LAD (QUANTILE=*value*, SILENT, TERSE) *dependent variable list of independent variables*;

Function:

LAD computes least absolute deviations regression, also known as L1 regression or Laplace regression. This type of regression is optimal when the disturbances have the Laplace distribution and is better than least squares (L2) regression for many other leptokurtic (fat-tailed) distributions such as Cauchy or Student’s t.

Usage:

To estimate by least absolute deviations in TSP, use the LAD command just like the OLSQ command. For example,

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LAD CONS,C,GNP ;
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estimates the consumption function using L1 regression instead of L2 (least squares) regression.

Options:

QUANTILE= quantile to fit. The default is .5 (the median).

SILENT/**NOSILENT** suppresses all printed output.

TERSE/**NOTERSE** suppresses all printed output except the table of coefficient estimates and the value of the likelihood function.

Output:

The usual regression output is printed and stored (see OLSQ for a table). The likelihood function (@LOGL) and standard error estimates are computed as though the true distribution of the disturbances was Laplace; this is by analogy to least squares, where the likelihood function and conventional standard error estimates assume that the true distribution is normal (with a small sample correction to the standard errors). The only additional statistic is @PHI, which contains the sum of the absolute values of the residuals. This quantity divided by the number of observations and squared is an estimate of the variance of the disturbances, and is proportional to the scaling factor used in computing the variances of the coefficient estimates.

Method:

The LAD estimator minimizes the sum of the absolute values of the residuals with respect to the coefficient vector b:

$$\min_b \sum_{i=1}^{N} |y_i - X_i b|$$

(This formula is modified slightly if a quantile other than .5 is used). The estimates are computed using the Barrodale-Roberts modified Simplex algorithm. A property of the LAD estimator is that there are K exact zero residuals (for K right-hand-side variables); this is analogous to the least squares property that there are only N-K linearly independent residuals. In addition, the LAD estimator occasionally produces a non-unique estimate of the coefficient vector b; TSP issues a warning message in this case.

The estimated variance-covariance of the estimated coefficients is computed as though the true distribution is Laplace:

$$V(\hat{b}) = \omega^2 (X'X)^{-1} \text{ , where } \omega^2 \text{ depends on } \tau \text{ , the quantile used.}$$

$$\omega^2 = \lambda^2 \quad \text{when } \tau = .5 \text{ ; } \quad \omega^2 = \frac{\tau(1-\tau)}{f(F^{-1}(\tau))^2} \quad \text{in general.}$$

$$\text{For } \tau < .5 \text{ , } \quad f(F^{-1}(\tau)) = \frac{\tau}{\lambda} \text{ , so } \omega^2 = \lambda^2 \frac{(1-\tau)}{\tau} \text{ ; for } \tau > .5 \text{ , } \quad f(F^{-1}(\tau)) = \frac{(1-\tau)}{\lambda} \text{ , so } \omega^2 = \lambda^2 \frac{\tau}{(1-\tau)} \text{ .}$$

$$\lambda \text{ is defined from the Laplace density: } \quad f(x) = \frac{e^{-\left|\frac{x}{\lambda}\right|}}{2\lambda} \text{ . } \quad \lambda = \text{@PHI/@NOB .}$$

This formula can also be derived as the BHHH estimate of the variance-covariance matrix if the first derivative of |e| is defined to be unity at zero, as it is everywhere else. The outer product of the gradients of the likelihood function will then yield the above estimate. It is also possible to define f(x) and F(x) via empirical density estimation, to provide a more "robust" @VCOV matrix, but this is not done by TSP at present.

See Judge et al (1988) for details on the statistical properties of this method of estimation. See Davidson and MacKinnon (1993) on testing for normality of the residuals in least squares.

References:

Barrodale, I., and F.D.K. Roberts, Algorithm #478, **Collected Algorithms from ACM Volume II**, Association for Computing Machinery, New York, NY, 1980.

Davidson, Russell, and James G. MacKinnon, **Estimation and Inference in Econometrics**, Oxford University Press, New York, NY, 1993, Chapter 16.

Judge, George, R. Carter Hill, Wiliam E. Griffiths, Helmut Lutkepohl, and Tsoung-Chao Lee. **Introduction to the Theory and Practice of Econometrics**. John Wiley & Sons, New York, Second edition, 1988, Chapter 22.

Koenker, R. W., and G. W. Bassett, "Regression Quantiles," **Econometrica** 46 (1978), pp. 33-50.