Economics 201b
Spring 2010
Problem Set 4
Due Thursday April 15

1. **Robinson-Crusoe in U.S.S.R.** Consider a "Robinson-Crusoe" economy with two-goods, one consumer and one firm. Firm is labor-oriented: it maximizes the profits per unit of labor, given the wage rate w and price of potatoes p (i.e. it maximizes $\frac{\pi(p,w)}{\ell}$ where ℓ is an amount of labor).

   (a) What is the definition of competitive equilibrium in this case? Give a formal definition. Call it P ("Proletariat") equilibrium. (Yes, P equilibria of the whole world, unite!).

   (b) When production function $f(z)$, where z is labor input, is strictly concave what is the set of all Pareto optimal allocations?

   (c) Continue to assume that $f(z)$ is strictly concave, under what condition on utility function does the P equilibrium exist? Give the description of the equilibrium in this case.

   (d) For an arbitrary production function check whether P equilibrium is Pareto efficient.

   (e) Now, suppose that in recognition of such great management innovation, firm receives an award from *Politburo* of the economy (i.e. local social planner). Thus, the firm maximizes now $\frac{\pi(p,w)+a}{\ell}$ where $a > 0$ is the fixed award amount. If the utility function U is quasi-concave, continuous and strictly monotone, are there any conditions on the production function such that P equilibrium exist? Prove or give counterexample.

2. **Robinson-Crusoe: back to Berkeley.** Consider again following "Robinson-Crusoe" economies with two-goods, one consumer and one firm. For each case, compute all Pareto optimal allocations and check whether or not the Second Welfare Theorem holds. Justify your answer.

   (a) $U(x_1, x_2) = \log x_1 + \log x_2,\ \omega = (24, 0),$
   $Y = \{(-y_1, y_2) : \ y_2 \le e^{y_1 - 1},\ y_1 \ge 0\}$

   (b) $U(x_1, x_2) = \log x_1 + \log x_2,\ \omega = (24, 0),$
   $$Y = \{(-y_1, y_2) : \ y_2 \le \begin{cases} \frac{3}{4} y_1 & \text{if } 0 \le y_1 \le 20 \\ y_1^2 + 15 & \text{if } 20 < y_1 \end{cases}$$

   (c) $U(x_1, x_2) = 3x_1^2 + e^{x_2},\ \omega = (24, 0),$
   $Y = \{(-y_1, y_2) : \ y_2 \le \log(y_1 + 1),\ y_1 \ge 0\}$

3. **Quasi-equilibrium to equilibrium in economy with production.** In lecture we have shown that with strict monotonicity of preferences any price quasi-equilibrium is also a price equilibrium in pure exchange economy. Now, you

need to prove that under our assumptions on preferences (continuous, convex and strongly monotone) as well as an additional assumption that $\exists y_j \in Y_j$: $\sum_j y_j + \bar{\omega} \gg 0$ this claim is also true in the Arrow-Debreu economy with production.

4. **"Tricky" Boundary Conditions.** A common misconception about the boundary condition on excess demand is to think that it says that if the price of a good goes to zero, then excess demand for *that good* goes to infinity. Although intuitively plausible, this is false even for very well-behaved preferences, since relative prices matter. Working this problem should help you avoid this misconception.

   Consider the preference relation on $\mathbb{R}^3_+$ represented by the utility function $U(x_1, x_2, x_3) = \sqrt{x_1} + \sqrt{x_2} + x_2 + \frac{x_3}{1+x_3}$, and let the consumer's initial endowment be $\omega = (1, 1, 1)$.

   (a) Show that U is strongly monotone, strictly concave, and continuous.

   (b) If $(x_1, x_2, x_3) \in \mathbb{R}^3_+$ and $x_3 > 0$, show that $U(x_1, x_2 + x_3, 0) > U(x_1, x_2, x_3)$

   (c) If $p = (p_1, p_2, p_3) >> 0$ and $p_2 = p_3$, show that $x_3(p) = 0$.

   (d) For each n, let $p^n = (1 - \frac{2}{n}, \frac{1}{n}, \frac{1}{n})$. Show that $x_3(p^n) = 0$ for each n (and thus that demand for x_3 remains bounded even though $p^n_3 \to 0$).

   (e) Show that $\lim\limits_{n\to\infty} x_2(p^n) = \infty$.

5. **Importance of Assumptions.** Consider a two good economy, and illustrate graphically four examples of functions $z : \Delta^o \to \mathbb{R}^2$ which demonstrate that if any one of the conditions

   (a) continuity,

   (b) Walras' Law,

   (c) boundedness below ($\exists x \in \mathbb{R}$ s.t. $z(p) \geq x \ \forall p \in \Delta^o$),

   (d) boundary condition (if $p^n \to p \in \Delta \backslash \Delta^o$, then $|z_l(p^n)| \to \infty$)

   fails, then there may not be a solution to $z(p) = 0$. That is, each function you draw should violate only one of the four conditions, and have the property that $\nexists p$ s.t. $z(p) = 0$.

6. **Continuity of correspondences.** Let $\psi : \Delta \to 2^\Delta$ is a correspondence. Show that ψ is uhc if it has a closed graph. Demonstrate graphically an example of correspondence $\psi : X \to 2^X$ such that ψ has a closed graph but ψ is not uhc.