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CDF (BIVNORM or CHISQ or DICKEYF or F or NORMAL or T or WTDCHI,  
 DF=*degrees of freedom for CHISQ or T*, DF1=*numerator degrees of freedom for F*,  
 DF2=*denominator degrees of freedom for F*,  
 NLAGS=*number of lags for augmented Dickey-Fuller test*,  
 NOB=*number of observations for unit root or cointegration test*,  
 NVAR=*number of variables for cointegration test*,  
 RHO=*correlation coefficient for BIVNORM*,  
 EIGVAL=*vector of eigenvalues for WTDCHI*,  
 LOWTAIL or UPTAIL or TWOTAIL,  
 CONSTANT, TREND, TSQ, INVERSE, PRINT)  
*test statistic [significance level];*  
 or  
*significance level [critical value];* (for INVERSE)  
 or  
*x value y value [significance level];* (for BIVNORM)

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**Function:**

CDF calculates and prints tail probabilities (p-value or significance level) or critical values for several cumulative distribution functions. This is useful for hypothesis testing.

**Usage:**

CDF followed by the value of a scalar test statistic is the simplest form of the command. In this case, a two-tailed probability for the normal distribution will be calculated and printed. If the INVERSE option is used, the first argument must be a probability level; a critical value will be calculated. Arguments need not be scalars; they can be series or matrices. Other distributions or tail areas may be selected through the options. For hypothesis testing with regression diagnostics, see the REGOPT(PVPRINT) command.

**Options:**

BIVNORM/CHISQ/DICKEYF/F/**NORMAL**/T/WTDCHI specifies the bivariate normal, chi-squared, Dickey-Fuller, F, standard normal, student's t, and weighted chi-squared distributions, respectively.

**DF**= the degrees of freedom for the chi-squared or student's t distribution, or the number of observations for Dickey-Fuller (the NOB= option is a better way of specifying this).

**DF1**= the numerator degrees of freedom for the F distribution.

**DF2**= the denominator degrees of freedom for the F distribution.

**RHO**= the correlation coefficient for the bivariate normal distribution.

**EIGVAL**= vector of eigenvalues for the weighted chi-squared distribution.

**NLAGS**= the number of lagged differences in the augmented Dickey-Fuller test. This number is used to compute the approximate finite sample P-value or critical value. The default is zero (assume an unaugmented test). The NOB=

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option must also be specified for the finite sample value.

**NOB**= the number of observations for the augmented Dickey-Fuller or Engle-Granger tests. This number is used to compute the approximate finite sample P-value or critical value. The default is zero (to compute asymptotic instead of finite sample value).

**NVAR**= the number of variables for an Engle-Granger/Dickey-Fuller cointegration test. The default is 1 (plain unit root test), and the maximum is 6.

**CONSTANT/NOCONST** specifies whether a constant term (C) was included in the regression for Dickey-Fuller. NOCONST is only valid for NVAR=1.

**TREND/NOTREND** specifies whether a trend term (1,2,...,T) was included in the regression for Dickey-Fuller.

**TSQ/NOTSQ** specifies whether a squared trend term (1,4,9,...) was included in the regression for Dickey-Fuller.

**LOWTAIL/TWOTAIL/UPTAIL** specifies the area of integration for the density function. TWOTAIL is the default for most symmetric distributions (normal and t), UPTAIL is the default for chi-squared and F, and LOWTAIL is the default for bivariate normal and Dickey-Fuller. TWOTAIL is not defined for bivariate normal.

**INVERSE/NOINVERSE** specifies the **inverse** distribution function (input is significance level, output is critical value). Normally the input is a test statistic and the output is a significance level. INVERSE is not defined for bivariate normal.

**PRINT/NOPRINT** turns on printing of results. PRINT is true by default if there is no output specified.

### Output:

If the PRINT option is on, the input and output values will be printed, along with the degrees of freedom. If a second argument is supplied, it will be filled with the output values and stored (see examples 4 and 6). Input and output arguments may be any numeric TSP variables.

### Examples:

1. To compute the significance level of a Hausman test statistic with 5 degrees of freedom:

```
CDF(CHISQ,DF=5) HAUS;  
produces the output-- CHISQ(5) Test Statistic: 7.235999 , Upper tail area: .20367
```

2. Significance level of the test for AR(1) with lagged dependent variable(s):

```
CDF @DHALT; or REGOPT(PVPRINT) DHALT; before the regression
```

3. Two-tailed critical value for the normal distribution:

```
CDF(INV) .05;  
produces the output-- NORMAL Critical value: 1.959964 , Two-tailed area: .05000
```

4. Several critical values for the normal distribution:

```
READ PX; .1 .05 .01 ;
```

```
CDF(INV,NORM) PX CRIT;
PRINT PX,CRIT;
```

## 5. F critical values:

```
CDF(INV,F,DF1=3,DF2=10) .05;
produces the output-- F(3,10) Critical value: 3.708265 , Upper tail area: .05000
```

## 6. Bivariate normal:

```
CDF(BIVNORM,RHO=.5,PRINT) -1 -2 PBIV;
produces the output-- BIVNORM Test Statistic: -1.0000 , -2.0000 , Lower tail area: .01327
```

## 7. Dickey-Fuller unit root test (same as UNIT(MAXLAG=0,NOWS) Y; ) :

```
TREND TIME;
SMPL 2,50;
DY = Y-Y(-1);
OLSQ DY TIME C Y(-1);
CDF(DICKEYF) @T(3);
```

## 8. Augmented Dickey-Fuller unit root test, with finite sample P-value (same as UNIT(MAXLAG=3,NOWS,FINITE) Y; ) :

```
TREND TIME;
SMPL 2,50;
DY = Y-Y(-1);
SMPL 5,50;
OLSQ DY TIME C Y(-1) DY(-1)-DY(-3);
CDF(DICKEYF,NOB=@NOB,NLAGS=3) @T(3);
```

## 9. Engle-Granger cointegration test (same as COINT(NOUNIT,MAXLAG=0,ALLORD) Y1-Y4; ):

```
TREND TIME;
OLSQ Y1 TIME C Y2 Y3 Y4; EGTEST;
OLSQ Y2 TIME C Y1 Y3 Y4; EGTEST;
OLSQ Y3 TIME C Y1 Y2 Y4; EGTEST;
OLSQ Y4 TIME C Y1 Y2 Y3; EGTEST;
PROC EGTEST;
SMPL 2,50;
DU = @RES-@RES(-1);
OLSQ DU @RES(-1);
CDF(DICKEYF,NVAR=4) @T;
SMPL 1,50;
ENDPROC;
```

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10. Verify critical values for Durbin-Watson statistic, for regression with 10 observations and 2 RHS variables:

```
SMPL 1,10; OLSQ Y C X1;
SET PI = 4*ATAN(1); SET F = PI/(2*@NOB); TREND I;
EIGB = 4*SIN(I*F)**2;
SELECT I <= (@NOB-@NCOEF); ? use largestlest eigenvalues for dL
CDF(WTDCHI,EIG=EIGB) .879; ? dL for 5% (n=10, k'=1)
SELECT (@NCOEF <= I) & (I <= (@NOB-1)); ? use smallest eigenvalues for dU
MMAKE dU 1.320 1.165 1.001; ? dU for 5%, 2.5%, 1% (n=10, k'=1)
CDF(WTDCHI,EIG=EIGB,PRINT) dU;
```

11. Reproduce exact P-value for Durbin-Watson statistic (can already be done by REGOPT):

```
SMPL 1,10; ? data from Judge, et al (1988) example: DW = 1.037, P-value = .0286
READ Y X1; 4 2 7 4 7.5 6 4 3 2 1 3 2 5 3 4.5 4 7.5 8 5 6 ;
REGOPT(DWPVAL=EXACT);
OLSQ Y C X1;
MMAKE X @RNMS;
MAT XPXI = (X'X)";
TREND OBS; SELECT OBS > 1;
DC = 0; DX1 = X1 - X1(-1);
MMAKE DX DC DX1;
MMAKE BVEC 2 -1; MFORM(BAND,NROW=@NOB) DDP = BVEC;
MAT DMDP = DDP - DX*XPXI*DX';
? Eigenvalues of DMD' = D*D' - DX*(X'X)"(DX)'
? (same as nonzero eigenvalues of MA, because A = D'D)
MAT ED = EIGVAL(DMDP);
CDF(WTDCHI,EIG=ED) @DW;
```

### Method:

BIVNORM: ACM Algorithm 462. Inverse is not supplied because it is not unique unless x or y is known, etc.

CHISQ: DCDFLIB method: Abramovitz-Stegun formula 26.4.19 converts it to Incomplete Gamma, and then use DiDinato and Morris(1986). Inverse by iteration (trying values of x to yield p -- faster methods are also known).

F and T: DCDFLIB method: Abramovitz-Stegun formula 26.6.2 converts it to Incomplete Beta, and then use DiDinato and Morris (1993), i.e. ACM Algorithm 708. Inverse by iteration.

NORMAL: ACM Algorithm 304, with quadratic approximation for  $E < -37.5$ . Inverse: Applied Statistics Algorithm AS241, from StatLib.

DICKEYF: Asymptotic values from Tables 3 and 4 in MacKinnon (1994). Finite sample critical values from Cheung and Lai (1995) [augmented Dickey-Fuller] or MacKinnon (1991) [Engle-Granger]. To convert these to finite sample P-values, a logistic interpolation is used with the .05 size and either the .01 or .10 size (whichever is closer to the observed test statistic). Such interpolated P-values are fine for testing at the .01, .05, or .10 sizes, but would be highly speculative outside this range.

WTDCHI: If  $w_i$  are the eigenvalues,  $c_i$  are chi-squared(1) variables, and d is the test statistic, WTDCHI computes the following probability:  $\Pr[\sum(w_i*c_i)/\sum(c_i) < d] = \Pr[\sum((w_i-d)*c_i) < 0]$ . This is useful for computing P-values for the Durbin-Watson statistic, other ratios of quadratic forms in normal variables, and certain non-nested tests (for example, Vuong(1989)). The Pan method is used when the number of eigenvalues is less than 90; otherwise the Imhoff method is used. If the absolute values of the smallest eigenvalues are less than  $1D-12$ , they are not used; otherwise duplicate eigenvalues are not checked for. The inverse of this distribution is not implemented.

**References:**

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