Econ 204 – Problem Set 5

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1 Taylor Theorem and Mean Value Theorem

Problem 1

Let $f : \mathbb{R} \to \mathbb{R}$ be twice differentiable. Suppose $f'(0) = f'(1) = 2$ and $\forall x \in [0, 1]$, $|f''(x)| \leq 4$.

a. Prove that $|f(1) - f(0)| \leq 4$.

b. Prove that $|f(1) - f(0)| \leq 3$.

(Hint: use Taylor's Theorem)

Problem 2

Let f be a function on $[a, b]$ that is differentiable at c. Let $L(x)$ be the tangent line to f at c. Prove that L is the unique affine function with the property that

$$\lim_{x \to c} \frac{f(x) - L(x)}{x - c} = 0$$

Problem 3

Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function. Prove that $f'(\mathbb{R})$, has an intermediate value property, that is if f' takes at least two values $a < b$ then for every $c \in [a, b]$ there exists $x : f'(x) = c$

2 Implicit and Inverse Function Theorems

2.1 Problem 4

Given $t \in \mathbb{R}$, the agent chooses $x \in \mathbb{R}$ to solve

$$\max_{x \in S(t)} f(x, t)$$

where $f : \mathbb{R}^2 \to \mathbb{R}$ and $S(t) \subset \mathbb{R}$ for all t. Under what conditions on f does the solution to the above maximization problem $x^*(t)$ increase in t? Try to come up with an "intuitive" argument to explain your assumptions.

Problem 5

a) Let $F : \mathbb{R}^2 \to \mathbb{R}$ be a C^1 function. Suppose $F(0,0) = 0$. What sufficient condition on F will guarantee that $F(F(x,y),y) = 0$ can be solved for y as a C^1 function of x in a neighborhood of $(0,0)$?

b) Prove that the function $F : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $F(x,y) = (e^x \cos y, e^x \sin y)$ has a local inverse at each point of its domain. Does it have a global inverse? Find a maximal open subset of $\mathbb{R}^2$ such that F has a differentiable inverse on its image.