Why Conservatives and Liberals Should Both
Support Investing Social Security Money in
Stocks

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I. INTRODUCTION

There is a wide belief on both the left and the right that the current pay-
as-you-go Social Security system needs renovation. Proposals from the right
have tended to favor a fully-funded system of individual retirement accounts,
managed by the individuals themselves. It is their money, after all, and others
are likely to be much worse agents than the individuals themselves. Proposals
from the left cover a wide range. Some focus on attempts to renovate the
system through small changes to attempt to preserve the existing pay-as-
you-go system. Others hope to make reforms like increasing the retirement age and shifting to a more funded system.

On the right, the principal argument for fully-funded individual and individually-managed accounts is a primarily political-economy argument. If there is anything the past two centuries have taught us, it is that governments can be extraordinarily inefficient at managing citizens’ money. Private property rights have tended to be much better than government programs and promises at preserving and securing wealth. The existing pay-as-you-go Social Security scheme was started under an extremely favorable demographic situation that no longer exists. The existing pay-as-you-go Social Security scheme is not secure: the assumption that the government will extract sufficient tax revenue to cover its promises made in the distant past is questionable. And the government has been spending on current services the money that ought to have been invested to create reserves and prefund the system to compensate for the adverse shifts in demography. On the right, the principal argument for a Social Security system based on fully-funded individual accounts is that such a system is the only way to guard against the major risks of government malfeasance and government fecklessness.
The purpose of this paper, however, is to focus on the arguments for Social Security privatization that should lead the left, on this issue, to agree with the right. I do not believe that the left should agree with the right’s reasons for favoring fully-funded and individually-managed accounts. The left has different priors about the inefficiency and goodwill of governments, and to some degree different values. I argue that the left should agree with the goal, but for different reasons.

The left takes the existence of the equity premium very seriously. The existence in the past and continued existence today of a large and powerful equity premium makes individual retirement accounts invested in equities very powerful and low-risk wealth builders. A fully-funded system of well-diversified individual retirement accounts, if properly insured, should be very attractive to the left.

The history of the stock market shows that (see Siegel (1995) for example), typically, stocks have been substantially undervalued in the sense that long-term investments in them have yielded very high average returns, on the order of six percent per year in real values. The history of the stock market also shows that due to the mean-reverting nature of asset returns—the long-term risk associated with a diversified equity portfolio is very low.
Certainly it appears lower than investments in nominal bonds, which bear inflation risk, or trust in government promises—the lavishness of which is a principal source of inflation risk.

The probability of the collapse of a well-diversified leveraged portfolio invested in equities is low. The probability that such a collapse would not be reversed by subsequent mean reversion in stock prices is even lower. Thus diversified long-term investments in stocks largely insure themselves. They carry little risk. How little risk? This paper uses a particular metric to evaluate this risk. It asks how expensive it would be to insure a long-term individual Social Security account invested in stocks against the risk that the portfolio’s value would collapse. It asks how expensive standard finance suggests purchasing a long-term put option on such a portfolio should be. The answer is that the Black-Scholes price of such a put option is surprisingly low. It is low even in this world in which the large size of the equity premium reveals that the market price of systematic risk is very high.

Therefore, this paper proposes a system in which the government requires that individuals buy insurance on their retirement portfolios at the time as they make their retirement contributions\(^1\). Such an insurance policy would

\(^1\)There is of course a complicated and difficult issue of transition to such a system, but
be incentive-compatible. Those who take extra risks will have to pay higher insurance premiums, thus reducing the key moral hazard problem associated with investing public pensions in individual accounts—that individuals can gamble and anticipate that the government will cover their losses.

II. PAST PERFORMANCE AND FUTURE RESULTS

It was in 1924 that the first analyst pointed out that the return/risk trade-off for investments in U.S. common stocks was extraordinarily favorable. Investment manager Edgar L. Smith, in a book called ”Common Stocks as Long-Term Investments” and in a previous article in the ”Atlantic Monthly,” (see Smith (1924)) calculated that historical returns suggested that based on then-historical return patterns there was essentially no long-run risk associated with diversified and sufficiently long-horizon investments in American equities.

Subsequent historical returns have confirmed the pattern first identified by Edgar L. Smith more than eighty years ago. Indeed, it would have taken only fifteen years for those who invested in the stock market at its peak in 1929 and who then held their positions to recoup their full initial real value. due to the lack of space it is not addressed here.
Since 1945, the longest it has taken for a diversified stock portfolio to recover its initial value is the current episode—from 2000 to 2006. And everybody is confident that even investors who bought in February 2000 will soon be in the black: a reasonable forecast of current real returns on stocks is the current earnings yield of six percent per year.

Siegel (2002) presents the most comprehensive argument that the high long-run cumulative returns on stocks present a very favorable risk/return trade-off, for the high average returns have not been offset by any running of huge risks. He finds that there is effectively no risk/return trade-off over the long run for diversified stock vs. bond portfolios in America today.

Siegel (2002) calculates that if an investor holds his portfolio for 20 years or longer — a holding period typical for pension investments — then stocks provide expected returns superior to those of bonds (6 percent real return vs. only .5 percent for the fixed-income instruments, (see Mehra and Prescott (1985) for more discussion)) and carry, in fact, lower risk.


TABLE 1

PERCENTAGE OF PERIODS WHEN RETURNS ON ONE ASSET EXCEED ANOTHER ASSET

<table>
<thead>
<tr>
<th>Holding Period</th>
<th>Stocks Outperform Bonds</th>
<th>Stocks Outperform T-Bills</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>60.5</td>
<td>61.5</td>
</tr>
<tr>
<td>2 years</td>
<td>64.9</td>
<td>65.5</td>
</tr>
<tr>
<td>5 years</td>
<td>70.2</td>
<td>73.3</td>
</tr>
<tr>
<td>10 years</td>
<td>79.6</td>
<td>79.6</td>
</tr>
<tr>
<td>20 years</td>
<td>91.5</td>
<td>94.3</td>
</tr>
<tr>
<td>30 years</td>
<td>99.4</td>
<td>97.00</td>
</tr>
</tbody>
</table>

As the holding period increases, the chance that not stocks will not have superior performance diminishes. With a 10 year holding period, stocks outperform bonds in 79.6% of the periods, and T-bills in 79.6% of the periods. With a 20 year holding period, stocks outperform bonds in 91.5% of the periods, and T-bills in 94.3% of the periods. And with a 30 year holding period, bonds

\(^{2}\text{Siegel (1998)}\)
period, stocks outperform bonds in 99.4% of the periods, and T-bills in 97.0% of the periods.

The extraordinary likelihood in the historical data that stocks will outperform bonds and bills in the long run is the product of two factors. First, the longer holding period diversifies away some of the risk over time, allowing for the higher average return to tell. Second, long-run mean reversion in stock prices means that the risk of holding stocks diminishes as the time horizon increases.

**TABLE 2**

HOLDING PERIOD RISKS FOR ANNUAL REAL RETURNS

<table>
<thead>
<tr>
<th></th>
<th>1 year</th>
<th>2 years</th>
<th>5 years</th>
<th>10 years</th>
<th>20 years</th>
<th>30 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stocks</td>
<td>18.2%</td>
<td>13.0%</td>
<td>7.3%</td>
<td>4.5%</td>
<td>2.7%</td>
<td>1.6%</td>
</tr>
<tr>
<td>Bonds</td>
<td>8.8%</td>
<td>6.9%</td>
<td>5.1%</td>
<td>4.1%</td>
<td>3.2%</td>
<td>2.3%</td>
</tr>
<tr>
<td>Fixed income</td>
<td>6.1%</td>
<td>5.2%</td>
<td>4.2%</td>
<td>4.4%</td>
<td>3.1%</td>
<td>2.4%</td>
</tr>
</tbody>
</table>

Specifically, with a 10 year holding period stocks, bonds, and fixed-income instruments have relative standard deviation equal to 4.5%, 4.1% and 4.4%

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3Siegel (2002)
respectively. With a 20 year holding period the gap shrinks dramatically: the numbers are 2.7%, 3.2% and 3.1% respectively. And with a 30 year holding period gap between stocks and bonds disappears completely: the numbers are 1.6%, 2.3% and 2.4% respectively\(^4\).

Fama and French (1988) and Poterba and Summers (1988) point out that due to the mean-reverting nature of stock prices, long periods of high stock returns are usually followed by long periods of low stock returns. There is little sign of the same process in bonds and fixed-income instruments. Instead, unpredictable inflation makes real returns on bonds and fixed-income instruments very volatile over the long run, while stocks have proved to provide very good protection against inflation in the long run.

\(^4\)Average returns and standard deviations are not complete descriptions of the relative return distribution. It is useful to consider the differences between the maximum and minimum returns offered by portfolios of different assets held for different holding periods (see Siegel (2002)). With a 10-year holding period, stocks, bonds and fixed-income instruments have differences between their maximum and minimum real holding period rates of return equal to 21.0%, 17.8% and 16.7% respectively. Then, as we move to a 20-year holding period, the gap shrinks rather dramatically: the numbers are 11.6%, 11.9% and 11.3% respectively. Finally, with a 30-year holding period, stocks are less risky according to this metric: the numbers are 8%, 9.4% and 9.4.% respectively.
As J. Siegel (2002) notes, stocks are claims on the earnings of real assets. Since inflation has roughly equal influences on both input and output prices, it should have little effect on profits and thus on stock values in the long run. Modigliani and Cohn (1979) point out, however, that in the short run stocks perform badly in periods of inflation.

The reasons that diversified portfolios of stocks provide higher average returns and lower risk when held are still in dispute.

Siegel (2002) points out that one partial possibility might be that bond holders did not expect high inflation and, therefore, accepted lower nominal returns than turned out to be required to compensate for the inflation of the 1970s and the 1980s. But the high equity premium is a broader phenomenon. The equity premium was high before World War II, under the gold standard, when gold’s nominal anchor to the price level greatly diminished chances of inflation. The high equity premium persists today: compare the 6% annual earnings yield of the S&P 500 to the less than 2.5% real yield today on U.S. Treasury Inflation Protected Securities. Inflation illusion is not a successful explanation.

A second possibility is that fear of a repeat of the 1929 crash led investors to avoid stocks and accept low returns on seemingly safer fixed-income in-
struments. During and after World War II interest rates on bonds were depressed by the bond support policies and interest rate controls of the Federal Reserve. Investors care not just about long-run returns but also about short-run safety, and bonds and other fixed-income investments do carry a smaller risk of significant short-run loss than do stocks. Investors are pursuing real benefits by bidding up the prices of bonds. The problem is that the gap between long-term returns seems large given the minimal differences in long-term risks.

A political-economy possibility is fear of a high US corporate income tax. When the corporate income tax was first introduced in 1909, it was only 1%. Then, as the size of the government increased, the corporate income tax reached as high as 50 percent by 1980. Corporate income taxes differentially affect stocks and bonds: fear that the corporate income tax might jump again could well establish an apparent equity premium. Moreover, many kinds of government regulation affect stockholders as the residual claimants but not bondholders. Antitrust, environment, the wars on the tobacco industry, a legal system that places the costs of social disasters like asbestos entirely on current shareholder and not on other past beneficiaries of fire insulation—all these might make shareholders leery. However, government regulation
that puts a disproportionate burden on equity holders is likely only when equity holders are a small class. If everybody’s Social Security trust fund was invested in equities, political enthusiasm for imposing disproportionate tax or regulatory burdens on equity holders would vanish.

The higher average returns and lower long-run risks of diversified investments in stocks are facts. The question is whether these historical patterns can be projected into the future. Past performance is no guarantee of future results. But it is hard to see a world in which the return on equities falls significantly.

Consider a Cobb-Douglas production function:

\[ y = K^a L^{1-a} \]

Then

\[ a \frac{y}{K} = r, \]

where \( r \) is the rate of return on capital, which must be closely tied to the rate of return on equities. To significantly reduce the rate of return on equities would thus require a huge increase in the savings-investment share and an explosion of the capital-output ratio. It is hard to see how such a
scenario could come about. Investments in stocks are likely to pay very high returns over the long run. Barro (2005) provides a very powerful argument for continuing equity premium by allowing low-probability disasters.

Suppose that the equity premium falls not through a decline in expected returns on stocks but a rise in expected returns on bonds. Would that be a problem for the argument of this paper? No. The key point is that investing the trust fund in individual accounts offers much higher returns than a continuation of the current pay-as-you-go system. If bonds as well as stocks offer such attractive risk/return possibilities, the case for private accounts is not weakened but strengthened.

III. MODEL

I start this section with a review of European put and call options for readers who don’t have the definitions at their fingertips. A European call option is a security that gives its holder the right to purchase a share of its underlying security at a fixed exercise price on the maturity date of the option. A European put option is a security that gives its holder the right to sell a share of its underlying security at a fixed exercise price on the maturity date of the option.
According to the proposed system, the government would require that individuals buy insurance on their retirement portfolios at the time as they make their retirement contributions. Thus, individuals invest $P_{jt}$ in some portfolio of assets and purchase a put option for a price $Put_{jt}$ with a strike price $S$ written on this portfolio.

Therefore, by investing $P_{jt} + Put_{jt}$ today, tomorrow investor obtains

$$W_{t+1} = P_{jt+1} + \max[0, S - P_{jt+1}]$$

How do we value this put option? Consider a two-period economy. At period $t$, the price of the asset $j$ $P_{jt}$, the risk-free rate $r_f$, and the strike price $S$ are observed. $\tilde{C}$ is the aggregate consumption. We assume that $\ln P_{jt+1}$ and $\ln \tilde{C}$ are bivariate normally distributed with means $\left( \mu_j, \mu_c \right)$ and the variance-covariance matrix:

$$\begin{pmatrix}
\sigma^2_j & k\sigma_j\sigma_c \\
 k\sigma_j\sigma_c & \sigma^2_c \\
\end{pmatrix},$$

where $k$ is the correlation coefficient between $\ln P_{jt+1}$ and $\ln \tilde{C}$. Preferences are specified as

$$U = \frac{1}{1-B}C_t^{1-B} + \rho \frac{1}{1-B}C_{t+1}^{1-B}$$
At period $t+1$ an option is either exercised or not, depending on the value of $P_{jt+1}$.

Then following Rubinstein (1976) the price of a call option is

$$Call_{jt} = P_{jt}[N(Z_S + \sigma_j)] - S \cdot \frac{[N(Z_S)]}{1 + rf}, \hspace{1cm} (1)$$

where

$$Z_S = \frac{ln\left(\frac{P_{jt}}{S}\right) + ln(1 + r_f)}{\sigma_j} - .5\sigma_j,$$

$$R_j = r_j + 1 = \frac{P_{jt+1}}{P_{jt}},$$

$$\sigma_j^2 = VAR[ln(R_j)],$$

$N(\cdot)$ is the cdf of a standard normal variable. This is famous Black-Scholes formula.

It could be easily shown that (with all other parameters fixed) a put option is a monotonically increasing function of the variance of the portfolio the put is written on. Indeed, the higher the variance, the higher the profit that can be realized by exercising the put. But the variance of the portfolio represent total risk of the portfolio.
I specifically interested in the effect of the systemic component of this risk on the value of this put option. For this purpose, I use the Capital Asset Pricing Model.

We know that according to CAPM

\[ R_j = R_f + \beta_j R_m + \epsilon_j, \quad (2) \]

where \( R_f = r_f + 1, R_m \) stands for the equity premium and \( E[\epsilon_j] = 0 \).

We need to find the derivative of \( Call_{jt} \) with respect to \( \beta_j \).

Now,

\[ Call_{jt} = f(P_{jt}, \sigma_j(\beta_j), S, R_f), \quad (3) \]

We need to find \( \sigma_j \) as a function of \( \beta_j \).

We know that

\[ \ln(R_j) \sim N(\mu_j, \sigma_j^2). \]

Hence, following Rubinstein and Cox (1985)

\[ \sigma_j^2 = e^{2\mu_j} [e^{2\sigma_j^2} - e^{\sigma_j^2}], \quad (4) \]

where
\[ \sigma_j^2 = \text{VAR}[\ln(R_j)], \]
\[ \sigma_j'^2 = \text{VAR}[R_j], \]
\[ \mu_j = E[\ln(R_j)]. \]

Also,

\[ \ln(E[R_j]) = \mu_j + 0.5\sigma_j^2 \quad (5) \]

and from (5) we obtain

\[ e^{2\mu_j} = \frac{E^2[R_j]}{e^{2\sigma_j^2}}. \quad (6) \]

Substituting (6) into (4) we get

\[ e^{\sigma_j^2} = \frac{E[R_j^2]}{E^2[R_j]}. \quad (7) \]

Using (2) we obtain

\[ R_j^2 = R_f^2 + \epsilon_j^2 + 2R_f\epsilon_j + \beta_j^2 R_m^2 + 2\beta_j R_m R_f + 2\beta R_m \epsilon_j \]

So, taking expectations we obtain

\[ E[R_j^2] = R_f^2 + \beta_j^2 E[R_m^2] + 2\beta_j R_f E[R_m] + E[\epsilon_j^2] \quad (8) \]

\[ E^2[R_j] = R_f^2 + \beta_j^2 E^2[R_m] + 2\beta_j R_f E[R_m] \quad (9) \]
Hence, substituting (8) and (9) into (7), we have

\[ e^{\sigma_j^2} = \frac{R_f^2 + \beta^2 E[R_m^2] + 2\beta R_f E[R_m] + E[\epsilon_j^2]}{R_f^2 + \beta^2 E[R_m^2] + 2\beta R_f E[R_m]} \]  

(10)

Set

\[ g(\beta) = \frac{R_f^2 + \beta_j^2 E[R_m^2] + 2\beta_j R_f E[R_m] + E[\epsilon_j^2]}{R_f^2 + \beta_j^2 E[R_m^2] + 2\beta_j R_f E[R_m]} \]  

(11)

We can conclude from (10) that

\[ \sigma_j = \sqrt{\ln\left(\frac{R_f^2 + \beta_j^2 E[R_m^2] + 2\beta_j R_f E[R_m] + E[\epsilon_j^2]}{R_f^2 + \beta_j^2 E[R_m^2] + 2\beta_j R_f E[R_m]}\right)} = \sqrt{\ln(g(\beta_j))} \]

Differentiating (3) we obtain

\[ \frac{dCall_{jt}}{d\beta_j} = \frac{df}{d\sigma_j} \frac{d\sigma_j}{d\beta_j} = \frac{1}{2} \cdot \frac{S \cdot n(Z_s)}{1 + r_f} \cdot \frac{S \cdot n(Z_s) \ln^{-\frac{1}{2}}(g(\beta))g(\beta)^{-1} \frac{dg}{d\beta_j}} \]

where, \( n(\cdot) \) is the pdf of a standard normal variable

Differentiating \( g \) with respect to \( \beta_j \), we obtain

\[ \frac{dg}{d\beta_j} = \]

\[ \frac{\beta_j^2 2R_f E[R_m] (E[R_m^2] - E^2[R_m]) + 2\beta_j (E[R_m^2] R_f^2 - E^2[R_m] (R_f^2 + E[\epsilon_j^2])) - 2R_f E[R_m] E[\epsilon_j^2]^2}{R_f^2 + \beta_j^2 E[R_m^2] + 2\beta_j R_f E[R_m]} \]
I claim that there exists a $\beta_j^*$ large enough such that

$$\frac{dg}{d\beta_j} > 0$$

for all

$$\beta > \beta^*$$

Indeed,

$$\frac{dg}{d\beta_j} > 0$$

if and only if

$$\beta_j^2 2R_f E[R_m](E[R_m^2] - E^2[R_m]) + 2\beta_j (E[R_m^2] R_f^2 - E^2[R_m](R_f^2 + E[\epsilon_j^2])) - 2R_f E[R_m] E[\epsilon_j^2] > 0$$

Define

$$A = 2R_f E[R_m](E[R_m^2] - E^2[R_m])$$

$$B = 2(E[R_m^2] R_f^2 - E^2[R_m](R_f^2 + E[\epsilon_j^2]))$$

$$C = -2R_f E[R_m] E[\epsilon_j^2]$$

$$D = B^2 - 4AC$$

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Now, because
\[ E[R_m^2] - E^2[R_m] = VAR[R_m] > 0 \]
we can conclude that
\[ A > 0 \]

We also know that
\[ C < 0 \]

Therefore,
\[ D > 0 \]

Hence,
\[ \beta_j^* = \frac{-B + \sqrt{D}}{2A} \]

Also note that
\[ Put_{jt} = \frac{S}{1 + rf} - P_{jt} + Call_{jt} \]

So, differentiating again we obtain
\[ \frac{dPut_{jt}}{d\beta_j} = \frac{dCall_{jt}}{d\beta_j} \]
According to the proposed system, individuals will be required to invest $P_{jt}$ in some portfolio of assets and also to "insure" by purchasing a put option written on this portfolio, with a strike price $S$ equal to the initial investment $P_{jt}$. The gross rate of return on this portfolio $\frac{P_{jt+1}}{P_{jt}}$ is lognormally distributed. So, consider investing $P_{jt} = $1 for 20 years in a portfolio with $\beta = 1$, with a real expected rate of return of 6.5% and with a standard deviation of .1 annually for 20 years. In addition, investors purchase a put option that guarantees that 20 years from now investors will realize at least a real rate of return of 0% on this portfolio. How likely is it that this put option will be exercised? It is only .69%! At the same time the probability that the initial investment of $1 is going to be more than doubled is overwhelming. It is 81.6%. As we move to a 30 years holding period the probability of exercising becomes even smaller. It is only .13%. The probability that the

\[ S = P_{jt}(1 + r_f)^T \]

Why not to set $S = P_{jt}(1 + r_f)^T$? Because this way we will be executing a very expensive strategy. A strategy that guarantees at least an annual risk-free rate with a very low probability and a 6.5% annual return with a very high probability. Because of the No Arbitrage condition this strategy is going to be so expensive that after factoring in all the costs, the resulting rate of return will be equal to $r_f$. 

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initial investment of $1 is going to be more than doubled becomes even larger. It is 95.88%. See Figure 1 above. These theoretical probabilities are very consistent with the empirical probabilities calculated in Siegel (1998) considered in Section II. As we shall see below, this low probability of an exercise will make the price of a put option very low indeed.
Figure 2:

Figure 2 above shows the investor’s end-of-worklife wealth $W_{t+1} = P_{jt+1} + \max[0, S - P_{jt+1}]$ thirty years from now as a function of the investor’s end-of-worklife market value $P_{jt+1}$ of this asset portfolio. As long as $P_{jt+1} \leq S = 1$, the put option will be exercised. Then the investor’s wealth $W_{t+1} = S$ will be $\$1$. If $P_{jt+1} \geq 1$, then the investor’s wealth will be $W_{t+1} = P_{jt+1}$,
represented by a 45 degree line on the figure.

So, how expensive would it be to insure a diversified equity portfolio of the kind appropriate for an individual Social Security retirement account against the risk of collapse in its value? I use here a particular metric to price this insurance. This metric is the Black-Scholes price of a long-term put option written on this diversified equity portfolio. The answer is that the price of such a put option is surprisingly low. It is low even in the world we live in, where price of risk is very high.

In our first set of calculations we fix the portfolio’s $\beta$ at 1. Let us calculate the price of a put option written on this portfolio, using the Black-Scholes formula, as a function of the annual standard deviation $\sigma$. 
Let a risk-free rate $r_f$ be at 2% and a time horizon $T$ at 30 years. The results are shown in Figure 3 above. As the annual standard deviation of the portfolio increases, the price of the put option also increases. However, the price of put option remains low. For an annual standard deviation of portfolios as high as 20%, the price of the put option is only 15% of the original investment.
For a risk-free rate $r_f$ at 1% and a time horizon $T$ at 30 years, the Black-Scholes price of put option is slightly higher but still remains low. The results are shown in Figure 4 above. For an annual standard deviation as high as 15%, the price of the put option is only around 15% of the original investment.

Figure 4:
For a risk-free rate $r_f$ at 2% and a time horizon $T$ at 40 years the Black-Scholes price of put option once again remains low. The results are shown in Figure 5 above. For an annual standard deviation as high as 22%, the price of the put option is only 15% of the original investment.
For a risk-free rate $r_f$ at 1% and a time horizon $T$ at 40 years the Black-Scholes price of put option is slightly higher but still remains low. The results are shown in Figure 6 above. For an annual standard deviation as high as 10%, the price of the put option is only 9% of the original investment.
In our second set of calculations, we fix the annual standard deviation \( \sigma \) of a diversified stock portfolio at 10%, but let the portfolio’s \( \beta \) to vary. Let us calculate the price of a put option written on this portfolio, using the Black-Scholes formula, as a function of the portfolio’s \( \beta \). Set the risk-free rate \( r_f \) at 2% and the time horizon \( T \) at 30 years. As the the portfolio’s \( \beta \) increases, the price of the put option also increases. However, the price of put option remains low. Results are shown in Figure 7 below. For \( \beta = 2 \) the price of the put option is only 12% of the original investment.
Figure 7:

$f=2\%$, Standard Deviation $=10\%$, $P_r=1$, 30 years
For a risk-free rate $r_f$ at 1% and a time horizon $T$ at 30 years and annual standard deviation $\sigma$ at 10% the Black-Scholes price of the put option is slightly higher, but still remains low. The results are shown in Figure 8 above. For $\beta = 1$, the price of the put option is only 7% of the original investment.
Figure 9:

For a risk-free rate $r_f$ at 2% and a time horizon $T$ at 40 years and an annual standard deviation $\sigma$ at 10% the Black-Scholes price of put option once again remains low. The results are shown in Figure 9 above. For $\beta = 1$, the price of the put option is only 2% of the original investment.
Figure 10:

For a risk-free rate $r_f$ at 1% and a time horizon $T$ at 40 years and an annual standard deviation $\sigma$ at 10% the Black-Scholes price of the put option is slightly higher, but still remains low. The results are shown in Figure 10 above. For $\beta = 1$, the price of the put option is only 7% of the original investment.
V. ARE PRIVATE COMPANIES CAPABLE OF INSURING RETIREMENT PORTFOLIOS?

The calculations performed in this paper above can be interpreted in two ways. First, they can be interpreted as yardsticks of how worried those investing their retirement wealth—specifically, their Social Security wealth—in equities should be of the downside risk that the stock market will permanently collapse. In a well-functioning market, the actuarially-fair (including risk aversion) price of insurance is a convenient yardstick for the seriousness of the potential loss that the insurance would cover. In this situation, the price of insurance is the initial cost of the put option. The fact that the put option is cheap relative to the value of the portfolio reveals that the risk is not of high order. Investors and households have other more important things they need to worry about than this source of risk.

The second way to interpret the calculations is as a trial run for the cost of a implemented system in which the government allows individuals to invest but requires them to buy insurance on their retirement portfolios so that the government is not left holding the bag, forced by simple humanity to compensate after the fact those who took extravagant risks that turned
out badly.

Some argue that the solvency of our Social Security system can only be guaranteed by the government acting as a lender of last resort. They use this argument to conclude that the Social Security system should never be privatized. However, insuring retirement portfolios against the permanent collapse of the stock market is the same as insuring against a low-probability catastrophic event. Potential losses associated with this collapse have a risk profile like that associated with events like earthquakes. According to Jaffee and Russell (2006), 1/3 of all residential earthquake policies are written by private insurance companies. Why could not we let private insurance companies insure retirement portfolios? Jaffee and Russell (2006) also point out that the US property casualty industry has a carrying capacity of approximately $400 billion dollars. Hence, insuring retirement portfolios should be well within the reach of the private sector.

Setting up explicit insurance policies for long-term stock market risk requires that insurance companies engage in dynamic long-term hedging strategies to lay off the risk associated with writing the policies equivalent to the put options. We are going to consider here how a private insurance company might design the delta-hedge to provide all the necessary funds to cover the
insured losses.

Consider portfolio \( Z_t = [\Delta, -1] \) consisting of \( \Delta \) shares of some asset with a price \( p_t \) and \( -1 \) shares of a put option written on this asset with a strike price \( S \), where

\[
\Delta = \frac{\partial \text{Put}_{jt}}{\partial p_{jt}} = N(Z_S + \sigma_j) - 1 < 0
\]

The resulting portfolio is going to be an instantaneously risk-free one. This hedging strategy is very cheap. Table below illustrates the situation.

<table>
<thead>
<tr>
<th>Risk-free Rate, ( r_f )</th>
<th>Time to Expiration, ( T )</th>
<th>Put’s, ( \Delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>.02</td>
<td>40</td>
<td>-.06</td>
</tr>
<tr>
<td>.01</td>
<td>40</td>
<td>-.17</td>
</tr>
<tr>
<td>.02</td>
<td>30</td>
<td>-.09</td>
</tr>
<tr>
<td>.01</td>
<td>30</td>
<td>-.21</td>
</tr>
</tbody>
</table>

Fix annual standard deviation \( \sigma \) at 10%, the initial stock price \( P_{jt} \) at 1 and the strike price \( S \) also at 1. For a risk-free rate \( r_f \) at 2% and a time horizon \( T \) at 40 years \( \Delta \) is only -.06. For a risk-free rate \( r_f \) at 1% and a time horizon \( T \) at 40 years \( \Delta \) is smaller -.17. For a risk-free rate \( r_f \) at 2% and
a time horizon $T$ at 30 years $\Delta$ is higher -.09. For a risk-free rate $r_f$ at 1% and a time horizon $T$ at 30 years $\Delta$ is only -.21.

The ability to execute this hedging strategy hinges critically on the infrastructural strength of our financial markets. This strength was seriously tested in October of 1987, when a simultaneous sell out of a massive number of shares triggered a meltdown of the DOT system making it impossible to trade. As a result of the market crash of 1987 the Presidential Task Force on Market Mechanisms (1988) was formed. This task force produced a set of recommendations otherwise known as the Brady Commission Report. The thorough implementation of these recommendations has made future infrastructural failures a very low probability event.

Events like the multibillion dollar collapse of Long-term Capital Management and that of Amaranth Hedge Fund earlier this year are very much isolated events. Markets have been not only able to survive them, but thrive despite of them.

A market for these long-term put options does not exist yet, but it could be created once a proposed system is implemented, in which the government requires that individuals buy insurance on their retirement portfolios at the time they make their retirement contributions.
VI. CONCLUSION

I believe that thoughtful economists on both the left and the right should be enthusiastic about Social Security privatization. On the right, economists should be enthusiastic because Social Security privatization both limits government power and enhances individual choice. On the left, economists should be enthusiastic because private accounts invested in diversified portfolios of equities promise extremely high and low-risk returns for that half of Americans who have effectively no involvement in the stock market.

The two great empirical regularities in the performance of the aggregate stock market are (a) the high average return earned by investments in equities and (b) the fact of long-run mean reversion in stock prices. Together, these mean that long-run investments in diversified portfolios of equities are both high return and low risk.

Many critics of Social Security privatization fear that retirement portfolios invested in equities are in fact overwhelmingly risky. They fear that such portfolios could not be made safe: that they could not be insured by the private sector because the level of systemic risk that they carry is, in fact, much above what an insurance company will consider unacceptable. They further
argue that if the stock market slumps, then those who had invested their Social Security money in risky securities could see their funds go bankrupt—leaving the public with the choice of either watching retirees starve on the street in their old age, or picking up the costs and thus subsidizing those who had made overly risky investments.

Yet, based on two hundred years of American financial history, the probability that a well-diversified leveraged portfolio will lose a substantial part of its value and will not be able to regain it within a short period of time is very low. Hence, insuring well-diversified leveraged retirement portfolio should be cheap. How cheap? We model the situation by assuming that the insurance premium that would be charged to make a diversified portfolio of equities safe has to be equal to the price of a Black-Scholes put option written on this portfolio. For reasonable values of real risk-free interest rates, market risks, time horizons of 30 and 40 years, the price of the put is not more than 10-15% of the value of the portfolio this put is written on.

There are two different ways to interpret calculations performed in this paper. First, they can be interpreted as a measure of how concerned investors are that their retirement portfolios invested in equities will permanently collapse. The low price of the put option reveals that these concerns are not
very serious.

The second way to interpret the calculations is as a trial run for a system in which the government would require that individuals buy private insurance on their retirement portfolios at the time they make their retirement contributions.

Private insurance has several advantages. First, the agents will be exposed to risks and will buy insurance against these risks according to their own degree of risk aversion. Second, insurance contracts can be structured to reduce Moral Hazard. By contrast, an explicit insurance guarantee by a government serves, in effect, as an enabler of Moral Hazard.

I believe that I have demonstrated that a combination of a high equity premium and a mean reversion in stock prices create a great potential for making our Social Security System a very safe and a productive vehicle for the retirement funds’ accumulation.

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