respect: It concentrates on the problems in designing a control system around a given set of decisions rather than on the methods for determining optimal decisions. Specifically, Ijiri's objective is to "derive criteria for better coordination between the two processes—planning and accounting for control." Accountants have long recognized the link between control systems and decision-making levels within an organization. However, this book is unique because it illustrates how a formal planning model may be used to define and assess a set of feedback measurements provided to management.