Econ 204 Summer 2009
Problem Set 3
Due in Lecture Friday August 7

1. **Cauchy Sequence**

   Suppose $\{x_n\} \in \mathbf{R}^n$ is a Cauchy sequence. It has a subsequence $\{x_{n_k}\}$ such that $\lim_{n_k\to\infty} x_{n_k} = x$. Show that $\lim_{n\to\infty} x_n = x$.

2. **Compactness**

   Use the open cover definition of compactness to show that the subset $\left\{\frac{n}{n^2+1},\, n = 0, 1, 2\ldots\right\}$ of $\mathbf{R}$ is compact.

3. **Completeness**

   a. Show that $(0, 1)$ is not complete in the Euclidean metric.

   b. Show that $\{\frac{n\sqrt{2}}{m},\, m \neq 0,\, n, m \in N\}$ is not complete in the Euclidean metric.

4. **Completeness and Compactness**

   $(\mathbf{R}, d)$ is a metric space where d is defined as follows:

$$d(x,y) = \begin{cases} 1 & if\ x \neq y \\ 0 & if\ x = y \end{cases}$$

   (a) Show that $(\mathbf{R}, d)$ is complete.

   (b) Is $(\mathbf{R}, d)$ bounded? Is $(\mathbf{R}, d)$ compact? Prove your answer.

5. **Continuous Function**

   Let $f: X \to Y$ be a continuous function (X and Y are metric spaces). Using the characterization of continuous functions in terms of open sets, the open cover definition of compactness, and the open set definition of connectedness:

   a. Prove that if X is compact then $f(X)$, the image of f, is compact.

   b. Prove that if X is connected then $f(X)$, the image of f, is connected.

6. **Upper Hemicontinuous**

   Let $F : C \times \mathbf{R}^p \to \mathbf{R}^1$ be a continuous function, where $C \subseteq \mathbf{R}^1$. Let $\Psi(\omega) = \{x \in \mathbf{R}^n : F(x,\omega) = 0\}$ be a correspondence. Show directly from the definition that if C is compact, then Ψ is an upper hemicontinuous correspondence. (Hint: The proof is by contradiction. Suppose that Ψ is not upper hemicontinuous at some ω_0; this tells you that there is a sequence $\omega_n \to \omega_0$ with certain properties.)