Economics 201b
Spring 2010
Problem Set 6
Due Thursday April 29

1. There are many foreign embassies in the Washington, DC. In fact, there is an area of the city where quite a few of them are very close to each other. As you walk along one of the streets you observe three embassies standing next to each other. Each embassy has a flagpole with its national flag flying in the wind. You know that the flagpole height chosen by each embassy depends continuously on the heights chosen by other two embassies. (For instance, having too tall a pole compared to the neighbors would be ostentatious, whereas having one too short would look stingy.) Moreover, having observed heights chosen by the neighbors, each embassy has a single, most favorite height to set. DC ordinance imposes an upper limit of 100 feet on flagpole heights of the embassies. The choices of flagpole heights are in equilibrium when no one wishes to change the height of their flagpole. Prove that there exists an equilibrium.

2. Consider a two-person, two-good exchange economy where all agents have the same utility function, $i = 1, 2$:

$$u(x_{1i}, x_{2i}) = \max\{2 \min\{2x_{1i}, x_{2i}\}, \min\{x_{1i}, 4x_{2i}\}\}.$$

   (a) Draw the indifference curves for one of the consumer. Are this consumer's preferences convex?

   (b) Draw the Edgeworth box for this economy, denoting Pareto set, individually rational and core allocations.

   (c) Now suppose we have an economy of $I \in \mathbb{N}$ identical consumers with $I \geq 2$, each of which has the same preferences as the consumers described above and endowments are $(\omega_1, \omega_2) = (1 + 3\alpha, 2 - \alpha)$ where $\alpha \in (0, 1)$. Find a necessary and sufficient condition on α that must be satisfied for there to exist a Walrasian equilibrium of this economy. Show that as I increases, the set of $a \in (0, 1)$ that satisfy the condition you found increases in size.

   (d) Explain in a few sentences how these results relate to Theorem 2 in the Lecture Notes 11. That is, relate your above results to the fact that, in this economy, we can show that $\forall \alpha \in (0,1) \quad \exists p^* \gg 0,\ \ p^* \in \Delta^0$ with $0 \in conE(p^*)$ and $x_i^* \in D_i(p^*)$ such that

$$\frac{1}{I} \sum_{\ell=1}^{L} p_\ell^* \left| \left(\sum_{i=1}^{I} x_i^* - \sum_{i=1}^{I} \omega_i \right)_l \right| \leq \frac{2L}{I} \max\{\|\omega_i\|_\infty : \ i = 1, ..., I\}$$

   Fix $\alpha = \frac{1}{3}$ and compute an explicit bound for the *market value* of the surpluses and shortages in the economy. Verify that the bound provided by the theorem is tight enough.

3. Consider four-person, two-good pure exchange economy where agents have endowments $\omega_1 = \omega_2 = (10, 10)$ and $\omega_3 = \omega_4 = (10, 30)$ and the same utility function $U_i(x_{1i}, x_{2i}) = \log x_{1i} + \log x_{2i},\ i = 1, 2, \ldots, 4$. For each allocation vector given below show whether the it is Pareto optimal; is in the core (if not, provide a blocking coalition); can be supported as a competitive equilibria for some price vector. Explain your reasoning.

   (a) $x_1 = x_2 = (7.5, 15)$ and $x_3 = x_4 = (12.5, 25)$.

   (b) $x_1 = x_2 = (\sqrt{50}, 2\sqrt{50})$ and $x_3 = x_4 = (20 - \sqrt{50}, 40 - 2\sqrt{50})$.

   (c) $x_1 = (8, 12), x_2 = (9, 11), x_3 = (12, 23)$ and $x_4 = (11, 29)$.

4. Give an example of a three-person, two-good pure exchange economy where all agents have the same utility function $U_i(x_{1i}, x_{2i}) = \log x_{1i} + \log x_{2i},$. Find a set of *integer* endowments for these agents along with a Pareto optimal, individually rational *integer* allocation that is not in the core.

5. Consider a pure exchange economy with $H = 2$ consumers and L goods, with social endowment $\bar{\omega} \in \mathbb{R}^L_{++}$. In this question, we will consider the n-fold replica of this economy. In the n-fold replica, there are $2n$ agents, of whom n (referred to as type 1 agents) have preferences and endowments identical to those of agent 1 in the original economy, and n (referred to as type 2 agents) have preferences and endowments identical to those of agent 2 in the original economy.

   (a) Let p^* be an equilibrium price vector for the original economy. Show that p^* is also an equilibrium price vector for the (larger) n-fold replica economy.

   (b) Now, lets consider a special case where there are two commodities and two type of agents. Type 1 is characterized as

$$U_1(x_{11}, x_{21}) = x_{11}x_{21}, \quad \omega_1 = (10, 0)$$

   and type 2 is characterized as

$$U_2(x_{12}, x_{22}) = (x_{12})^{\frac{1}{2}}(x_{22})^{\frac{1}{2}}, \quad \omega_2 = (0, 10).$$

   Show that the allocation $x_1 = x_2 = (5, 5)$, (x_1 to agents of type 1 and x_2 to agents of type 2) is in the core for all levels of replication n.

   (c) Continue to assume two-good, two-agent type economy given above. Show that the allocation $x_1 = (9, 9),\ \ x_2 = (1, 1)$, is in the core for the original economy with one agent of the each type and is *not* in the core for the n-fold replica with $n \geq 2$. Discuss.

6. Give an example of acyclic preference relation that is *not* transitive.