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LECTURE NOTES: Robinson Crusoe Meets Walras and Keynes*

Once upon a time on an idyllic South Seas Isle lived a shipwrecked sailor, Robinson Crusoe, in solitary splendor. The only product of the island, fortunately adequate for Robinson's sustenance, was the wild yam, which Robinson found he could collect by refraining from a life of leisure long enough to dig up his dinner. With a little experimentation, Robinson found that the combinations of yams and hours of leisure he could obtain on a typical day (and every day was a typical day) were given by the schedule shown in Figure 1.

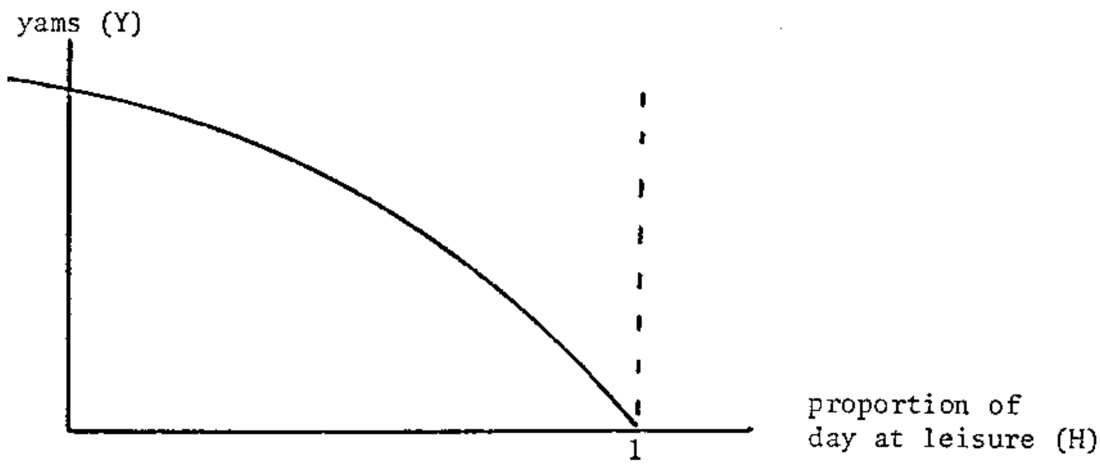

FIGURE 1

Being a rational man, Robinson quickly concluded that he should on each day choose the combination of yams and hours of leisure on his production schedule which made him the happiest. Had he been quizzed by a patient psychologist, Robinson would have revealed the preferences

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*With a little help from Axel Leijonhufvud and Hal Varian.

illustrated in Figure 2, with $I_1, I_2, \ldots$ representing indifference curves and I_1 worse than I_2, etc.

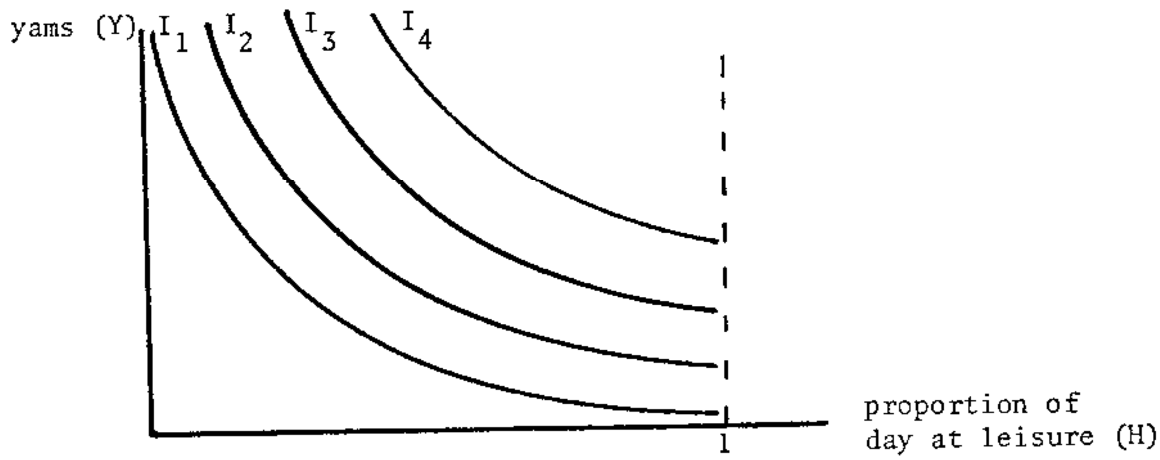

FIGURE 2

The result of Robinson's choice is seen by superimposing Figures 1 and 2; clearly, this combination of yams and leisure, denoted E in Figure 3, is the best point he can reach given the technology at his disposal.

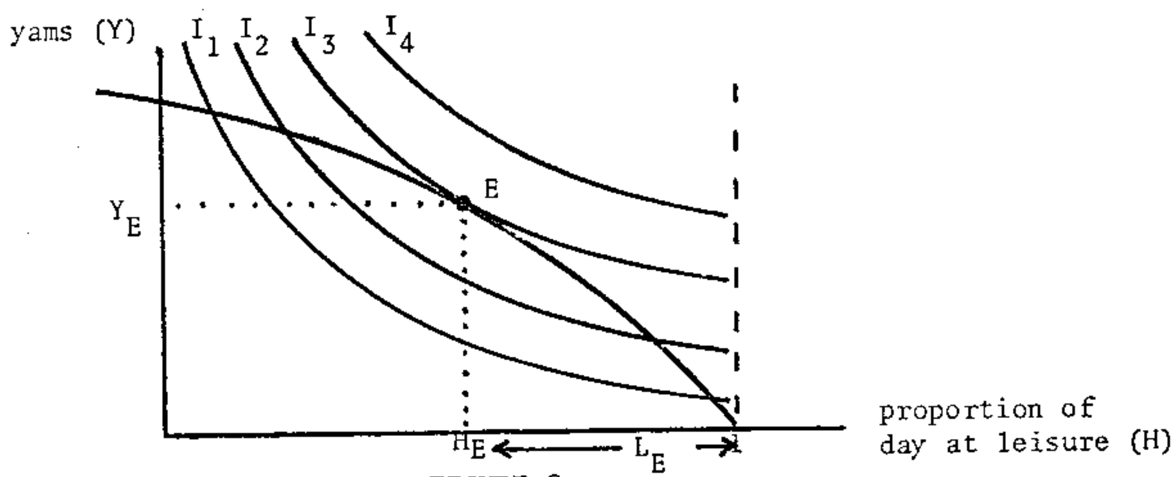

FIGURE 3

Robinson consumes Y_E yams at this point and spends H_E proportion at leisure out of his available endowment of 24 hours per day. The remaining time, $L_E = 1 - H_E$, is spent at labor digging up yams.

Robinson was reconciled to a peaceful, if tedious, existence consuming at the combination E each day. However, he was startled one morning to find wading through the breakers a rather scholarly gentleman in bedraggled formal attire who introduced himself as León Walras. It was quickly established that Walras was a student of the organization of economic activity. Walras was at first content to sit on the beach all day, watching Robinson dig yams and making occasional helpful comments on the use of clam shells to sharpen digging sticks. (Walras never partook of yams himself, but disappeared every evening. Robinson suspected Walras of seeking the companionship and table of Gauguin, who was leading a more bohemian existence on a neighboring island.)

One day, Walras remarked "It must be difficult when engaging in a mindless activity like digging yams to remember exactly what yam-leisure combination maximizes your preferences." Robinson really had no trouble at all, but to be agreeable he assented. Walras then said "You know, the way you run your life is rather feudal. If you would like, I will help you reorganize using the most modern techniques, direct from Paris*." Robinson was rather suspicious of this offer, since he had noted that like most economists, Walras was better at giving advice than digging yams. However, this promised a diversion from his daily drudgery, and he was anxious not to appear old-fashioned to his distinguished visitor. So he agreed to go along with the experiment.

Walras then outlined his proposal. "You, Robinson, should form a yam-digging company. Call it Crusoe, Inc. Let me introduce an acquaintance, Mr. Friday, who is standing just behind that palm tree, and whom I recommend you appoint as manager of Crusoe, Inc." Robinson was alarmed at this point at the appearance of a young man dressed, like most profesional managers, in a grey flannel Brooks Brothers suit. However, Mr. Friday's

*Walras was not actually from the Paris school, but when dealing with simple folk avoided raising the possibility that Frenchmen could live outside Paris. (ed.'s note) ready knowledge of yam-digging and corporate finance quickly reassured Robinson, particularly after Mr. Friday explained that his management fee would be exactly equal to the additional yams dug per hour under his direction. This meant that Robinson would in effect have the same leisure-yams transformation schedule in Figure 1 as before, and that he would not have to worry about where to dig. So Robinson agreed to the arrangement, and asked Walras to finish describing his plan.

"Each morning," said Walras, "I will call out a wage rate in yams per hour. You should instruct Mr. Friday, as your manager, to offer to hire the amount of labor and produce the quantity of yams which maximize your dividends as owner of Crusoe, Inc. He should report these offers to me, and should inform you of the dividends he expects to pay you. Given this dividend income and the wage rate I have called out, you should inform me of the amount of yams you want to demand. If your supplies and demands don't coincide with Crusoe, Inc., then I will call out another wage rate, and we'll start over. When we finally hit a wage rate where supply equals demand, then I'll stop and you and Mr. Friday can trade the amounts of labor and yams you offered."

"Could you run through that again from the top?" asked Robinson.

"Certainly," said Walras. "Let's start with the instructions to Mr. Friday. Suppose I call out a wage rate w, say w = 10 yams per hour. Suppose we draw a graph (Figure 4) of various combinations of labor in and yams out available to Crusoe, Inc."

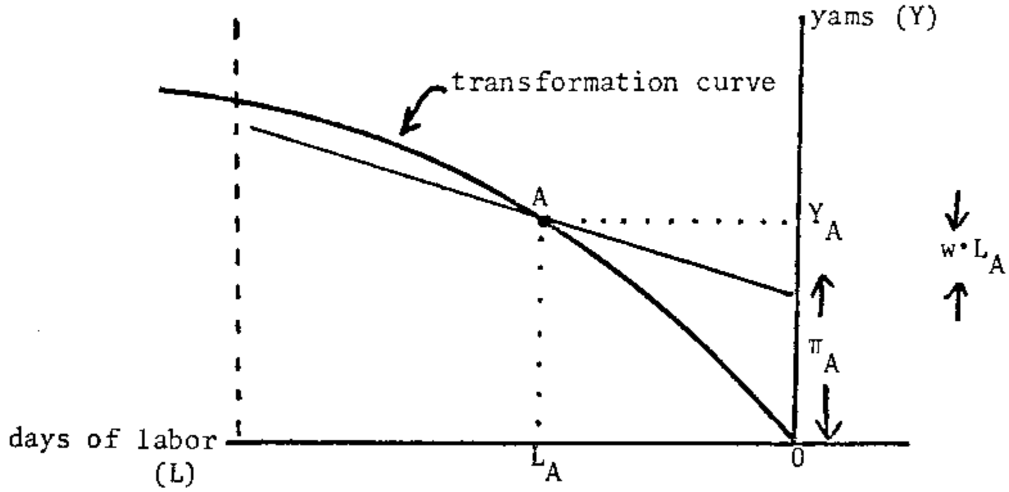

FIGURE 4

Robinson noted that this was the same graph as his Figure 1, except that instead of placing the origin at zero leisure and measuring leisure to the right, Walras was placing the origin at zero labor (or 24 hours of leisure) and measuring labor to the left. Walras continued, "Let us see what dividend income you would receive if Mr. Friday offered to operate the firm at point A. He would supply Y_A yams and demand L_A hours of labor. His profit, which you as owner would receive as dividend income, would be the value of his output less the value of his input. Since we are pricing things in yams, profit is

$$\pi_A = Y_A - w \cdot L_A \quad ,$$

in symbols." Walras explained that π was the Greek letter "pi," used by economists as a symbol for profits. Since Robinson looked somewhat puzzled, Walras went on to point out that if a line with slope $-w$ were drawn on the graph through A, then the points A, Y_A, and π_A formed a triangle whose horizontal side was L_A, and whose vertical side then had to be $w \cdot L_A$, and the cost of labor. (Walras reminded Robinson that the slope of a line was given by the ratio of the vertical to the horizontal sides of any right angle whose hypotenuse lay on the line; thus slope $= -w =$ vertical/horizontal $= -wL_A/L_A$.) Then the quantity of yams labeled π_A in the graph equaled the amount of profit defined by the formula. "Clearly," said Walras, "given your instructions to maximize profit, Mr. Friday will choose to operate at the point B in Figure 5." With a little scribbling, Robinson convinced himself that any combination of labor and yams in the line through B and π yielded profit in yams of π,

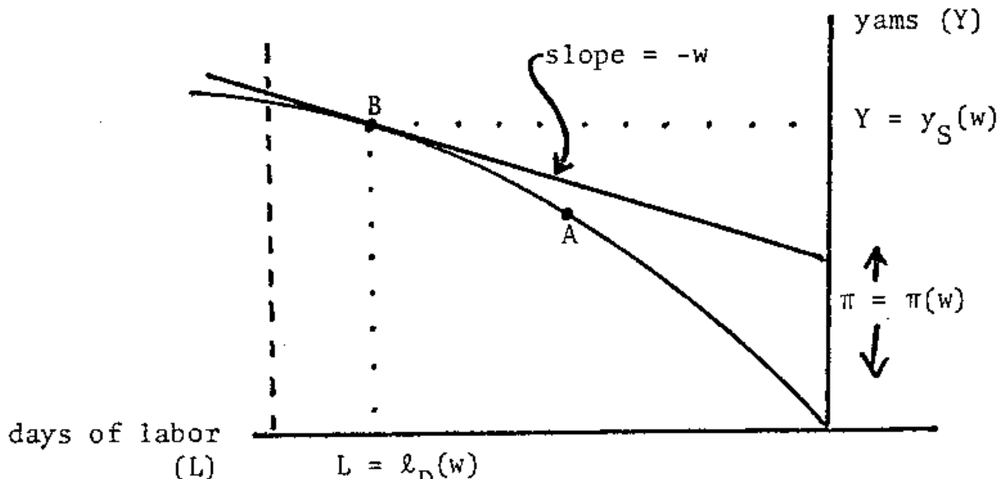

FIGURE 5

that profit increased when one had more yams or less labor, and hence that B represented the best place to operate the firm to maximize profits. Walras said, "Please note that the amount of labor demanded, yams supplied, and maximum profits depend on the wage rate w. As I call out various wage rates, Mr. Friday will respond (in accord with your instructions to maximize profits) with schedules something like those graphed in Figure 6 (labor demand), Figure 7 (yam supply), or Figure 8 (profits)." Walras told Robinson that as a consumer he need not worry about the shape of these curves, as he would need to respond only to non-wage income and the wage rate. However, Walras suggested that if Robinson were curious, he could

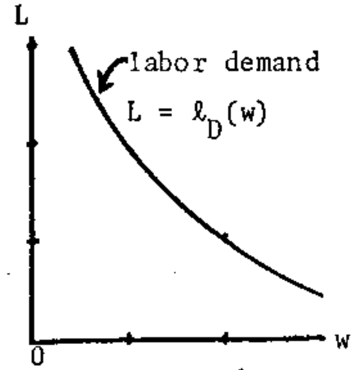

FIGURE 6

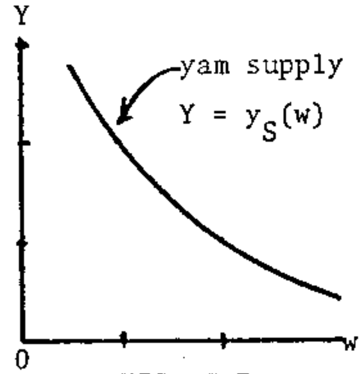

FIGURE 7

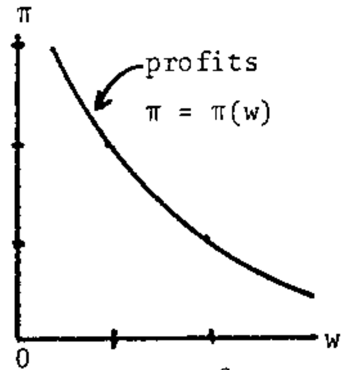

FIGURE 8

look at the offers Mr. Friday would make at different wage rates, as in Figure 9. Robinson made a brief show of interest, but soon Walras was

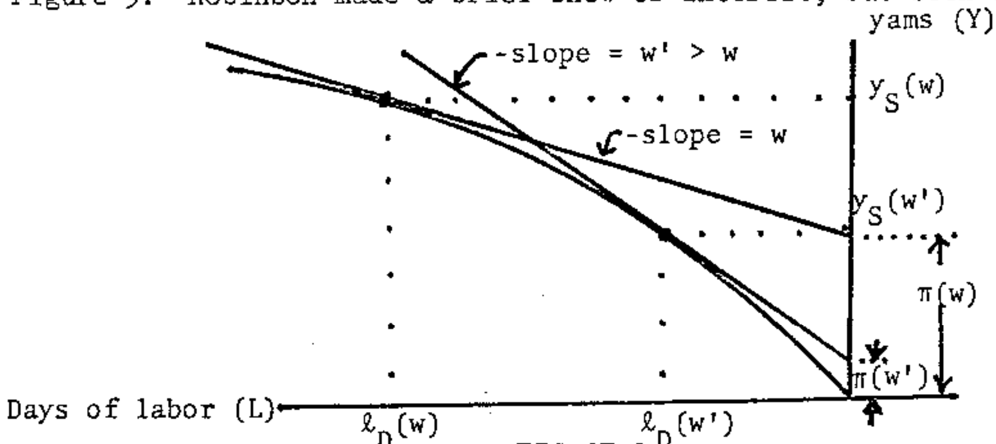

FIGURE 9

off again. "Now you, Robinson, will face the wage rate w I call out and the non-wage or dividend income $\pi(w)$ reported to you by Mr. Friday. Your opportunities will be represented by the budget line in Figure 10. If you

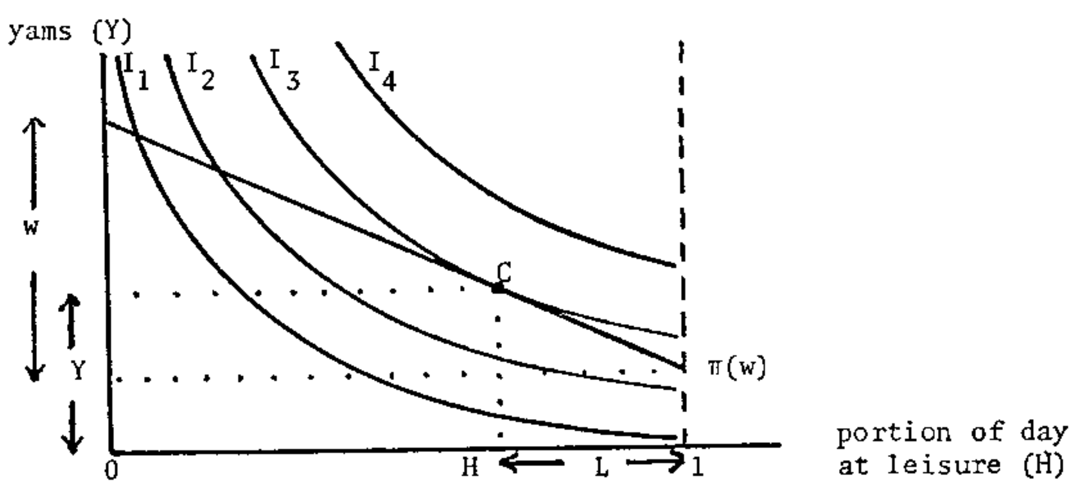

FIGURE 10

supply no labor, then you receive $H = 1$ day of leisure and an amount of yams $Y = \pi(w)$ equal to your dividend income. For each hour of leisure you give up to dig yams, you receive w additional yams worth of labor income. As a rational man, you will clearly want to choose the yam-leisure combination C which maximizes your preferences. Then you will offer to supply the amount of labor $L = 1 - H$, the difference between the total amount of leisure you are endowed with (1 day) and the amount you choose to consume, H. You will demand an amount of yams Y."

"Now wait a minute," said Robinson, "I may not be from Paris, but I wasn't born yesterday. I know that there are points on that budget line which couldn't possibly work. If I don't supply any labor, then Crusoe, Inc. can't produce any yams, and its profits will be zero, not $\pi(w)$."

"Not to worry," replied Walras, "the whole point of organizing your economy this way is that you don't <u>need</u> to worry about whether Crusoe, Inc. can actually provide bundles on your budget line. I just need information from you on what <u>you</u> would like <u>if</u> you had this budget line, your supplies and demands. It is <u>my</u> job to see that when we reach a final wage rate, your desires will be consistent with what Crusoe, Inc. provides." Walras continued "Don't you see what I have done for you? I have freed you from the nagging anxiety that your choices might not be consistent with the production possibilities of your economy. All you have to do is act like a modern rational consumer, calling out your supplies and demands for each budget line I mention." Robinson suspected he was being conned. However, Walras looked like he was prepared to go into a lengthy economic discourse on the topic, so Robinson said "Oh yes, I agree. Will I have demand and supply schedules like Mr. Friday's?"

"Yes indeed," said Walras, "but of course of somewhat different shape. When the wage rate is very high, your dividend income will be low, but your potential wage income is very high. If you are like most people, you will then choose to work only a small amount. Because you would be so well paid, you would end up consuming a great deal of both yams and leisure. An economist would say that your income effect, which makes you want to consume both yams and leisure in greater quantities when your income rises, has outweighed your substitution effect, which makes you want to consume more yams and less leisure (i.e., supply more labor) when the wage rate, which is also the relative price of leisure, rises.

"On the other hand, if the wage rate is very low, your dividend income is very high and the relative price of leisure is very low, leading both your income and substitution effects to push you in the direction of consuming a great deal of leisure, and supplying very little labor. Your yam demand will be high because of the income effect.

"At intermediate wage levels, you are likely to offer somewhat more labor and get less for it, so that your consumption of both yams and leisure will be lower than at the extreme wages. The result is that your demand curve for yams and supply curve for labor are likely to look like Figures 11 and 12, respectively."

"You can try varying the wage rate w and (as a consequence) dividend income $\pi(w)$ in Figure 10," continued Walras, "and see if your own preferences give yam demand and labor supply curves like these. Of course, the exact shape, and the question of how they behave at extreme

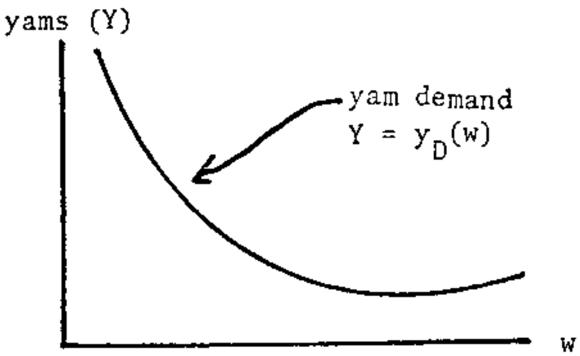

FIGURE 11

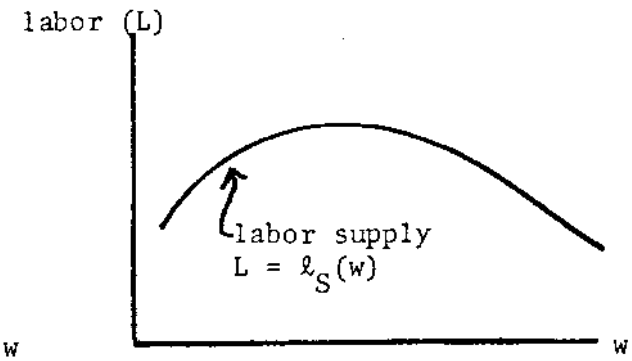

FIGURE 12

wage rates are very sensitive to the degree to which you are willing to substitute leisure for yams, and whether your tastes for both yams and leisure are normal in the sense that you want more of both when income rises.

"Perhaps," Walras said enthusiastically, "you would like me to run through my microeconomics course that I give in France. Then I can show you precisely how we define income and substitution effects, and how consumers in your situation with varying tastes might behave."

"I wouldn't want to put you to any trouble," Robinson hastily. "Perhaps we could postpone that and try out your new organization." The hour was late, so Robinson, Walras, and Mr. Friday agreed to start the following morning.

Early the next day, Walras called Robinson and Mr. Friday together, and began calling out wages. "One-half bushels of yams per day," said Walras. Mr. Friday responded "I want to buy 1 day of labor and sell 1 bushel of yams, and I will deliver 1/2 bushel of yams of dividend income to the consumer." After a quick calculation, Robinson replied "I don't want to sell any labor, and I will buy 1/2 bushel of yams with my dividend income."

"Aha!" said Walras, "demand for labor exceeds the supply of labor. Therefore, I am going to have to quote a higher wage." After several iterations, Walras found that at a wage rate of $\frac{1}{\sqrt{2}}$ yams per day , the labor demand of Crusoe, Inc. and the labor supply of Robinson were both 1/2 day, and demand equaled supply at $\frac{1}{\sqrt{2}}$ yams. He then told Robinson and Mr. Friday to trade these amounts. Figures 13 and 14 show the demand and supply curves Walras actually discovered by calling our various wage rates.

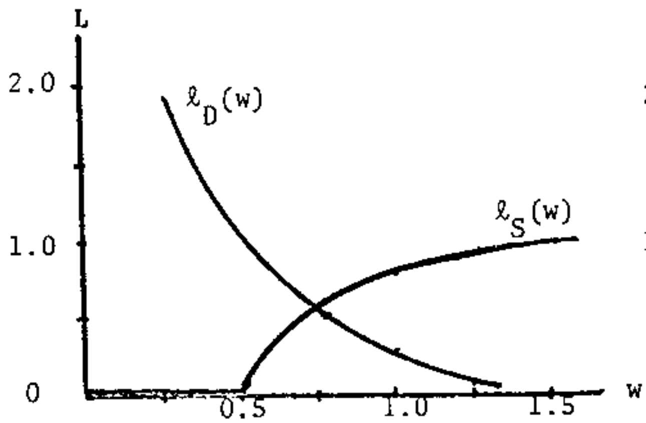

FIGURE 13

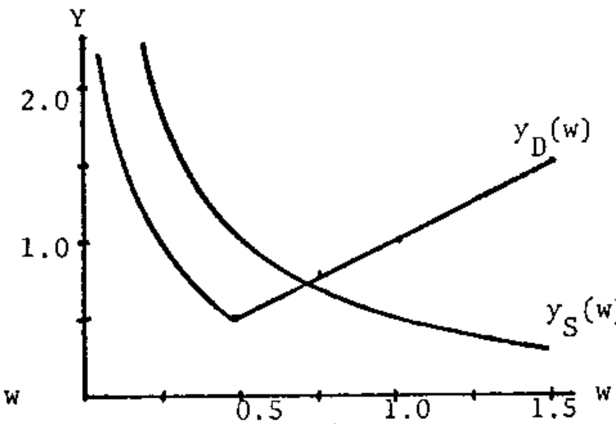

FIGURE 14

"Both the labor market and the yam market have supply and demand equal at the same price," said Robinson. "What would you have done if they had balanced at different prices?"

"Oh, that can't happen," said Walras, "because of a law I discovered. Look at the formula for the profit of Crusoe, Inc.

$$\pi(w) = y_S(w) - w\ell_D(w) \quad .$$

"Now look at the formula for your budget constraint,

$$y_D(w) = w\ell_S(w) + \pi(w) \quad ,$$

or expenditures on yams equals wage income plus dividend income expressed in yam units. If you substitute the first formula for $\pi(w)$ in the second formula, you get

$$y_D(w) = w\ell_S(w) + y_S(w) - w\ell_D(w) \quad .$$

"Rearranging this, we get

$$y_D(w) - y_S(w) = w(\ell_S(w) - \ell_D(w)) \quad .$$

"This formula holds for every value of w. It says that if for some w, supply equals demand in one of the markets, making one side of the equation zero, then the other side of the equation must also be zero, and supply must equal demand in the second market."

"Very interesting," said Robinson, "so the fact that supply equals demand at $w = \frac{1}{\sqrt{2}}$ in both Figure 13 and Figure 14 was no accident."

"Right! It always works out that way," said Walras. "By the way, are you happy with the yam-leisure combination you finally obtained under my scheme?"

"Why, it's just the same as the combination I was choosing before you arrived," said Robinson. "I guess I am exactly as happy as I was then, although it is of course interesting to have Mr. Friday telling me where to dig yams."

"That's no accident either," said Walras, "and that is the beauty of my scheme. Let me show you why it works. In Figure 15, I have re-drawn Figure 3 which showed the yam-leisure combination you chose when you were completely on your own. Suppose I draw a straight line through the mutual

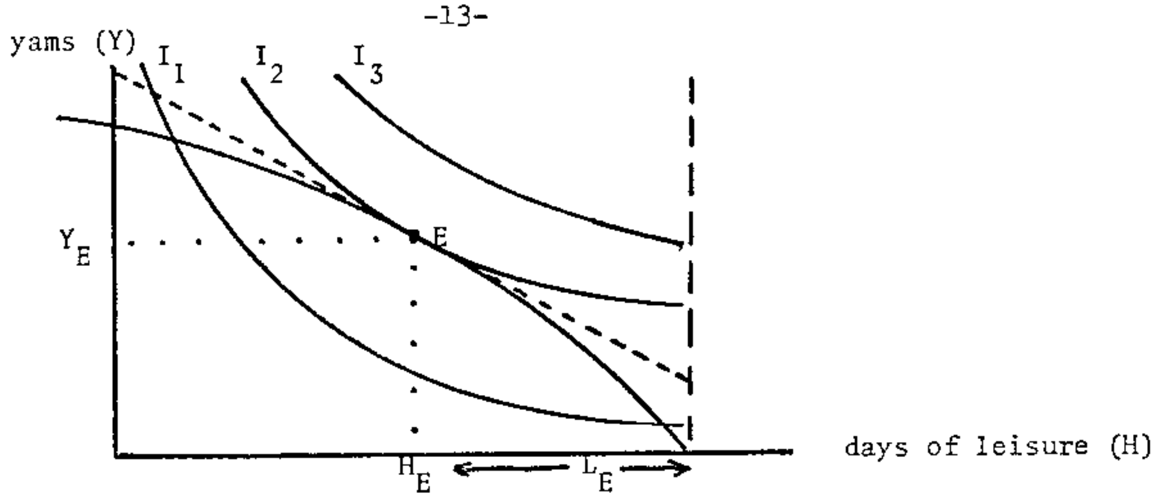

FIGURE 15

tangency point E. If you look at that line from Mr. Friday's point of view, the point E corresponds to a profit maximum for the wage rate given by the slope of the line, and π is the dividend income. From your own point of view, E is the best point on the budget defined by this line. Hence, both you and Mr. Friday will want to trade $\frac{1}{\sqrt{2}}$ unit of labor and $\frac{1}{\sqrt{2}}$ units of yams, and we have an equilibrium with supplies equal to demands.

"Suppose, on the other hand, that I call out some other price, as in Figure 16. Then, Mr. Friday will want to go to A, and with the

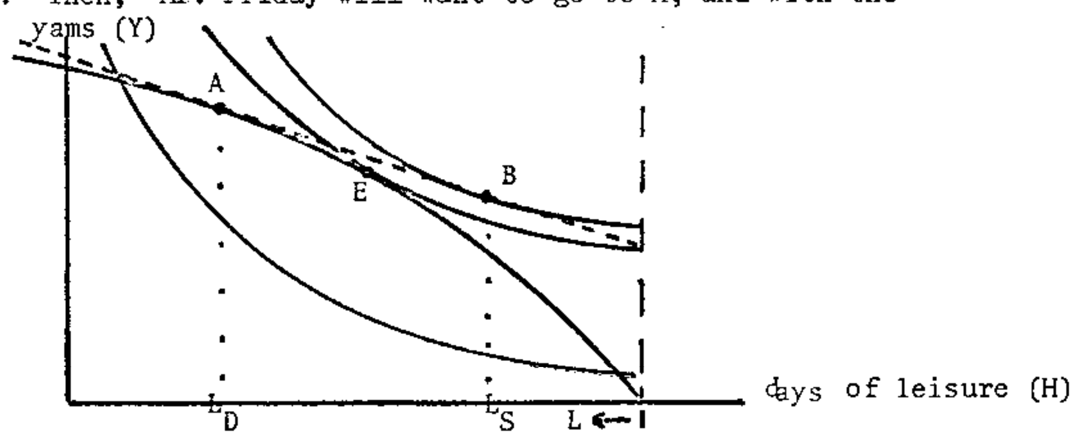

FIGURE 16

resulting budget line you will want to go to B. In this case, labor demand exceeds labor supply and yam supply exceeds yam demand, and we cannot have an equilibrium."

"Well, it is a relief to me that your scheme works," said Robinson. "Frankly, I was skeptical at first. However, I know that before this started, I was as well-off as I could possibly be on this island. I am happy your markets get me back to the same place. Tell me, will your scheme always work?"

"It will in all nice economies like yours," replied Walras. "However, if you look back at Figure 15, you will see that a key to finding a price to clear the market is that the budget line fitted through the tangency E is not cut anywhere else by the Crusoe, Inc. transformation curve or by your indifference curve. The way these curves are shaped in your economy this could never happen. However, in an economy with what economists call 'increasing returns' in production or 'non-convex' preferences, my market scheme may break down. In some of the advanced economies there may also be a problem with things like pollution, noise, and congestion which lead to what economists call 'externalities.' In this case, my market system may lead to an equilibirum, but in failing to take account of externalities may leave consumers less well-off than they might possibly be under centralized economic controls."

"Fascinating," said Robinson, munching on a yam, "some other time you must tell me more about it."

Under Walras's supervision, the markets for labor and yams were operated each morning. As the months went by, Robinson found himself satisfied in every way. Variations from day to day in his tastes for yams or the digging prospects of Crusoe, Inc., were quickly accommodated by the markets, and Robinson always found himself in the comfortable situation of not having to search outside the market for ways to improve his lot.

Then, one morning, Robinson and Mr. Friday arrived at the beach to find no Walras. A quick search turned up a bottle floating in the surf with a note: "Tied up organizing a market for artist's models. Back tomorrow. León."

"What shall we do?" Mr. Friday asked Robinson.

"I suppose we could try trading at the wage rate we traded at yesterday," said Robinson. Mr. Friday agreed, and each quickly calculated his demands and supplies. Because of shifts in tastes and production possibilities, the old equilibrium wage rate led to a situation like that shown in Figure 17. Mr. Friday offered to hire labor and buy yams at point B, Robinson at point A. The result was an excess supply of labor (Figure 18) and an excess demand for yams (Figure 19) at this price.

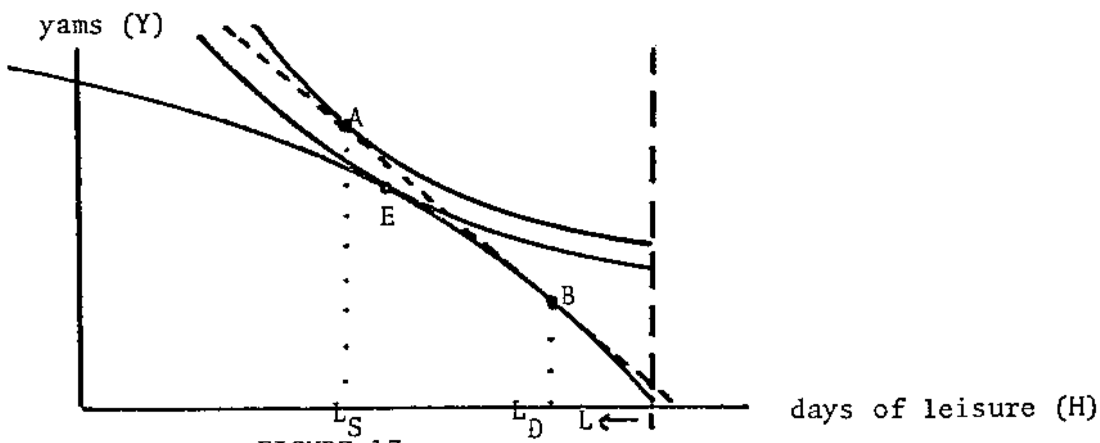

FIGURE 17

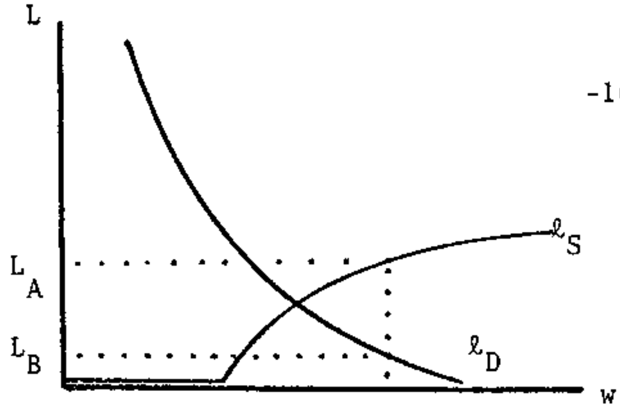

FIGURE 18

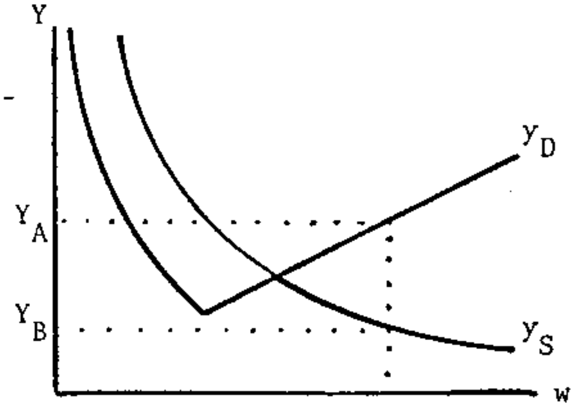

FIGURE 19

"I'm afraid I can use only the amount of labor L_B," said Mr. Friday. "I'll have to lay you off for the rest of the time $(L_A - L_B)$ you want to work. In recognition of your many months of meritorious service, however, Crusoe, Inc. would like to present you with this digging stick with a mahogany handle."

"That's quite a shaft," said Robinson.

"Thank you," replied Mr. Friday.

"Crusoe, Inc. also regrets that due to the high cost of labor, we are unable to fill your order for Y_A yams," said Friday.

"That's just as well," said Robinson. "Since I can't work as much as I would like, I don't have the income to buy yams that I thought I would. I can only afford Y_B yams."

"How fortunate, that's exactly what we can provide," said Mr. Friday, "it's been a pleasure to serve you." This seemed to settle the matter, but Robinson was unhappy. He would really have preferred to work a little more and have more yams to eat. He began to long for the good old days, when he was his own boss. "Perhaps you would like to barter a little labor for a few more yams," he said to Mr. Friday. "I'll make you a good price, lower than the wage rate Walras left us with."

"That's very appealing," said Mr. Friday, "and certainly in the spirit of free enterprise. Crusoe, Inc. is always interested in seeing lower wages. Our motto is 'What is good for Crusoe is good for Robinson'."

"Perhaps we could operate the markets ourselves, then," said Robinson, "using the bidding and arbitrage activities of competitive forces to lead to market clearing prices. If supply of labor exceeds demand, as now, the wage will be bid down. On the other hand, if demand exceeds supply, the wage will be bid up."

"Bid up?" said Mr. Friday, "I want to make it perfectly clear that Crusoe, Inc. stands four-square behind free enterprise and the preservation of capitalism. Consequently, we are agaisnt cutthroat competition which would drive up the cost of labor, starting us down the road to Socialism."

Robinson was sure that the bidding process he had suggested would work, and that Mr. Friday's rhetoric was only an attempt to get Robinson to agree to laws and regulations which would give Crusoe, Inc. an advantage in bargaining. However, to avoid further argument, he said "Perhaps we should stop trading for today, and let Walras resume tomorrow." Mr. Friday agreed.

The next day, Walras returned. "I hope you gentlemen were able to trade in my absence," he said.

"I was unable to sell all the labor I wanted to," said Robinson.

"The wage rate was too high," said Walras. "Such things can happen when conditions in the economy are changing so fast that the market manager can't catch up, or when the economy adopts rules and regulations such as wage and price controls which keep prices from adjusting to equate supply and demand. Or, of course, when the market adjustment mechanism is very sluggish, due to difficulties in passing along information, as happened yesterday when I was away. These are the classical reasons that prices might fail to adjust rapidly, leading to trade outside Walrasian equilibrium."

"What's Walrasian equilibrium?" asked Robinson.

"Oh, that's what I call the combination of prices and quantities where supplies and demands are equal," said Walras, "and these supplies and demands come from the schedules of profit-maximizing firms and preference-maximizing consumers. As your experience has shown, in Walrasian equilibrium the consumer is as well-off as he could possibly be. However, in trading situatons outside of equilibrium, this will usually not be true. Further, you are likely to find that your demand and supply schedules don't tell the whole story about your trades when you are out of equilibrium."

"That's true," said Robinson. "When I found out I couldn't work as much as I wanted, I realized that I couldn't afford to buy the amount of yams I had originally offered."

"You might say that your effective or ex post demand was unequal to your notional or ex ante demand," said Walras.

"You took the words right out of my mouth," said Robinson.

The following week, Walras disappeared again for a day, leaving instructions to trade at the previous day's wage rate. This time, the changes in tastes and technology led to too low a wage rate, as shown in Figure 20. Crusoe, Inc. offered point B, while the consumer offered point A. Labor demand L_B exceeded labor supply L_A,

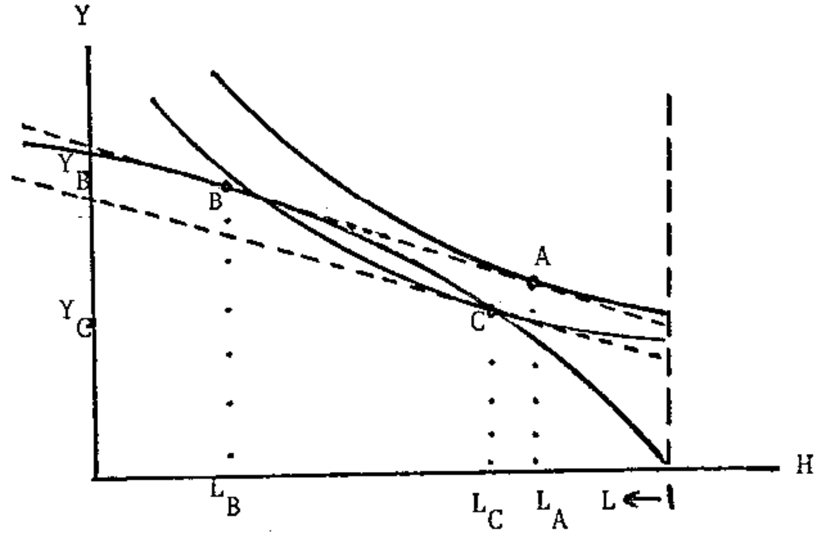

FIGURE 20

and yam supply exceeded yam demand. This time Robinson refused to work more than L_A. Soon, he received a note from Crusoe, Inc. "Due to circumstances beyond our control, labor shortages will cause dividends for this period to fall below the previously anticipated level." Faced with less non-wage income than he had expected to receive, Robinson made some further calculations and revised his offer of labor supply. After an exchange of communications, it was agreed that Robinson would supply an amount of labor L_C, and would exactly use up his available income to purchase the available supply of yams Y_C.

This would have been the end of the matter had not Mr. Friday noted with some alarm the gap between the amount of yams Y_B he had planned to sell at the quoted wage w, and the amount Y_C he was actually able to sell. From his vantage point, it seemed entirely possible that with trade at prices which did not clear all markets, he could contract to hire a great deal of labor, say L_B, to produce a large amount of yams that would be left unsold to spoil. He was concerned that he would end up in the position illustrated in Figure 21. Instead of achieving B which would yield maximum profits for the firm, his hiring of the

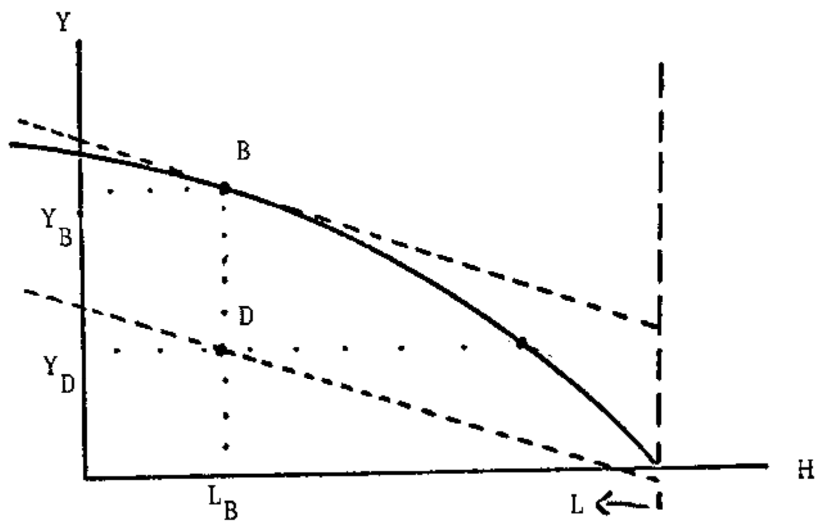

FIGURE 21

input L_B would lead to sales of Y_D rather than Y_B and result in profits near zero. Mr. Friday feared that such a performance could endanger his standing with Robinson, and possibly ruin his promising career as a business executive in the yam industry. Therefore, he formulated a plan and presented it to Robinson at the next stockholder's meeting.

"The management wishes to report that it has secured an attractive rate of dividends without taking undue risks during the past year," said Mr. Friday. "However, because of market fluctuations in the recent past, the management proposes to institute policies to avoid commitments.

to produce yams in quantities which are in excess of prospective sales, thereby protecting the interests of the owners against danger of excessive inventories of unsold goods. The management emphasizes that under this policy the company will continue to pay high dividends with minimum risk."

"Would you repeat that in plain English?" asked Robinson.

"The management can respond to stockholders questions only if they are submitted in advance in writing," replied Mr. Friday.

"Come off it, Friday," said Robinson.

"Very well," Friday said reluctantly, "I don't intend to offer to produce more than I think I can sell."

"That seems reasonable enough," said Robinson. "You have my blessing."

The following day, Mr. Friday put his plan into action. On the basis of past experience and the first wage rate w called out by Walras, he decided that an amount of yams Y^* was the most he could reasonably expect to sell. He decided that as the calling out of prices and offers went along during the morning, he would follow a very simple rule for changing Y^*: If the consumer's demand for yams exceeded Y^*, then he would gradually increase Y^*, and if demand were less than Y^*, then he would slowly decrease Y^*. A mathematician would describe his adjustment process in the following way. Let t denote the "clock time" during which Walras is conducting the auction to determine the real wage, and suppose the seconds are measured $t = 1,2,3,\ldots.$ Then, the change in Y^*, defined as $\Delta Y^*_t = Y^*_{t+1} - Y^*_t$, is a sign-preserving function of the algebraic difference of Y_{Dt} and Y^*_t. In fact, Mr. Friday changed Y^* in proportion to $Y_{Dt} - Y^*_t$, leading to the linear function

$$\Delta Y_t^* = \alpha(Y_{Dt} - Y_t^*) \quad ,$$

where α is a positive constant. This is termed a difference equation.

Walras's method of adjusting the real wage can also be described by a simple rule, the change in the wage $\Delta w_t \equiv w_{t+1} - w_t$ is proportional to the excess demand for labor, $L_{Dt} - L_{St}$:

$$\Delta w_t = \beta(L_{Dt} - L_{St}) \quad ,$$

where β is a positive constant.

Each morning, Robinson, Mr. Friday, and Walras would start from the previous day's wage and expected maximum sales, and let this adjustment process run on until w and Y* approached limiting values. Then, trade would occur at these values.

The results were alarming! Instead of always achieving the Walrasian equilbrium where Robinson was as well off as he could possibly be, the adjustment process often led to situations in which Robinson was offered a very low wage, and as a result chose to work very little and consume only a few yams. He complained bitterly to Walras that something was going wrong, and he was clearly not as happy as in the good old days.

"Very strange," said Walras, "supply equals demand in both markets. You must be mistaken, and just think you are less than happy. My economic theory shows you must be just as happy as possible."

"I'm hungry, and I can't eat economic theories," retorted Robinson. "You must do something!"

"I never ran into anything like this in my classroom in France," said Walras. "Come to think of it, I'm rather homesick for Paris."

"Robinson is just lucky that things aren't worse," interjected Mr. Friday. "Due to my astute management, I have avoided saddling him with losses occasioned by the production of goods which can't be sold."

"You mean you base your production decisions on what you think is the maximum amount you can sell, rather than what would maximize profits on the assumption that anything produced could be sold?" asked Walras.

"Exactly," answered Mr. Friday proudly.

"Robinson, I believe we have located your problem," said Walras, "Expectations."

"I'm hungry," responded Robinson.

"I believe you deserve a full explanation of expectations and how they might influence equilibrium," said Walras. "I could of course provide the complete story, even though the topic is out of my area. However, I really must be getting back to Paris, and fortunately I see just approaching in a fast cutter a young economic colleague who has thought deeply about these matters. He will be happy I am sure to give you the details."

With this, Walras jumped up, dashed into the surf, and swam briskly in the direction of Paris, soon disappearing over the horizon. Robinson's attention was now diverted to the cutter, which rapidly approached the shore and rode gracefully up on the sand, assisted by two lanky seamen.

A sprightly gentleman bounced out. "I am Lord Keynes," he said, "and these seamen, A.L. and H.V., are my interpreters."

"Do you have any food?" asked Robinson. "I am in a very depressed state."

"Depression!" exlaimed Lord Keynes, "How interesting. You must tell me more." Robinson recounted his history on the island, first as a centralized economy, then operated under the guidance of Walras, and then finally the coming of darker days with low wages and little production. He concluded by stressing how he could think better on a full stomach, and by mentioning Walras's cryptic comment about expectations.

"A most unusual case," said Lord Keynes, "one that I have only hinted at in my theories."

"Oh, you said it, sir, after your fashion," chorused A.L. and H.V.

"You are getting ahead of me gentlemen," replied Lord Keynes. Turning to Robinson, he asked, "Would you like all this explained to you?" "I'd rather eat," said Robinson.

"By all means," said Lord Keynes, "first food for the mind, then for the body." He then proceeded, with occasional interjections from A.L. and H.V., to a description of Robinson's recent experiences and his explanation for them. Robinson listened mutely, omitting only an occasional low moan of hunger. Here is Lord Keynes's story:

The figures below illustrate some of the cases in which Robinson and Mr. Friday found themselves. In Figure 22, the wage w and the expected maximum sales are both low. If the firm were not constrained by maximum expected sales, it would choose to produce at point C.

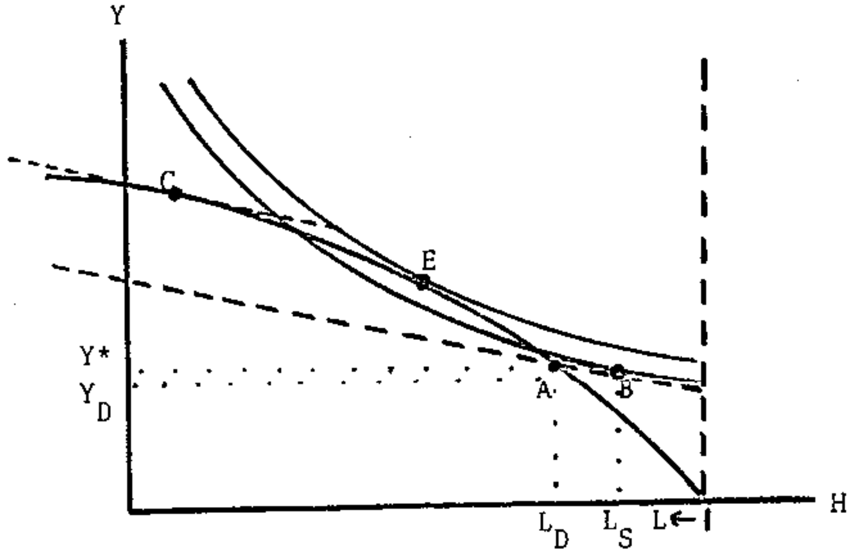

FIGURE 22

However, given that it expects to be able to sell at most Y*, it chooses instead point A, offering to hire L_D and produce Y*. Robinson, when faced with the income budget line through A, chooses point B, with labor supply L_S, to maximize his preferences. The result is that labor demand exceeds labor supply, leading to a rise in the wage, and output demand is less than the expected maximum sales Y*, leading to a fall in Y*. Note that E is the original Walrasian equilibrium. In Figure 23, the firm offers the sales-limited point A rather than the maximum profit point C, while the comsumer offers the point B. The result is

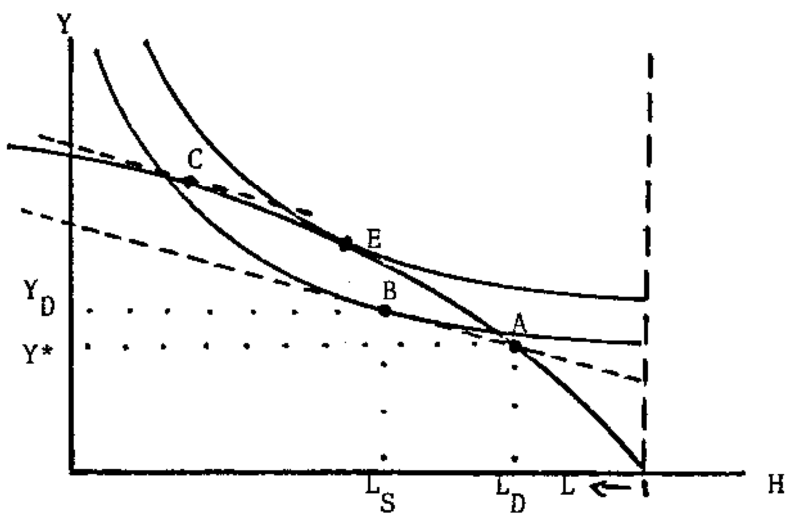

FIGURE 23

labor supply exceeding labor demand, leading to a fall in the wage rate, and output demand exceeding maximum expected sales, leading to an increase in expected sales.

Comparing Figures 22 and 23, one sees that for any maximum expected sales less than the amount of output prevailing in Walrasian equilibrium E, one can find a wage rate at which labor supply equals labor demand. Further, the wage which achieves this , as illustrated in Figure 24, is lower than the wage which would prevail in Walrasian equilibrium.

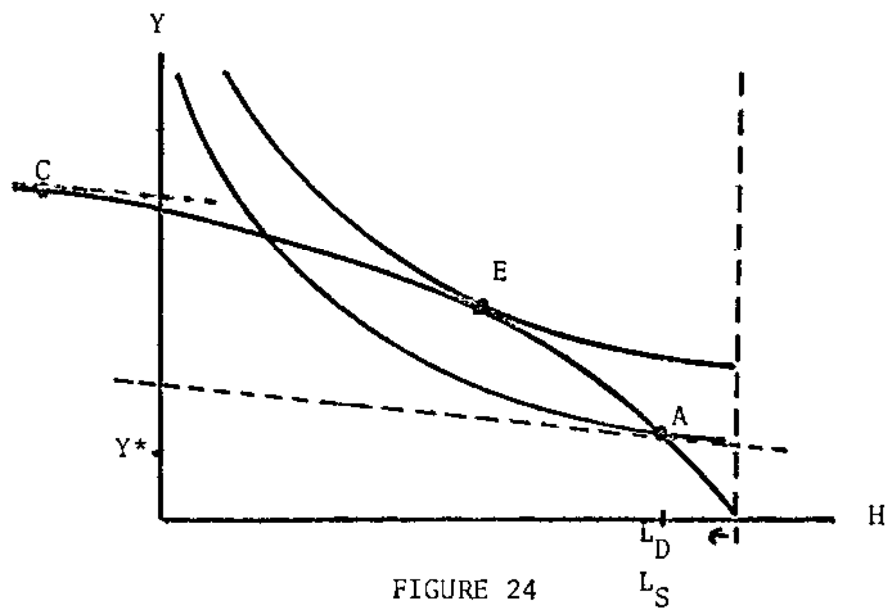

FIGURE 24

In this figure, the wage rate which makes the budget line tangent to Robinson's indifference curve through A leads to labor supply equal to labor demand and maximum expected sales equal to yam demand. This is an <u>equilibrium</u> for the economy in the sense that if it reaches A, with the budget line through A shown in the diagram, then both Robinson and Mr. Friday will choose to stay at this point. This is easy to see for

Robinson: given the budget line in the diagram, his preferences are maximized at A, and this is the offer he will make. On the other hand, Mr. Friday expects he can sell at most Y*, and this is exactly what he does sell, so he has no information which leads him to change his expectations. Of course, Mr. Friday would offer to produce at C rather than A if he thought he could sell the output, and Robinson would be better off at the point E than he is at A. Clearly, if Robinson and Mr. Friday abandon the strategies they are using to determine their behavior, they could soon find mutually advantageous barters from A in the direction of E. We saw earlier that Mr. Friday was too anal-compulsive to accept an advantageous barter in a previous circumstance when the wage rate was too high and too little was being produced. One might expect that the pressures of competition would eventually make it impossible for Mr. Friday to indulge his psychosis, since eager young MBA's would step in and offer Robinson a better deal. In the simple Robinson Crusoe economy, where the advantages of barter are obvious to all parties, we should not be surprised to see this happen eventually. However, in a more complex economy, where Robinson worked for the Crusoe Aircraft Engine Company, a barter in which Robinson would work additional hours in exchange for a few bags of Crunchy Granola would involve many parties, including air frame manufacturers, air lines, travel agents, junketing Granola executives and employees, etc. It is easier to imagine here such a multilateral barter would be hard to carry off. Robinson would have a difficult time setting it up, and it is quite plausible that Crusoe Aircraft Engine Company could fail to see how putting Robinson to work a few more hours a week would lead to an increase in the demand for aircraft engines. In any case, an economic outcome on a South Seas island that appears only because of a business psychosis may occur in a complex economy when businesspersons are perfectly "rational," just not sufficiently omnicient to see all the possibilities for multilateral barter in the economy.

Returning to an examination of Robinson, Mr. Friday, and yams, note first from Figure 24 that there are a whole range of equilibria in the sense of point A, depending on the level of expected maximum sales Y*. This is illustrated in Figure 25. At any point along the portion of the production possibility frontier EAF, say the point D, we can have an equilibrium if the expected maximum expected sales of the firm are equal to the output at D, and the wage rate is set so that the slope of

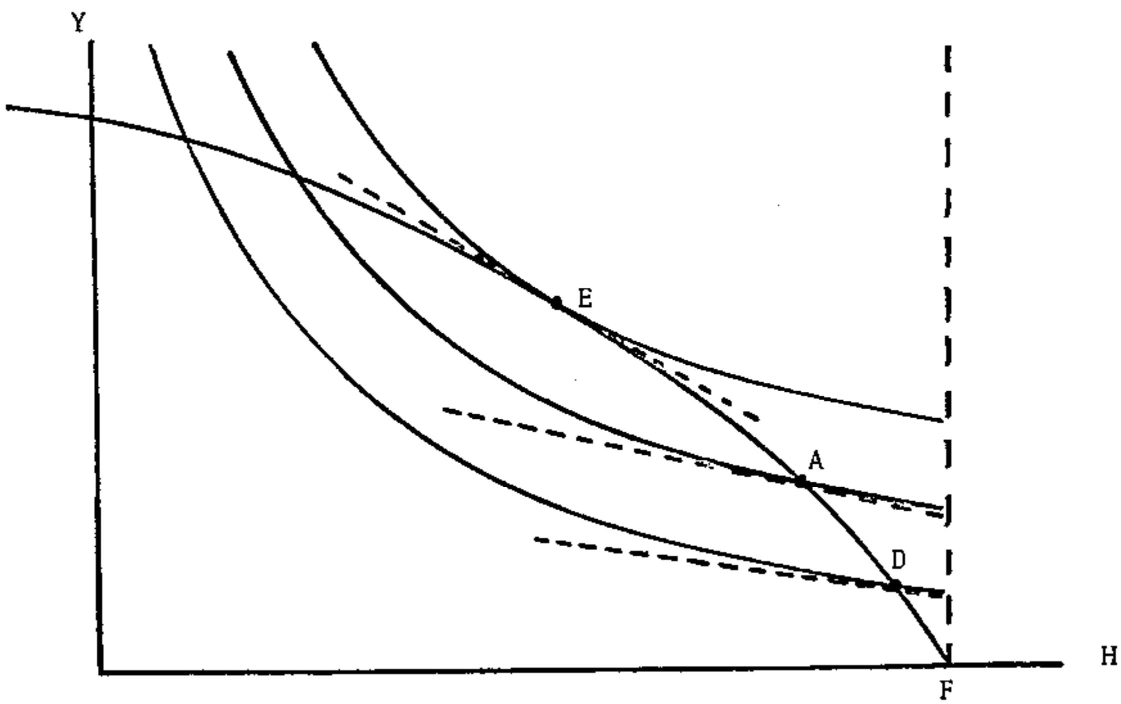

FIGURE 25

the budget line through D is set to be tangent to Robinson's indifference curve through D. At each point such as A, D, or F, the equilibrium wage will be less than at the Walrasian equilibrium E.

Thus far, we have considered cases where maximum expected sales is effective in limiting the output offered by Crusoe, Inc. Alternately, expected sales Y* may be sufficiently high so that the firm's output is determined instead by classical considerations of profit maximization. This will correspond generally to high w and Y*. The first of these cases is illustrated in Figure 26. The profit maximum in the absense of sales constraints is at point C, which gives an output below the maximum Y* the firm thinks it can sell. At this budget, Robinson chooses A.

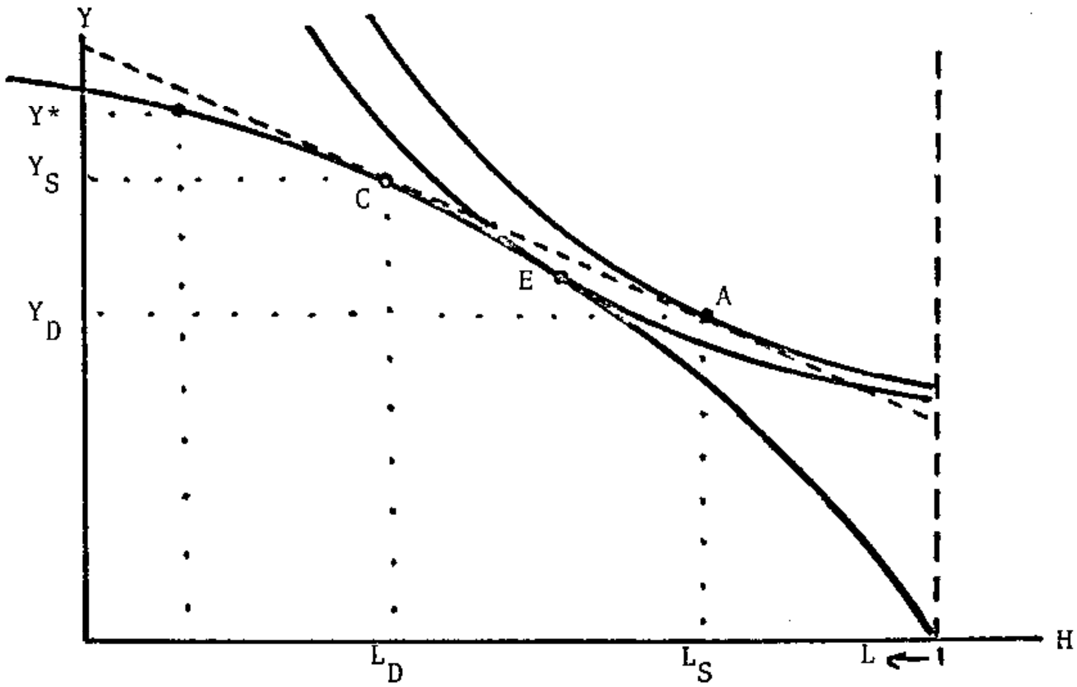

FIGURE 26

The result is Y_D less than Y*, causing Y* to fall, and L_D greater than L_S, causing the wage rate to rise.

Figure 27 illustrates a case where the profit maximum C gives an output below Y*, and the point A chosen by Robinson leads to labor supply exceeding labor demand, but Y* exceeding Y_D. Hence, Y* falls and w falls.

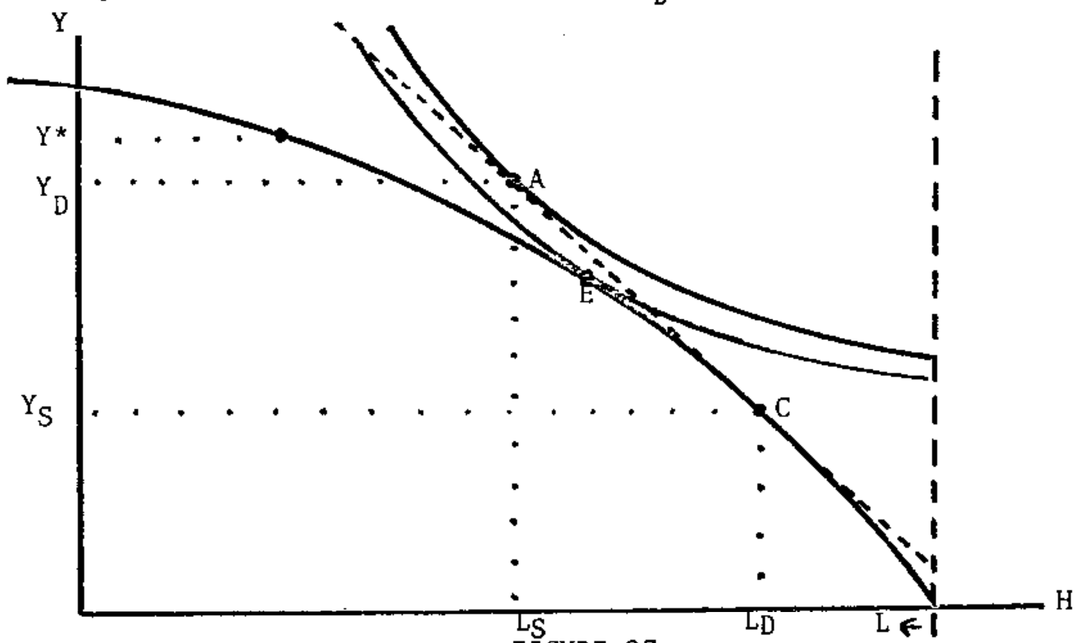

FIGURE 27

Finally, Figure 28 illustrates the case where Y_D exceeds Y* and L_S exceeds L_D, leading to a rise in Y* and a fall in w.

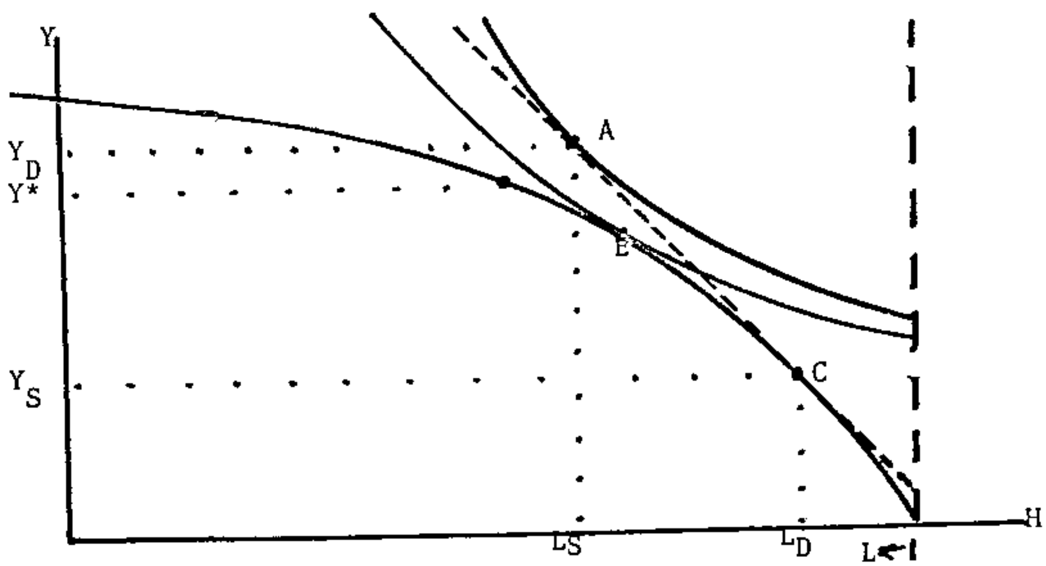

FIGURE 28

We can piece together, from Figures 22, 23, 26, 27, and 28, a description of the path taken through time by the wage rate and the level of maximum expected sales as the adjustment process goes on. In Figure 29, with w on the vertical axis and Y* on the horizontal axis, first plot the locus of w & Y* combinations such that at the wage w, Y* is a profit-maximum (without sales constraints) for the firm, i.e., for each w, Y* is the value of Y maximizing $\pi = Y - wL$ for Y & L combinations on the transformation curve in Figure 4. This curve is

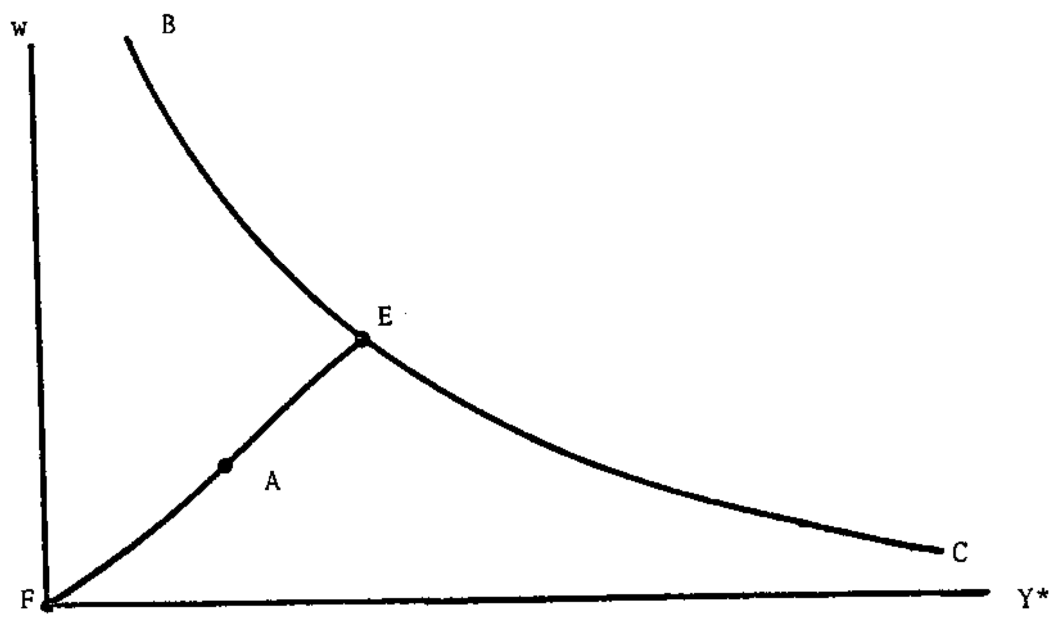

FIGURE 29

labeled BEC, with E corresponding to the Walrasian equilibrium in Figure 2 or Figure 15. As one can see by looking back at Figure 7 and the argument leading to it, this is just the yam supply curve of the firm when sales are not limiting.

Next, we note from Figure 25 that for any output level ranging from the Walrasian quantity at point E down to zero at point F, there is a wage rate at which one attains a sales-limited equilibrium. Further, as one moves from E, through A, to F in Figure 25, the wage rate is falling. Hence, the locus of equilibrium w & Y* points plotted in Figure 29 declines from E through A to F, with w and Y* decreasing together.

Figure 30 reproduces Figure 29, and adds two elements. The first is a horizontal line extending to the right from E, corresponding to the Walrasian equilibrium wage rate. Provided one is northeast of the curve BEC, so that the sales constraint is not limiting, one has labor supply greater than the labor demand derived in Figure 6 when w exceeds w_E, and vice versa for w below w_E. Also, when one is northeast of BEC and the wage rate is very high for a given Y*, Robinson's yam demand will exceed Y*, while at the wage rate w_E it will be less than Y*. Hence, there is a curve extending northeast from E which is the locus of w & Y* points where Robinson's yam demand equals Y*. These curves break Figure 30 into five regions, which have been labeled with circled numbers

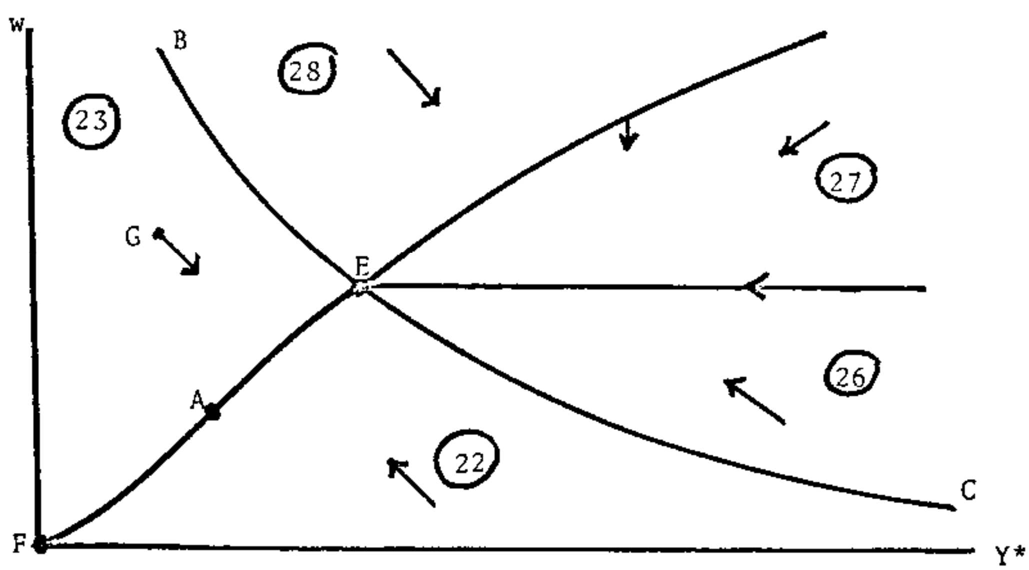

FIGURE 30

corresponding to Figure 22, 23, 26, 27, and 28. The qualitative behavior of w and Y* in each of these regions then corresponds to that described in the corresponding figures. For example, suppose one happens historically to be at a w & Y* combination G in region 23. Then, output is limited by expected maximum sales. At the relatively high wage rate, Robinson offers to supply more labor than is demanded, and to purchase more yams than the expected maximum sales. Hence, w falls and Y* rises, just as in Figure 23.

"Is all this clear so far, Robinson?" asked Lord Keynes.

"Fine, fine," mumbled Robinson, "are we ready for the part about feeding the body now?"

"Now, now," replied Lord Keynes, "you must work through the other cases for me, so I am sure you understand."

With an urgency which the economist found commendable, Robinson rushed through a check of what happens in regions 22, 26, 27, and 28.

Lord Keynes continued his explanation: When adjustment on a given day starts in regions 22 or 23, the dynamic process will tend to lead to an equilibrium like A, where wages and output are lower than would be best for Robinson. Technically, there is no unemployment at A, since at this low wage Robinson is unwilling to supply more labor than the small amount demanded. However, the economy is in another sense at "less than full employment" if we interpret the latter to be the labor supply at the point E where Robinson is as well off as possible.

"Would you like a numerical example?" asked Lord Keynes.

"Later, in an appendix, after we eat," replied Robinson.

"Very well," said Lord Keynes, "but before we stop, I had better make some suggestions on how to correct your problem. Clearly, you run into difficulty when Mr. Friday's expectations on maximum sales are too low. Therefore, the answer is to keep those expectations high. I recommend that you establish a government which plays some direct role in your economy. You might introduce money, and give yourself more purchasing power when output is low. Or you might have the government become a net purchaser of yams, which it could feed to you as its employee. Then Mr. Friday could count on government contracts to keep his maximum sales high. You might even nationalize Crusoe, Inc. Why, I could write a book about the possibilities open to you."

"You have, sir," chorused A.L. and H.V.

"Why so I have," said Lord Keynes. "Well, then, read the book, Robinson. For now, let's go find a few yams to take care of your empty stomach."

"Best idea I've heard in months," replied Robinson, and off the troup went to the digging ground.

APPENDIX

Here is the post-lunch numerical example Lord Keynes gave Robinson. Measure labor L in man-days per day, and yams in bushels. Suppose the firm has a production function $Y = \sqrt{L}$, faces a real wage w, and expects it can sell at most Y*. The profit maximum in the absence of a sales constraint occurs at $L_C = 1/4w^2$, $Y_C = 1/2w$, and $\pi_C = 1/4w$. This is obtained by considering profits as a function of L, $\pi = \sqrt{L} - wL$, and setting $d\pi/dL = 0$.

If $Y^* \geq Y_C$, then the sales constraint is non-binding, and the firm will offer L_C, Y_C, and π_C. If $Y^* < Y_C$, the the firm will set $Y = Y^*$ and obtain L so that $Y^* = \sqrt{L}$, or $L = Y^{*2}$. Also, $\pi = \sqrt{L} - wL = Y^* - wY^{*2}$. In summary,

$$L_D = \begin{cases} 1/4w^2 & \text{if } w > 1/2Y^* \\ Y^{*2} & \text{if } w \leq 1/2Y^* \end{cases}$$

$$Y_S = \begin{cases} 1/2w & \text{if } w > 1/2Y^* \\ Y^* & \text{if } w \leq 1/2Y^* \end{cases}$$

$$\pi = \begin{cases} 1/4w & \text{if } w > 1/2Y^* \\ Y^* - wY^{*2} & \text{if } w \leq 1/2Y^* \end{cases}$$

Suppose the consumer has the utility function $u = \log Y - L$ which he maximizes subject to the budget constraint $Y = \pi + wL$. Substituting $\pi + wL$ for Y, one obtains

$$u = \log (\pi + wL) - L \quad .$$

Then,

$$du/dL = \frac{w}{\pi + wL} - 1 \quad .$$

If $w \leqq \pi$, then $du/dL < 0$ and the consumer will offer no work ($L = 0$) and a purchase of yams equal to profit income ($Y = \pi$). If $w > \pi$, then $du/dL = 0$ at $L = 1 - \pi/w$, and $Y = \pi + wL = w$.

From the firm, we have π as a function of w and Y^*, with two cases. Since there are two cases for the consumer, we have four cases in all. Yam demand for each case is found by substituting the expression for π:

$$Y_D = \begin{cases} w & \text{if case (a): } w > 1/2Y^* \text{ and } w > \pi = 1/4w \\ w & \text{if case (b): } w \leqq 1/2Y^* \text{ and } w > \pi = Y^* - wY^{*2} \\ 1/4w & \text{if case (c): } w > 1/2Y^* \text{ and } w \leqq \pi = 1/4w \\ Y^* - wY^* & \text{if case (d): } w \leqq 1/2Y^* \text{ and } w \leqq \pi = Y^* - wY^{*2} \end{cases}$$

Similarly, the supply of labor satisfies

$$L_S = \begin{cases} 1 - \pi/w = 1 - 1/4w^2 & \text{if case (a)} \\ 1 - \pi/w = 1 - (Y^*/w) + Y^{*2} & \text{if case (b)} \\ 0 & \text{if case (c)} \\ 0 & \text{if case (d)} \end{cases}$$

Combining supply and demand,

$$\frac{1}{\alpha}\Delta Y^* = Y_D - Y^* = \begin{cases} w - Y^* & \text{if case (a)} \\ w - Y^* & \text{if case (b)} \\ (1/4w) - Y^* & \text{if case (c)} \\ -wY^{*2} & \text{if case (d)} \end{cases}$$

$$\frac{1}{\beta}\Delta w = L_D - L_S = \begin{cases} (1/2w^2) - 1 & \text{if case (a)} \\ (Y^*/w) - 1 & \text{if case (b)} \\ 1/4w^2 & \text{if case (c)} \\ Y^{*2} & \text{if case (d)} \end{cases}$$

Plotting the curves shown in Figure 30 for this example, we obtain Figure 31. The heavy line EF is a continuum of equilibria. Starting from points such as T or T''' in regions 22 and 23, one tends to move to some equilibrium in the EF line. Starting from points

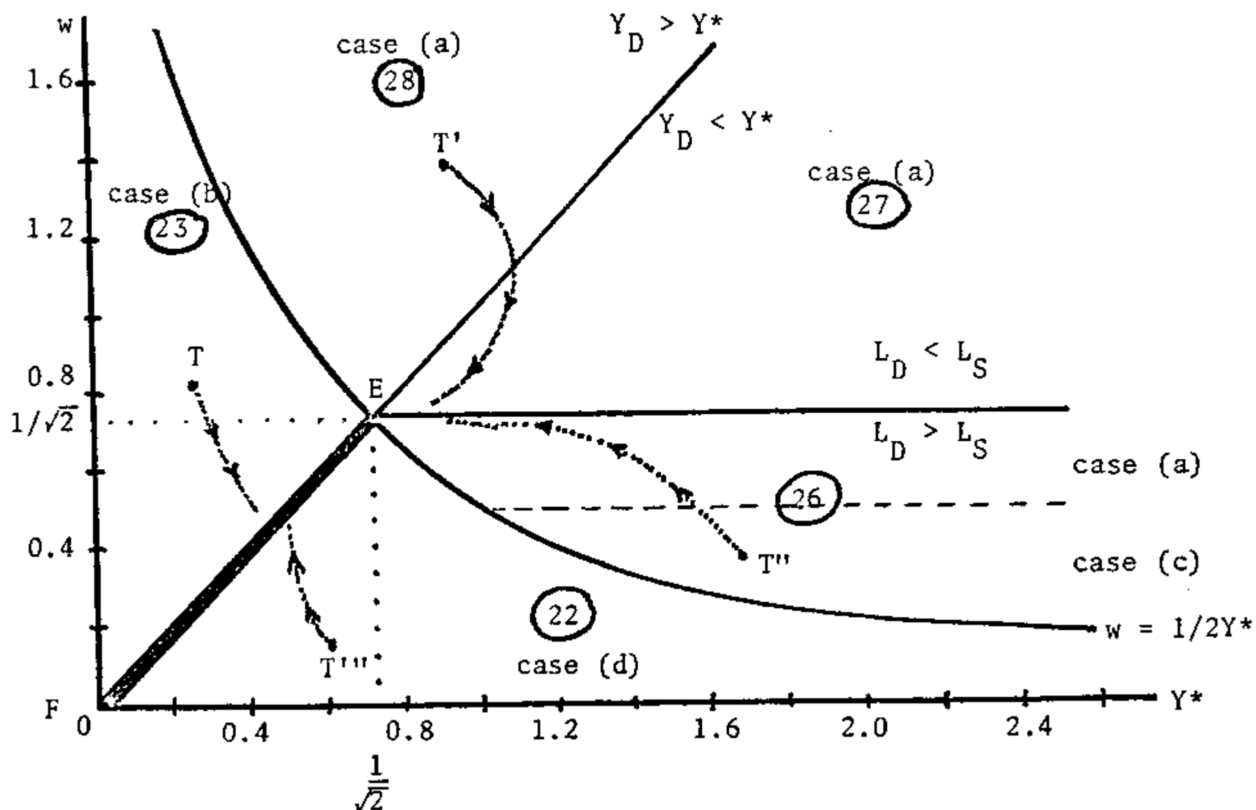

FIGURE 31

such as T' or T" in regions 28, 26, or 27, one tends to move to E. Paths starting from region 23 near the boundary w = 1/2Y* may cross into region 28 if β is not too large relative to α, and vice versa if β is much larger than α. Similarly, paths will tend to move from the edge of 22 into 26 if β is not too small relative to α, and from the edge of 26 into 22 if β is small relative to α. Finally, if either α or β are large, then the difference equations may exhibit cyclic or explosive behavior.

The following APL program and trial runs illustrate the paths that can occur in this example. In this program, PAR [1] is α and par [2] is β.