Econ 204 Summer 2009
Problem Set 1
Due in Lecture Friday, July 31 2009

1. **Cardinality**

   For each pair of set A and set B, show that A and B are numerically equivalent. (Hint: Show that there exists a bijection $f : A \to B$, i.e. f is one to one and onto.)

   (a) $A = (-1, 1)$ $B = (-\infty, +\infty)$

   (b) $A = [0, 1]$ $B = (0, 1)$

   (c) A is an infinite uncountable set, $B = A \cup C$ where C is an infinite countable set.

2. **Induction**

   Using mathematical induction, show the following: $n = 1, 2, 3, \ldots$

   (a) $\sum_{i=1}^{n} k^{-i} = \frac{1-\frac{1}{k^n}}{k-1}$, $k \neq 1$.

   (b) $\sum_{i=n}^{\infty} (k-1)k^{-i} = k^{1-n}$, $k > 1$.

   (c) $\sum_{i=1}^{n} \frac{1}{\sqrt{i}} \geq \sqrt{n}$

3. **Bijection**

   Suppose $f : X \to Y$ is a bijection, i.e. f is one to one and onto. Show that for any $A, B \subset X$, $f(A \cap B) = f(A) \cap f(B)$.

4. **Supremum Property and Completeness Axiom**

   Use the Completeness Axiom to prove that every nonempty set of real numbers which is bounded below has an infimum.

5. **Limit of Decreasing Sequence**

   Show that every decreasing sequence of real numbers that is bounded below converges to its infimum. (Hint: you can directly use the result of question 4)

6. **Metric Space**

   (a) $\rho(x, y) = \begin{cases} 1 & \text{if } x \neq y \\ 0 & \text{otherwise} \end{cases}$, prove whether or not it is a metric on $\mathbf{R}^n$.

   (b) $\rho(x, y) = \sum_{i=1}^{n} |x_i - y_i|$, prove whether or not it is a metric on $\mathbf{R}^n$.

   (c) Suppose (S_1, d_1) and (S_2, d_2) are metric spaces. Show that $(S_1 \times S_2, \rho)$ is a metric space, where $\rho\left(\left(x_1, x_2\right), \left(y_1, y_2\right)\right) = max\left\{d_1\left(x_1, y_1\right), d_2\left(x_2, y_2\right)\right\}$ for all $x_1, y_1 \in S_1$ and all $x_2, y_2 \in S_2$.