Improving Revealed Preference Bounds on Demand Responses

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Handout

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Background Motivation

- Improve power of test of rationality
  - for both experimental and observational data.

- to consider rationality over groups of decisions and over periods of time - to characterise changing tastes.

- to provide tight bounds on welfare costs of relative price and tax changes.

- in this presentation the focus is on demand responses - to provide tight bounds on demand responses (and elasticities) and on the distribution of demands (quantiles).

Data

- continuous micro-data on incomes and expenditures

- finite set of observed price and/or tax regimes

- discrete demographic differences across households

- use this information alone, together with revealed preference theory to assess consumer rationality and to place ‘tight’ bounds on behavioural responses

Data: Observational; Experimental

- Is there a best design for experimental data?

- Blundell, Browning and Crawford (2003) develop a method for choosing a sequence of total expenditures that maximise the power of tests of RP conditions with respect to a given preference ordering.

  – the sequential maximum power (SMP) path

    Suppose that the sequence

    $$\{\mathbf{q}_s(x_s), \mathbf{q}_t(x_t), \mathbf{q}_u(x_u), ..., \mathbf{q}_v(x_v), \mathbf{q}_w(x_w)\}$$

    rejects RP. Then the SMP path also rejects RP.

  – also develop a method of bounding true cost of living indices.

What would be the best design in the observational case?

- individual data allows us to describe local expansion paths - nonparametric Engel curves

- use the nonparametric expansion paths to mimic the experimental design - differ across markets - by time period (and location)

- empirically acceptable and theoretically sound method for pooling over types

  - shape invariant or shape similar specification for demographics

  - unobserved heterogeneity?

- endogeneity with nonparametric regression

Assumption 1. For each agent there exists a set of demand functions $\mathbf{q}(\mathbf{p}, x) : \Re_{++}^{J+1} \rightarrow \Re_{++}^{J}$ which satisfy adding-up: $\mathbf{p}'\mathbf{q}(\mathbf{p}, x) = x$ for all prices $\mathbf{p}$ and total outlays x.

- Thus we are implicitly assuming that preferences are strictly convex and locally non-satiated. For a given price vector $\mathbf{p}_t$ we denote the corresponding J-valued function of x as $\mathbf{q}_t(x)$ (with $q_t^j(x)$ for good j) which we shall refer to as an *expansion path* for the given prices. We shall also have need of the following assumption:

Assumption 2. Weak normality: if $x > x'$ then $q_t^j(x) \geq q_t^j(x')$ for all j and all $\mathbf{p}_t$.

Bounds on Demand Responses

Suppose we observe a set demand vectors $\{\mathbf{q}_1, \mathbf{q}_2, ...\mathbf{q}_T\}$ which record the choices made by a consumer when faced by the set of prices $\{\mathbf{p}_1, \mathbf{p}_2, ...\mathbf{p}_T\}$.

- new price vector $\mathbf{p}_0$ with total outlay x_0.

- best *support set* $S^V(\mathbf{p}_0, x_0)$ is given by:

$$\left\{ \mathbf{q}_0 : \begin{array}{c} \mathbf{p}_0'\mathbf{q}_0 = x_0, \ \mathbf{q}_0 \geq \mathbf{0} \text{ and} \\ \{\mathbf{p}_t, \mathbf{q}_t\}_{t=0...T} \text{ satisfies RP} \end{array} \right\}$$

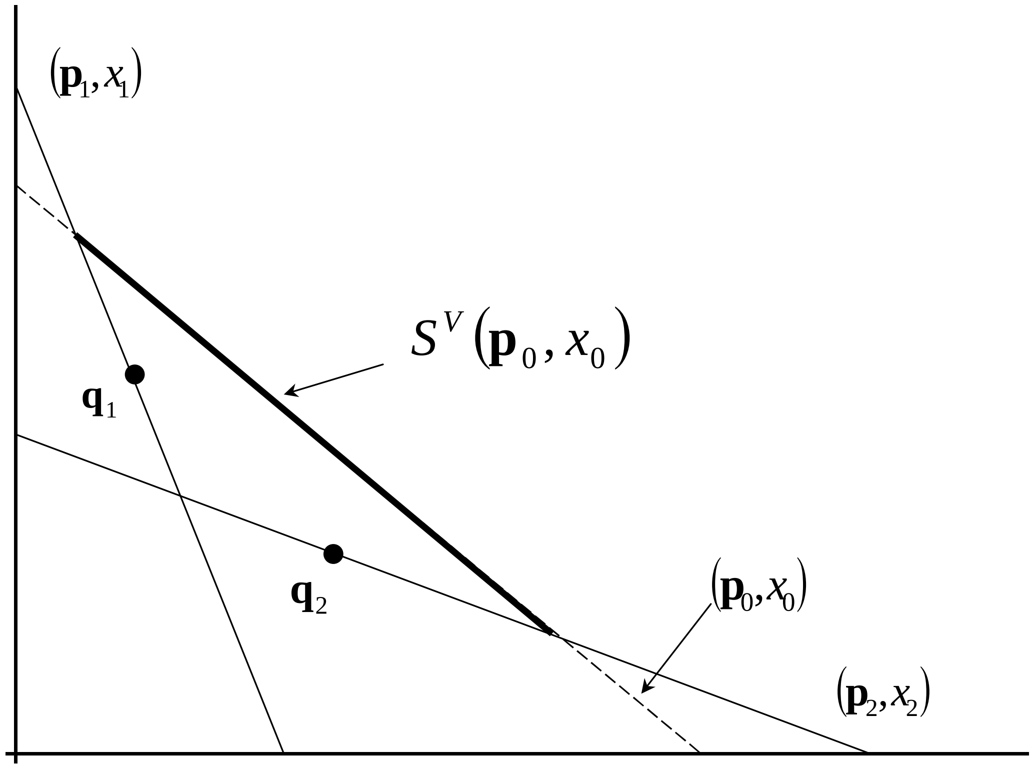


Figure 1. The Support Set with RP

E-Bounds on Demands

- Suppose we have a set on nonparametric expansion paths $\mathbf{q}_t(x)$ for each of T price regimes.

- To derive the best support set using expansion paths (*E-Bounds*) we identify the T demand vectors such that

$$\mathbf{q}_0 R^0 \mathbf{q}_t(\tilde{x}_t)$$

  where each $\tilde{x}_t$ solves the implicit equation

$$\mathbf{p}_0' \mathbf{q}_t(\tilde{x}_t) = x_0$$

- These budget levels $\{\tilde{x}_t\}_{t=1,\ldots,T}$ give the precise budget levels on each expansion path at which the new demand vector will be directly revealed preferred to the $\mathbf{q}_t(\tilde{x}_t)$ demands -$>$

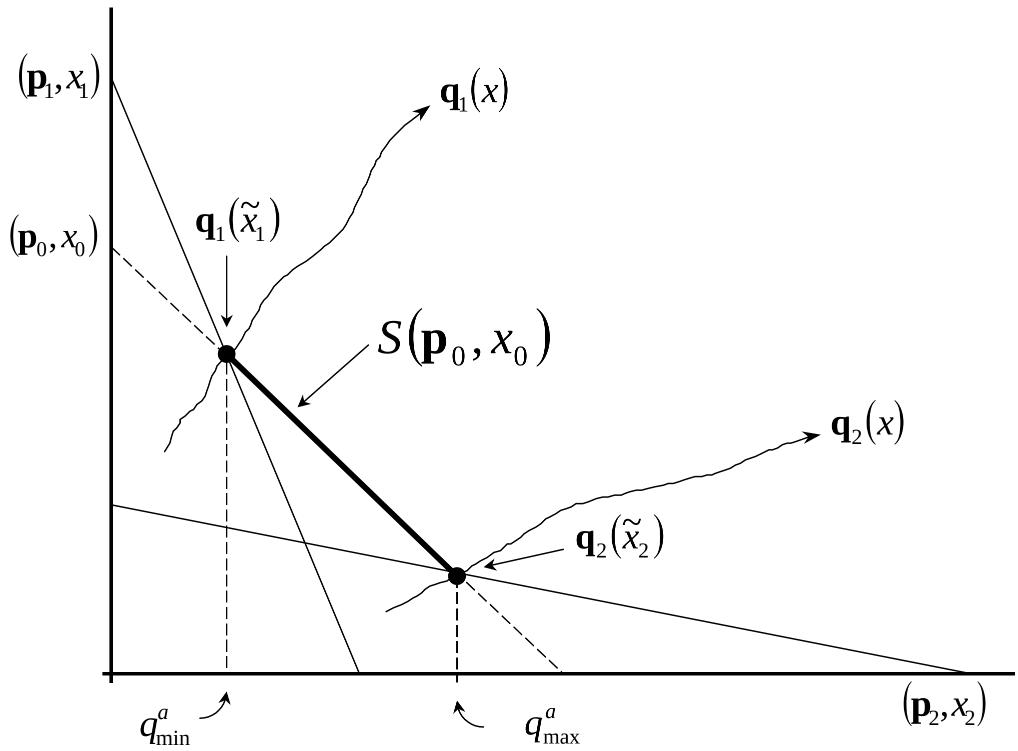
$\{\mathbf{q}_t(\tilde{x}_t)\}_{t=1,\ldots,T}$ *intersection demands*

The **support set** is given by $S\left(\mathbf{p}_0, x_0\right)$

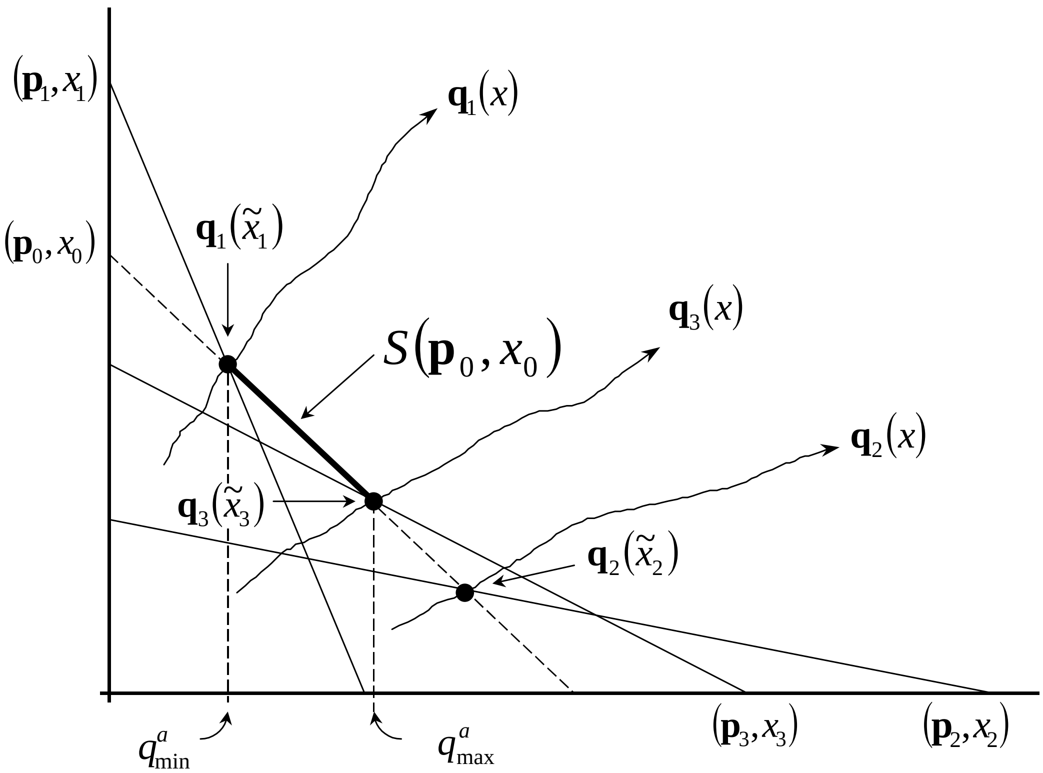
$$\left\{ \mathbf{q}_0 : \begin{array}{c} \mathbf{q}_0 \geq \mathbf{0},\ \mathbf{p}_0'\mathbf{q}_0 = \mathbf{x}_0 \\ \text{and } \mathbf{p}_t'\mathbf{q}_0 > \mathbf{p}_t'\mathbf{q}_t(\tilde{x}_t) \text{ for } t = 1, 2 ... T \\ \text{such that } \left\{\mathbf{p}_0, \mathbf{p}_t; \mathbf{q}_0, \mathbf{q}_t\left(\tilde{x}_t\right)\right\} \text{ satisfy GARP} \\ \text{where } \tilde{x}_t \text{ is such that } \mathbf{p}_0'\mathbf{q}_t\left(\tilde{x}_t\right) = x_0 \end{array} \right\}$$

Proposition: *For any* $\left(\mathbf{p}_0, x\right)$, *if the intersection demands* $\left(\mathbf{p}_t, \mathbf{q}_t\left(\tilde{x}_t\right)\right)_{t=1...T}$ *satisfy GARP*
A. The set $S\left(\mathbf{p}_0, x_0\right)$ *is non-empty.*
B. *The set* $S\left(\mathbf{p}_0, x_0\right)$ *is convex.*
C. *For any point on the new budget line that is not in* $S\left(\mathbf{p}_0, x_0\right)$, *the intersection demands and this point fail GARP.*

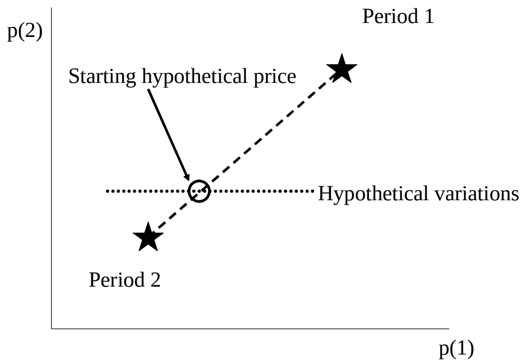
- these best support sets (E-bounds) can be used to
  - tighten the bounds on complete demand responses
  - local to each point in the income distribution



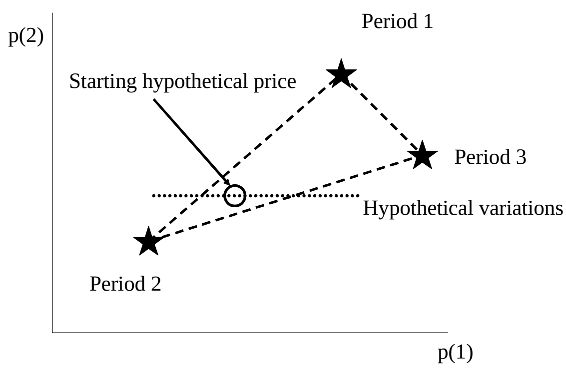
The Support Set with Expansion Paths and RP



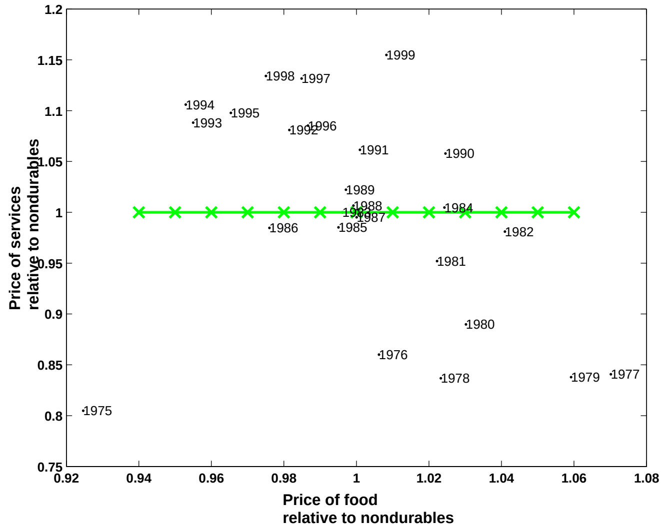
Support Set with RP and Many Prices



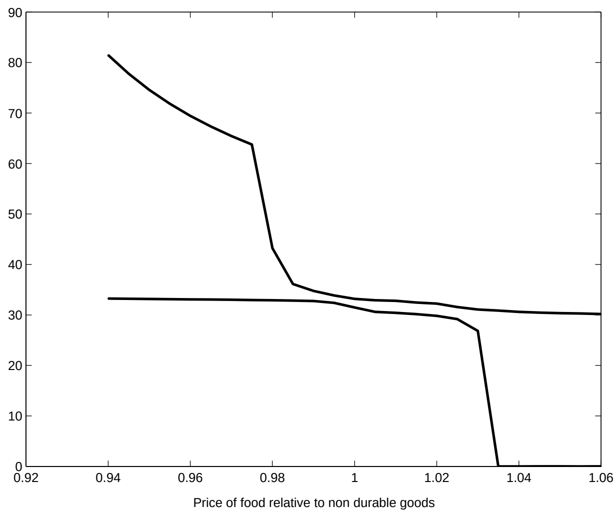
Convex combination



Convex Hull



The time series of relative prices



E-Bounds to Demand Responses

Local Perturbations and Changing Tastes

- How should we characterise changing tastes?

- Allow local perturbations to preferences to describe the degree of taste changes, through a shift in marginal utility.

  – differ across individuals with different incomes.

  – assess the direction of taste change and how tastes change for rich and poor.

- Slowly changing tastes would be reflected by a systematic evolution of these perturbations.

- Let G denote the set of RP-consistent data sets

$$G = \{\mathbf{Q} : \{\mathbf{P}, \mathbf{Q}\} \text{ satisfies RP}\}$$

if the intersection demand data violate RP then

$$\widetilde{\mathbf{Q}} \notin G$$

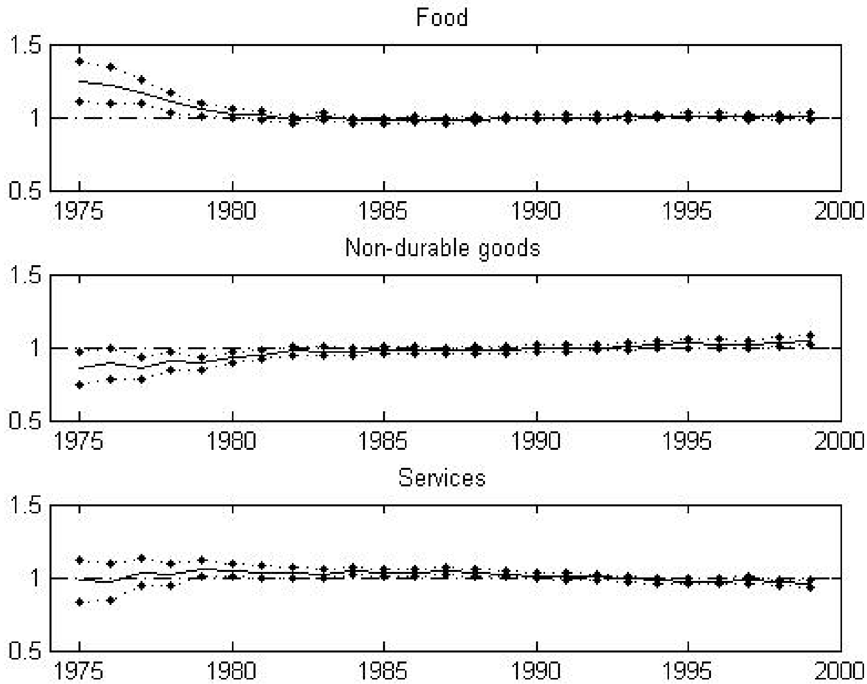
- Suppose we now define $\mathbf{Q}^* = \mathbf{E} \odot \mathbf{Q}$ where $\mathbf{E}$ are a set of perturbations to preferences

$$\min_{\mathbf{Q}^*} f(\mathbf{Q}^*) = \text{vec}\left(\mathbf{E} - \mathbf{1}_{(J \times T)}\right)' \mathbf{\Omega}^{-1} \text{vec}\left(\mathbf{E} - \mathbf{1}_{(J \times T)}\right)$$

subject to

$$\begin{aligned} \mathbf{Q}^* &\in G \\ \mathbf{Q}^* &\geq \mathbf{0}_{(J \times T)} \\ \mathbf{p}_0' \mathbf{Q}^* &= x_0 \mathbf{1}_{(1 \times T)} \end{aligned}$$

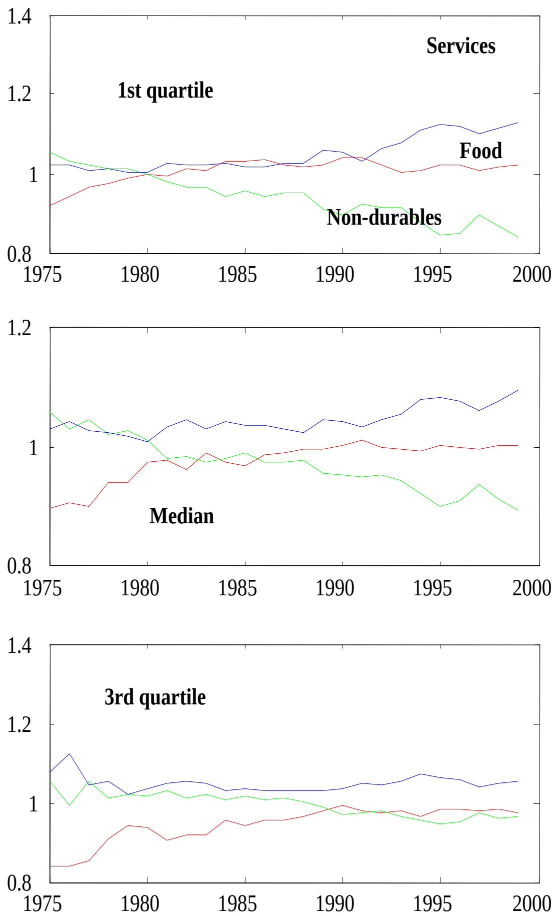
-$>$ GARP-consistent, non-negative, intersection demands.



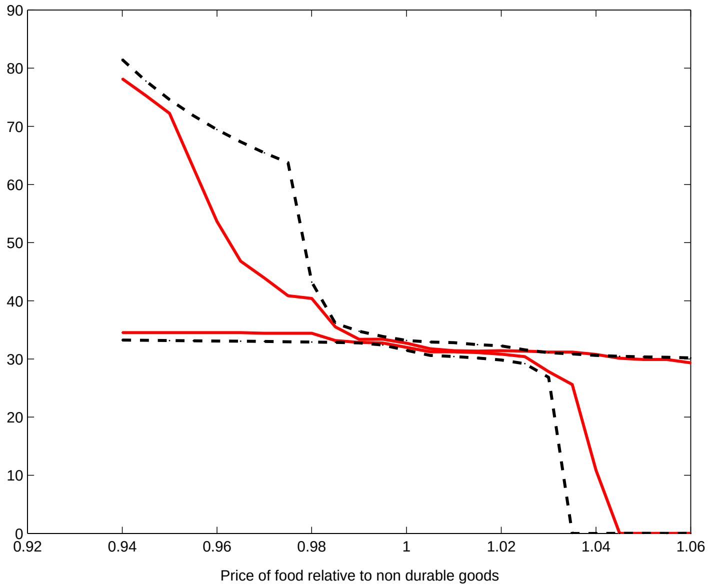
Changing Tastes

- periods of taste stability for some types of consumers over certain groups of goods

- tastes evolve differently across the income distribution



Taste Changes Across the Income Distribution



Improved E-Bounds on Demand Responses

Non-separable Heterogeneity

- Let demands $\mathbf{q}$ be written

$$\mathbf{q} = d(\mathbf{p}, x, \boldsymbol{\varepsilon})$$

  where $\boldsymbol{\varepsilon}$ is a $J-1$ vector of unobservable heterogeneity variables. Since the budget constraint $\mathbf{p}'\mathbf{q} = x$ holds there are $J-1$ independent demands.

- Aim: to bound demand responses local to quantiles of $\boldsymbol{\varepsilon}$ and x.

  – impose RP conditions local to quantiles of $\boldsymbol{\varepsilon}$ and x.

  – impose RP conditions across the distribution of demands

- Assume sufficient conditions for $d(.)$ to be invertible in ε. Let $\varepsilon = m(\mathbf{q}, \mathbf{p}, x)$ denote the inverse of $d(.)$ with respect to ε conditional on $(\mathbf{p}, x)$.

- global invertibility is necessary for global nonparametric identification of $U(\mathbf{q}, \boldsymbol{\varepsilon})$

- random utility $U(\mathbf{q}, \boldsymbol{\varepsilon})$ where $\mathbf{q} \in \mathbb{R}_+^J$ and $\boldsymbol{\varepsilon} \in \mathbb{R}^{J-1}$

- demand functions $d(\mathbf{p}, x, \boldsymbol{\varepsilon})$ for $J - 1$ inside goods $\mathbf{q}_{-J} = (q_1, \ldots q_{J-1})'$ solve

$$\mathbf{p} = MRS(\mathbf{q}_{-J}, x - \mathbf{p}'\mathbf{q}_{-J}, \boldsymbol{\varepsilon})$$

where $MRS(\mathbf{q}, \boldsymbol{\varepsilon}) = \left[\frac{\partial}{\partial x_j} U(\mathbf{q}, \boldsymbol{\varepsilon}) / \frac{\partial}{\partial x_J} U(\mathbf{q}, \boldsymbol{\varepsilon})\right]_{j=1,\ldots,J-1}$

Maximization of random utility => the conditional residuals $\nu(\mathbf{p}, x, \boldsymbol{\varepsilon}) = d(\mathbf{p}, x, \boldsymbol{\varepsilon}) - E[d(\mathbf{p}, x, \boldsymbol{\varepsilon})|\mathbf{p}, x]$ functionally dependent on $\mathbf{p}$ and x.

For scalar heterogeneity $\varepsilon \in \mathbb{R}$, global invertibility follows from strict monotonicity of d with respect to ε

Assumption A1: For each ε, $U \in \mathcal{U}$ is continuous in its arguments, continuously differentiable in $\mathbf{q}, \varepsilon$ strongly monotone, concave and strictly quasi-concave in $\mathbf{q}$.

Assumption A2: The $(J-1)\times(J-1)$ matrix $\nabla_{\epsilon} MRS(\mathbf{q}, \varepsilon)$ has full rank $J-1$ for all ε.

Assumption A3: The bordered Hessian satisfies

$$\begin{vmatrix} \nabla_{\mathbf{ww}'} U(\mathbf{q}, \varepsilon) & \nabla_{\mathbf{w}} U(\mathbf{q}, \varepsilon) \\ \nabla_{\mathbf{w}'} U(\mathbf{q}, \varepsilon) & 0 \end{vmatrix} \neq 0$$

for all $\mathbf{w}' = (\mathbf{q}', \varepsilon')$.

Assumptions A1 - A3 guarantee that the reduced form system of stochastic demands $d(\mathbf{p}, x, \varepsilon)$ is a system of continuously differentiable demand functions - unique value of $\mathbf{q}_{-J}$ with any $\mathbf{p}$,x and ε, i.e. it has a well-defined reduced form $\mathbf{q}_{-J} = d(\mathbf{p}, x, \varepsilon)$. Assumptions A1-A3, thus, amount to coherency conditions.

Assumption A5: $MRS(\mathbf{q}, \boldsymbol{\varepsilon})$ is multiplicatively separable with respect to ϵ:

$$MRS(\mathbf{q}, \boldsymbol{\varepsilon}) = v(\mathbf{q}) + K(\mathbf{q})\psi(\boldsymbol{\varepsilon}),$$

where $v(\mathbf{q})$ is a $(J-1) \times 1$ vector of nonnegative functions, $K(\mathbf{q})$ is a $(J-1) \times (J-1)$ matrix with full rank, and $\psi : \mathbf{R}^{J-1} \to \mathbf{R}^{J-1}$.

Lemma: *Suppose A1, A2, A3, A4 and A5 hold. Then, for any* **p** *and x,* $d(\mathbf{p}, x, \boldsymbol{\varepsilon})$ *is globally invertible for all* $\mathbf{q}_{-J} \in B_{-J}(\mathbf{p}, x)$*, and, hence,* $\mathbf{q}_{-J}$ *has a non-degenerate distribution on* $B_{-J}(\mathbf{p}, x)$*, given any* **p** *and x.*

Consider the random demand system

$$\begin{aligned}
q_1 &= d_1(x, \varepsilon_1, ...\varepsilon_{J-1}) \\
q_2 &= d_2(x, \varepsilon_1, ...\varepsilon_{J-1}) \\
&. \\
&. \\
q_{J-1} &= d_{J-1}(x, \varepsilon_1, ...\varepsilon_{J-1}) \\
q_J &= x - \Sigma_{k=1}^{J-1} p_k q_k
\end{aligned}$$

We can write this in an equivalent way, using a transformation of the demand system and the distribution ε as

$$\begin{aligned}
q_1 &= s_1(x, \eta_1) \\
q_2 &= s_2(x, \eta_1, \eta_2) \\
&. \\
&. \\
q_{J-1} &= s_{J-1}(x, \eta_1, ...\eta_{J-1}) \\
q_J &= x - \Sigma_{k=1}^{J-1} p_k q_k
\end{aligned}$$

$\eta_k' s$ are independent across $k-$ not structural random terms but allow us to identify each demand observation with point that corresponds to a multidimensional quantile.

- estimate the s functions and the distribution of $\boldsymbol{\eta}$ using either the normalisation on the s functions or a normalisation on the distribution of $\boldsymbol{\eta}$.

- consider normalising the distribution of $\boldsymbol{\eta}$ to be $\cup(0,1)$.

- let $r_1, r_2\ ,\ , r_{J-1}$ denote the inverse functions of $s_1...s_{J-1}$ with respect to $\eta_1, ...\ \eta_{J-1}$

$$
\begin{aligned}
\eta_1 &= r_1(x, q_1) \\
\eta_2 &= r_2(x, \eta_1, q_2) \\
&. \\
&. \\
q_{J-1} &= r_{J-1}(x, \eta_1, ...q_{J-1})
\end{aligned}
$$

- given arbitrary functions $r_1, ..., r_{J-1}$ and observations $\{q^i, x^i\}$ calculate, recursively

$$
\begin{aligned}
\eta_1^i &= r_1(x^i, q_1^i) \\
\eta_2^i &= r_2(x^i, \eta_1^i, q_2^i) \\
&. \\
&. \\
\eta_{J-1}^i &= r_{J-1}(x^i, \eta_1^i, ...q_{J-1}^i)
\end{aligned}
$$

Use $\{\eta^i, x^i\}$ to estimate the joint distribution of $(\eta_1, ., \eta_{J-1},$ on a grid $\{(\eta^1, x^1), ., (\eta^S, x^S)\}$.

The estimator for the functions r_1, $., r_{J-1}$ can be defined as the one that minimises

$$\sum_{s=1}^{S} \left[\begin{array}{c} \widehat{F}_{\eta_1,\eta_2,..,\eta_{J-1},x}(\eta_1^s \eta_2^s, .., \eta_{J-1}^s, x^s; r) \\ -\eta_1^s \eta_2^s \cdot \cdot \eta_{J-1}^s \widetilde{F}(x^s) \end{array} \right]^2$$

where $\widehat{F}_{\eta_1,\eta_2,..,\eta_{J-1},x}(\eta_1^s \eta_2^s, .., \eta_{J-1}^s, x^s; r)$ is the nonparametric estimator of the joint distribution of (η, x) when the functions are $r_1, ...r_{J-1}$;

$\widetilde{F}_I(x^s)$ is a nonparametric estimator for the marginal distribution of x,

$\eta_1^s \eta_2^s, .., \eta_{J-1}^s$ is the value of the marginal distribution of $\boldsymbol{\eta}$ (that is, $F_{\eta_1,\eta_2,..,\eta_{J-1}}(t_1, ..., t_{J-1}) = t_1 t_2 \cdots t_{J-1}$).

Summary

- considered rationality over groups of decisions and over periods of time
  - characterising changing tastes.

- provided the best nonparametric bounds on demand responses (and elasticities) under RP.
  - local to quantile of the income distribution

- allowed for non-separable heterogeneity and to study distribution of demands for any income quantile consistent with RP.

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