Econ 204 – Problem Set 3

Due 11:59pm Friday August 8 on Gradescope

1. For each of the following subsets of $\mathbb{R}$, either prove it is compact directly from the definition, or show that it is not compact by finding an open cover with no finite subcover:

   (a) $\varnothing$

   (b) $\{c\}$ for some $c \in \mathbb{R}$

   (c) $(-1, 1)$

2. Show that every open covering of $\mathbb{R}^n$ has a countable subcovering. *Hint: The countable union of finite sets is countable.*

3. Let (X, d) be a metric space. Given a subset $Y \subseteq X$, define

$$Y^\varepsilon = \{x \in X : d(x, Y) < \varepsilon\}. \tag{1}$$

   Let $K \subseteq X$ be compact and A an open set such that $K \subseteq A$. Show that there exists $\varepsilon > 0$ such that $K^\varepsilon \subseteq A$.

4. Let X be a compact space and $\Gamma : X \to 2^X$ be a nonempty, compact-valued, upper hemicontinuous correspondence. Show that if $C \subseteq X$ is compact, then $\Gamma(C)$ is compact.

5. Let (X, d) be a compact metric space.

   (a) Show that there exists $A \subseteq X$ that is at most countable and satisfies $\overline{A} = X$.

   (b) We say that $x \in X$ is an isolated point if there exists $\delta > 0$ such that $B(x, \delta) = \{x\}$. Show that the set of isolated points of X is empty, finite or countable.