

Economics 296, Statistics 260
Quantitative Risk Management I
Modeling and Measuring Financial Risk

Syllabus

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Course Objectives

Recent events have brought risk management to the forefront of investing. This semester is intended to be the first course in a three-course cluster, currently under development. The cluster is intended to study quantitative risk measurement, modeling and management from theoretical and practical perspectives.

This first semester gives an overview of quantitative risk measurement and an in-depth treatment of linear factor modeling for major asset classes. The second semester is intended to be divided into two tracks, which can be taken in parallel. The first track provides a high-level view of derivative and credit risk, simulation and stress testing, liquidity measurement and regime shift models. The second track is concerned with risk management and evaluation of financial decisions; topics include financial indices, liquidity management, asset allocation, market imperfections and performance evaluation.

Target Audience

This a 3-unit graduate level course and is intended for Masters and PhD students in Economics, Statistics, Mathematics, and Industrial Engineering and Operations Research who are interested in quantitative finance and financial economics. Undergraduates may be admitted by permission of the instructor: see more information under Prerequisites, below.

Prerequisites

Students need a basic understanding of the principles of finance as well as a strong quantitative background that includes courses in multi-variable calculus, linear algebra, probability and multi-variable statistics. Programming skills in a high level language such as Matlab are essential, and experience with C^{++} or Java and relational databases is desirable. Collaboration is part of the curriculum: a successful student needs to be an exceptional team player and an exceptional individual contributor.

Required courses: Economics 136 or UGBA 133, Math 54, Statistics 134, Statistics 135 or Economics 141, or equivalent.

Recommended courses: Math 104, Math 110, IEOR 221

Note to Masters students in Statistics: This class is particularly appropriate for Masters students in Statistics. If you are interested in the class, but lack the Economics prerequisites, the course may still be appropriate for you; please discuss your situation with the instructor.

Note to Undergraduates majoring in Statistics: This class may be used to fulfill a 150-level major requirement in Statistics. Admission to the class is by permission of the instructor, on a case-by-case basis. Students should have a major GPA of A- or better and have successfully completed Stat 133. Econ 136 or UGBA 133 are preferred as prerequisites, but may be waived for students with Econ 101A or UGBA 101A and a strong statistical background.

Note to Undergraduates majoring in Mathematics or Applied Mathematics: This class may be used to satisfy the Economics cluster requirement for Applied Mathematics majors. Admission to the class is by permission of the instructor, on a case-by-case basis. Students should have a major GPA of A- or better. Econ 136 or UGBA 133 are preferred, but may be waived for students with Econ 101A or UGBA 101A and a strong statistical background.

Note to Undergraduates in Economics: This class is open to Economics undergraduates by permission of the instructor. It will generally be limited to those intending to pursue a Ph.D. in Economics or Finance, who have completed Econ 201A with a grade of A- or better.

Requirements

The course will meet for three hours of lecture and two hours of section per week. There will be six graded problem sets, which will require a substantial amount of empirical work. The course grade will be determined as follows:

- Problem sets 60% (10% each)
- Final 40%

Students are encouraged to work collaboratively on problem sets, but each student must write up and submit his/her own problem set.

Course Books

Lectures will be drawn from the books listed here as well as a number of articles in the bibliography.

- Connor, Gregory, Lisa R. Goldberg and Robert Korajczyk, *Portfolio Risk Analysis*
- Föllmer, Hans and Alexander Schied, *Stochastic Finance: An Introduction in Discrete Time*, Second Edition

These books are supplemental to the course:

- Akerlof, George A. and Robert J. Shiller, *Animal Spirits: How Human Psychology Drives the Economy, and Why it Matters for Global Capitalism*
- Bookstaber, Richard *A Demon of our Own Design*
- Campbell, John Y., Andrew W. Lo and Craig MacKinlay, *The Econometrics of Financial Markets*
- Campbell, John Y. and Luis M. Viceira, *Strategic Asset Allocation*
- Cochrane, John H., *Asset Pricing*
- Duffie, Darrell and Kenneth J. Singleton, *Credit Risk: Pricing, Measurement, and Management*
- Grinold, Richard and Ronald Kahn *Active Portfolio Management*
- Hasbrouck, Joel *Empirical Market Microstructure: The Institutions, Economics, and Econometrics of Securities Trading*
- Jorion, Phillipe, *Value at Risk: The New Benchmark for Managing Financial Risk*
- Lewis, Michael, *The Big Short: Inside the Doomsday Machine*
- Luenberger, David G., *Investment Science*
- Mandelbrot, Benoit and Richard Hudson, *The (mis)Behavior of Markets*
- McNeil, Alexander J., Rudiger Frey and Paul Embrechts, *Quantitative Risk Management*
- Pearson, Neil D., *Risk Budgeting: Portfolio Problem Solving with Value-at-Risk*
- Rebonato, Riccardo, *Plight of the Fortune Tellers: Why We Need to Manage Financial Risk Differently*
- Schönbucher, Philipp J., *Credit Derivatives Pricing Models: Models, Pricing and Implementation*

Course Schedule

(1) Measuring Financial Risk (3 weeks, 01/22/13 - 02/07/13)

By formulating an investment decision as a tradeoff between portfolio expected return (good) and variance (bad), Markowitz (1952) generates a simple formula for allocating funds to securities. In this setting, portfolio variance is the quantitative embodiment of risk. Subsequently, in response to mounting evidence that variance does not provide a complete description of risk, the ideas in Markowitz (1952) have been expanded to include many additional quantitative risk measures. Examples include moment-based measures such as volatility, skewness and kurtosis, quantile-based measures such as value at risk, mixed quantile/moment measures such as shortfall, and crisis measures such as drawdown and default probability. We begin the study of these measures in the context of their applications to financial risk management, with an emphasis on where they have added value and where they have failed. An important theme for this section is the relationship between quantitative risk measures and diversification, and we examine this relationship in the context of examples. We look briefly at the axiomatic framework for measuring risk described in Artzner et al. (1999) and Föllmer and Schied (2004).

Readings:

- (a) Artzner et al. (1999)
- (b) Föllmer and Schied (2004, Chapter 4)
- (c) Markowitz (1952)

Problem set 1 is due in class on 02/12.

(2) Modeling Financial Risk: Linear Factor Models of Equity Markets (3 weeks, 02/12/13 - 02/28/13)

Data constraints mandate that a successful model of equity risk require must be based on a manageable number of factors or “risk drivers” that are common to securities. These factors lie at the core of the model, and the risk of a portfolio of equities is expressed in terms of the risk of the factors. The most basic example is the (linear, single-factor) market model, in which excess equity return, R_t , can be expressed:

$$R_t = a + \beta m_t + \epsilon_t \tag{1}$$

where m_t is the excess market return, β_t is the sensitivity of the equity. We look at expansions of Formula (1) to include Fama-French-Carhart factors (Fama and French (1992a), Fama and French (1992b), Carhart (1997)) and we discuss the architecture of multi-factor generalizations based on statistical, fundamental and macro-economic factors. We look at the connection between linear risk models and linear asset pricing models such as CAPM and APT, and we explore the very important issue of determining model efficacy and stability.

Readings:

- (a) Bertsimas, Lauprete, and Samarov (2004)
- (b) Carhart (1997)
- (c) Markowitz (1952)
- (d) Ross (1976)
- (e) Sharpe (1964)

Problem set 2 is due in class on 03/05.

(3) Modeling Financial Risk: Linear Factor Models of Interest Rate and Spread Risk (2 weeks, 03/05/13 - 03/14/13)

Historically, the risk in developed-market sovereign bonds has largely been driven by interest rate fluctuations. We study the term structure of interest rates and standard bond characteristics including duration and convexity. We look at as key-rate and shift-twist-butterfly factors for interest rate risk. We begin the study of defaultable bonds by extending the linear models using spreads based on sector and agency ratings.

Reading:

- (a) Connor, Goldberg and Korajczyk (2010, Chapter 6)

Problem set 3 is due in class on 03/19.

(4) Modeling Financial Risk: Linear Factor Models of Currency Risk (2 weeks, 03/19/13 - 04/04/13)

An investor with a global mandate must decide how much currency risk to hedge. From a modeling perspective, currencies have attractive features. There are large, liquid markets for currency forwards and futures, and relatively long data histories of prices and exchange rates. However there are structural irregularities in currency markets such as pegs, rebases, politically generated discontinuities and consolidation (e.g., into the Euro). There is also an interesting and ongoing debate about whether currency exposure is a free lunch. We look at covered and uncovered interest rate parity, differences between futures and forwards and the carry trade.

Readings:

- (a) Bhansali (2007)
- (b) Black (1989)
- (c) Connor, Goldberg and Korajczyk (2010, Chapter 7)
- (d) Perold and Schulman (1988)

Problem set 4 is due in class on 04/09.

(5) **Modeling Financial Risk: Integrating Linear Factor Models (2 weeks, 04/09/13 - 04/18/13)**

You are the chief risk officer (CRO) of a large pension fund and you need to assess the risk of a portfolio with thousands of equity and fixed income positions held in numerous developed markets. The portfolio is managed by a team of asset and market specialists, each of whom has a relevant linear factor model for his portfolio. We look at different schemes for aggregating the individual factor models into a single “über-model.” We examine some of the (numerous) issues that come up in aggregate models based on disparate data histories and frequencies, and risk factors with different statistical properties. Even in the best case when these so-called technicalities do not arise, data constraints impose a tradeoff between specificity (agreement of the über-model with its components for a narrow portfolio) and data constraints.¹

Readings:

- (a) Connor, Goldberg and Korajczyk (2010, Chapter 8)
- (b) Shepard (2008)

Problem set 5 is due in class on 04/23.

(6) **Modeling Financial Risk: Attributing and Budgeting in Linear Factor Models (2 weeks, 04/23/13 - 05/02/13)**

To first order, return to a portfolio can be expressed as a value-weighted sum of asset returns, but the risk of a portfolio is not the weighted sum of the asset risks. However, the Euler formula:

$$f(x_1, x_2, \dots, x_n) = \sum_{i=1}^n x_i \frac{\partial f}{\partial x_i}$$

for differentiable, positive linearly homogenous functions can be applied to all coherent risk measures including volatility, value at risk and expected shortfall.

Readings:

- (a) Goldberg, Menchero, Hayes, and Mitra (2010)
- (b) Menchero and Davis (2011)

Problem set 6 is due 05/07. Please put it in the GSI’s mail box by 5:00 PM.

¹An elegant analysis of this point is in Shepard (2008).

References

- Carlo Acerbi. Spectral Measures of Risk: A Coherent Representation of Subjective Risk Aversion. *Journal of Banking & Finance*, 26(7):1505–1518, 2002.
- Carlo Acerbi and Giacomo Scandolo. Liquidity Theory and Coherent Measures of Risk. *Quantitative Finance*, 8(7):681–692, 2008.
- George A. Akerlof and Robert J. Shiller. *Animal Spirits: How Human Psychology Drives the Economy, and Why it Matters for Global Capitalism*, Princeton University Press, 2009.
- Yakov Amihud and Haim Mendelson. Asset Pricing and the Bid-Ask Spread. *Journal of Financial Economics*, 17:233–249, 1986.
- Philippe Artzner, Freddy Delbaen, Jean-Marc Eber, and David Heath. Coherent Measures of Risk. *Mathematical Finance*, 9(3):203–228, 1999.
- Cliff Asness, Andrea Frazzini, and Lasse H. Pedersen. Leverage Aversion and Risk Parity. *Financial Analysts Journal*, 68(1):47–59, 2012.
- Vineer Bhansali. Volatility and the Carry Trade. *The Journal of Fixed Income*, Winter, 2007.
- Dimitris Bertsimas, Geoffrey J. Lauprete, and Alexander Samarov. Shortfall as a Risk Measure: Properties, Optimization and Applications. *Journal of Economic Dynamics & Control*, 28:1353–1381, 2004.
- Fischer Black. Currency Risk and Reward in International Equity Portfolios. *Financial Analysts Journal*, 45(4):16–22, 1989.
- Richard Bookstaber. *A Demon of Our Own Design*, John Wiley & Sons, 2007.
- Michael J. Brennan and Avanidhar Subrahmanyam. Market Microstructure and Asset Pricing: On the Compensation for Illiquidity in Stock Returns. *Journal of Financial Economics*, 17:233–249, 1996.
- John Y. Campbell, Andrew W. Lo, and A. Craig MacKinlay. *The Econometrics of Financial Markets*, Princeton University Press, 1997.
- John Y. Campbell and Luis M. Viceira. *Strategic Asset Allocation*, Oxford University Press, 2002.
- John Y. Campbell. Measuring the Risks of Strategic Tilts for Long-Term Investors. In *The New World of Pension Fund Management*. CFA Institute, Charlottesville, VA, 12–23, 2004.
- Mark M. Carhart. On Persistence in Mutual Fund Performance. *Journal of Finance*, 52(1):57–82, 1997.

- John H. Cochrane. *Asset Pricing*, Revised Edition, Princeton University Press, 2005.
- Gregory Connor, Lisa R. Goldberg, and Robert A. Korajczyk. *Portfolio Risk Analysis*, Princeton University Press, 2010.
- John Cox, Jonathan Ingersoll, and Stephen Ross. A theory of the term structure of interest rates. *Econometrica*, 53:385–408, 1985.
- Darrell Duffie and Kenneth J. Singleton. *Credit Risk: Pricing, Measurement, and Management*, Princeton University Press, 2003.
- Eugene F. Fama and Kenneth R. French. Common Risk Factors in the Returns of Stocks and Bonds. *Journal of Financial Economics*, 33:3–56, 1992a.
- Eugene F. Fama and Kenneth R. French. The Cross-Section of Expected Stock Returns. *Journal of Finance*, 47(2):427–465, 1992b.
- Hans Föllmer and Alexander Scheid. *Stochastic Finance: An Introduction in Discrete Time*, Second Edition, Walter de Gruyter, 2004.
- Lawrence R. Glosten and Paul Milgrom. Bid Ask and Transaction Prices in a Specialist Market with Heterogeneously Informed Agents. *Journal of Financial Economics*, 14:71–100, 1985.
- Lisa Goldberg, Jose Menchero, Michael Hayes, and Indrajit Mitra. Extreme Risk Analysis. *Journal of Performance Measurement*, 14(3):17–30, 2010.
- Richard C. Grinold and Ronald N. Kahn. *Active Portfolio Management*, Second Edition, McGraw-Hill, 2000.
- Joel Hasbrouck. Order Characteristics and Stock Price Evolution: An Application to Program Trading. *Journal of Financial Economics*, 41:129–149, 1996.
- Joel Hasbrouck. Intraday Price Formation in the Market for U.S. Equity Indexes. *Journal of Finance*, 58:2375–2399, 2003.
- Joel Hasbrouck. *Empirical Market Microstructure: The Institutions, Economics, and Econometrics of Securities Trading*, Oxford University Press, 2007.
- David Heath, Robert Jarrow, and Andrew Morton. Bond Pricing and the Term Structure of Interest Rates: A New Methodology for Contingent Claims Valuation. *Econometrica*, 60(1):77–105, 1992.
- Phillipe Jorion. *Value at Risk: The New Benchmark for Managing Financial Risk*, Second Edition, McGraw-Hill, 2001.
- Michael Lewis. *The Big Short: Inside the Doomsday Machine*, W.W. Norton & Company, 2010.
- Andrew W. Lo and Mark T. Mueller. Warning: Physics Envy May be Hazardous to Your Wealth. *Journal of Investment Management*, 8(2), 2010.

- David G. Luenberger. *Investment Science*, Oxford University Press, 1998.
- Benoit B. Mandelbrot and Richard L. Hudson. *The (mis)Behavior of Markets*, Basic Books, 2004.
- Harry Markowitz. Portfolio Selection. *Journal of Finance*, VII(1):77–91, 1952.
- Alexander J. McNeil, Rüdiger Frey, and Paul Embrechts. *Quantitative Risk Management*, Princeton University Press, 2005.
- Jose Menchero and Ben Davis. Risk Contribution Is Exposure Times Volatility Times Correlation: Decomposing Risk Using the X-Sigma-Rho Formula. *Journal of Portfolio Management*, 37(2):97–106, 2011.
- Lubos Pastor and Rober F. Staum. Are Stocks Really Less Volatile in the Long Run? *EFA 2009 Bergen Meetings Paper, AFA 2010 Atlanta Meetings Paper*, 1-55, 2010.
- Andrew J. Patton and Kevin Sheppard. Evaluating Volatility and Correlation Forecasts. In *Handbook of Financial Time Series*. Springer Berlin Heidelberg, 801–838, 2009.
- Neil D. Pearson. *Risk Budgeting: Portfolio Problem Solving with Value-at-Risk*, John Wiley & Sons, 2002.
- André F. Perold and Evan C. Schulman. The Free Lunch in Currency Hedging: Implications for Investment Policy and Performance Standards. *Financial Analysts Journal*, 3, 1988.
- Riccardo Rebonato. *Plight of the Fortune Tellers: Why We Need to Manage Financial Risk Differently*, Princeton University Press, 2007.
- Richard Roll. A Simple Implicit Measure of the Effective Bid-Ask Spread in an Efficient Market. *Journal of Finance*, 39:1127–1139, 1984.
- Stephen Ross. The Arbitrage Theory of Capital Asset Pricing. *Journal of Economic Theory*, 13(3):341–360, 1976.
- Philipp J. Schönbucher. *Credit Derivatives Pricing Models: Models, Pricing and Implementation*, John Wiley & Sons, 2003.
- Peter Shepard. Integrating Multi-Market Risk Models. *Journal of Risk*, 10(2):25–45, 2008.
- William F. Sharpe. Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk. *Journal of Finance*, 19:425–442, 1964.
- William F. Sharpe. Adaptive Asset Allocation Policies. *Financial Analysts Journal*, 66(3):46–59, 2010.
- Oldrich Vasicek. An Equilibrium Characterization of the Term Structure. *Journal of Financial Economics*, 5:177–188, 1977.

Problem Sets

(1) Measuring Financial Risk

We will investigate various risk measures and their use in quantifying risk. Go to bSpace and download the file *returns.zip*. The data files of interest are:

- *crspdata.txt*, this is an ascii file of dimension 2040×3 . The first column contains the name of the index corresponding to a return. The ticker is either *CRSP-Stocks*, which indicates monthly returns for a value weighted index comprised of all stocks in the CRSP database, or *CRSPBonds*, which indicates monthly returns for an amount outstanding weighted index of all Treasury bonds in the CRSP database. The second column contains the return dates and the third column contains the returns. The time period covered is January 1926 through December 2010.
 - *indexdata.txt*, this is an ascii file of dimension 1200×3 . The first column contains the name of the index corresponding to a return. The ticker is either *RUS3000*, which indicates monthly returns for the Russell 3000, *GSCOMM*, which indicates monthly returns for the Goldman Sachs Commodity Price Index, *USCORPB*, which indicates monthly returns for a U.S. AAA Corporate Bond Index, or *USGOVNB*, which indicates monthly returns for a 10-Year U.S. Government Bond Index. The second column contains the return dates and the third column contains the returns. The time period covered is January 1986 through December 2010.
- (a) Define a *coherent* (in the sense of Artzner et al (1999)) risk measure, i.e., what properties must a coherent risk measure satisfy? Indicate whether each of the following risk measures is or is not coherent (if it is not coherent, explain why):
- (a) standard deviation
 - (b) value-at-risk (VaR)
 - (c) semi-variance (either upper or lower)
 - (d) expected shortfall (ES)
- (b) Consider an investment universe of 100 defaultable corporate bonds with face value \$100. Assume that the defaults of different bonds are independent and that the default probability is 2% for each bond. If there is no default, each bond pays 105 in one year. If there is a default, the bond pays nothing in one year. Hence, in the default state the *loss*, L_i , of each bond is given by $L_i = 100$. In the no default state, we have $L_i = -5$. Thus, the L_i form a sequence of iid random variables with $\mathbb{P}(L_i = -5) = 0.98$ and $\mathbb{P}(L_i = 100) = 0.02$.

Consider two portfolios with current value equal to \$10,000. Portfolio A consists of 100 units of bond one and portfolio B consists of 1 unit of each of the 100 bonds. Calculate VaR at the 95% confidence level for each portfolio. Which portfolio has the higher VaR? Does this support or contradict your economic intuition about which portfolio carries more risk? Which property of a coherent risk measure is the key to this result?

- (c) Find closed form expressions for VaR and ES, given any confidence level $\alpha \in (0, 1)$, for the following loss distributions:
 - (a) normal (with parameters μ and σ)
 - (b) exponential (with parameter λ)
 - (c) power law (with parameter α)
 - (d) standardized student-t (with ν degrees of freedom)
 - (e) non-standardized student-t (with parameters μ , σ , and ν)
- (d) Next we examine the effect of different distributional assumptions on estimates of VaR and ES, using the data you downloaded from bSpace. For each data series (there are six in all) do the following:
 - (a) Assume the returns follow a *normal* distribution. Estimate the parameters of the distribution (μ and σ). Based on the estimated parameters and the distributional assumption, estimate VaR and ES.
 - (b) Assume the returns follow a *non-standardized student-t* distribution with ν degrees of freedom. Estimate the parameters of the distribution (μ , σ , and ν). Based on the estimated parameters and the distributional assumption, estimate VaR and ES.
 - (c) Compare your results from (a) and (b) with estimates of VaR and ES based on the *empirical* distribution.

Using either the CRSP data or the index data, form a portfolio (a convex combination of the asset classes) and repeat (a)-(c) for this portfolio.

- (e) Estimate correlations among pairs of asset classes over different economic regimes, using different time horizons and exponential weights, and by varying the return horizon. Relate your observations to relevant market conditions and events, and formulate hypotheses about the best way to estimate correlations to support effective financial decisions.
- (f) Re-examine the idea that expected return is good and risk is bad in the context of the last fifty years of market behavior.
- (g) Provide examples that illustrate the strengths and weaknesses of particular coherent risk measures. Discuss to what extent the axiomatic approach to measuring risk is valuable.
- (h) Provide data-driven arguments for or against the assertion that financial returns are normally distributed. Give examples of the implicit use of the normal distribution in financial decisions or regulation.

(2) Linear Factor Models in Equity Markets

- (a) We will first investigate the issue of how many stocks are required to *track* a major market. Go to bSpace and download the file *equity.zip*. This file contains data on the 1877 largest stocks in the U.S. equity market. The time period covered is January 2, 2004 through December 31, 2009 (1511 trading days). The data files of interest are:
- *secddata.txt*, this is an ascii file of dimension $(1511 * 1877) \times 3$. The first column is an integer corresponding to the tickers that can be found in the *ticker.txt* file, i.e., 1 corresponds to the first ticker in *ticker.txt*, 2 corresponds to the second ticker, and so on. The second column contains the stock returns, and the third column contains the market capitalizations.
 - *ticker.txt*, this is an ascii file containing a list of all the stock tickers in the data set.
 - *retdate.txt*, this is an ascii file containing a list of all the dates in the data set.
- (a) Plot the cumulative return to the market (as represented by the 1877 stocks in our data set). If you are using Matlab, the file *benchmark.m* might be helpful. If you are not using Matlab, it might still be useful to peruse *benchmark.m* to get some hints on how to easily manipulate the *secddata.txt* file.
- (b) Pick 50 stocks from the data set, using any criteria you choose. Plot the cumulative return to both a capitalization weighted portfolio and an equally weighted portfolio. Estimate the realized *tracking error* of your portfolios against the market, where we define tracking error as the root-mean-square of the difference between the 1-day simple return of the portfolio and the 1-day simple return of the market – this is also referred to as root-mean-squared error (RMSE). Calculate the RMSE over the entire sample period. Can you reduce the RMSE by choosing a different portfolio of 50 stocks?
- (c) Build 100 n -stock portfolios of *randomly* chosen stocks. Calculate the mean RMSE over the 100 portfolios (where the RMSE is calculated with respect to the entire sample period). Repeat for $n = 10, 20, 30, 40, 50, 60, 70, 80, 90, 100$, and plot the mean RMSE as a function of the number of securities in the portfolios. What do you find? Choose an n , and plot the cumulative returns to all 100 portfolios, and to the market (in the same figure).
- (d) Your goal is to minimize the RMSE and at the same time minimize the number of stocks in your tracking portfolio. Doing this “right” generally involves solving an optimization problem. You are not being asked to do that here. Can you find a heuristic approach to choosing stocks that does a reasonable job of meeting the criteria of minimizing the RMSE, for a given number of stocks? Use your approach to build several n -stock portfolios, where $n = 10, 20, 30, 40, 50, 60, 70, 80, 90, 100$, and plot the RMSE as a function of the number of stocks in the portfolios. How do your results compare to the results using randomized portfolios? Choose an n , and

plot the cumulative returns to your portfolio and to the market (in the same figure).

- (b) We now consider market models. All of the risk forecasts below should be estimated in three ways: expanding window, 63-day rolling window, and a 21-day half-life with exponential weights.
 - (a) Use a market model (beta of each stock with respect to the market) to forecast the risk of the individual securities in our data set. For a handful of securities, plot the market model risk forecasts and the risk forecasts based strictly on returns to each security. What characterizes stocks for which the market model works well? What characterizes stocks for which the market model works poorly? Base your determination of “works well” and “works poorly” on the visual evidence.
 - (b) Build 100 50-stock portfolios of *randomly* chosen stocks. Using the period January 2, 2004 through December 29, 2006, estimate the beta of each portfolio with respect to the market. Plot the security market line (mean excess returns against market model predicted expected excess returns). Does the market model appear to hold for this period? Compute the Gibbons, Ross, and Shanken (GRS) test.² Is the market model formally rejected? Repeat for the period January 2, 2007 through December 31, 2009, and for the entire sample period. What do you find?
 - (c) For a handful of the 50-stock portfolios from the previous part, plot the market model risk forecasts and the risk forecasts based strictly on returns to each portfolio. Can you say anything about portfolios for which the market model works well? What about portfolios for which the market model works poorly? Again, base your determination of “works well” and “works poorly” on the visual evidence.
- (c) Can we improve on the market model?
 - (a) Add the Fama-French size (SMB) and value (HML) factors to the market model and repeat part 2. Estimate sensitivities of securities and portfolios to SMB and HML via OLS.
 - (b) Evaluate your risk forecasts (over the entire *out-of-sample* period, as well as the sub-periods ending December 29, 2006 and December 31, 2009) using RMSE, QLIKE,³ and bias tests.⁴ Do the tests reveal a dominant forecasting methodology?

²See Campbell, Lo, and MacKinlay, Chapter 5.

³See Patton and Sheppard (2009).

⁴See Connor, Goldberg, and Korajczyk, Section 14.3.

(3) Linear Factor Models in Fixed Income Markets

- (a) In this part, we will investigate shift, twist, and butterfly “shapes” for three countries, Singapore (SGP), Great Britain (GBR), and the United States (USA). Go to bSpace and download the file *fixedincome.zip*. Look for the following three files:

- *yields_daily.txt*
- *yields_weekly.txt*
- *yields_monthly.txt*

All of these files have the same structure. Column 1 is the yield date, column 2 is the country (see *yields_countries.txt* for the integer equivalents of the countries), column 3 is the term-to-maturity of the yield, and column 4 is the yield. If you are using Matlab, the file *getyields.m* will help you extract yield data from these files. If you are not using Matlab, it might still be useful to peruse *getyield.m* to get some hints on how to easily manipulate the *yields_*.txt* files.

- (a) For each country, choose a date and calculate the continuously compounded yields. In the files, the yields with a term-to-maturity greater than 1 year are semi-annually compounded, the yields with a term-to-maturity less than 1 year are simply compounded, and the yields with a term-to-maturity equal to 1 year are simply compounded for the United States and Singapore, and semi-annually compounded for Great Britain.
- (b) Using the weekly data for each country, plot the principal components corresponding to the three largest eigenvalues of the covariance matrix of yield *changes*, against the yield terms-to-maturity (also commonly known as yield curve *vertices*). What fraction of the key rate covariance matrix is accounted for by the first three principal components? Are the shapes of the first three principal components similar across countries?
- (c) Repeat part (a) using the daily and monthly data. Does the data frequency make a difference to the fraction of the key rate covariance matrix accounted for by the first three principal components? Does it alter the shapes of the first three principal components?
- (b) For this part, please use the following files (found in *fixedincome.zip*):
- *spot_weekly.txt*, which has the same structure as *yields_weekly.txt*, but only contains U.S. rates. In addition, the rates in this file are *continuously* compounded rates (by comparison, the rates in *yields_weekly.txt* are *semi-annually* compounded).
 - *bonds_weekly.txt*, which contains weekly data for all non-callable U.S. Treasury bills, notes, and bonds, over the sample period January 3, 2001 through December 29, 2010. It has the following structure:
Column 1: integer id of bond (with respect to the quote date)
Column 2: quote date
Column 3: total return (period $t - 1$ to t , where t is the quote date)

- Column 4: total return (period t to $t + 1$)
- Column 5: number of cash flows (ncf)
- Columns 6 through $5+ncf$: time of cash flows (real-time)
- Columns $6+ncf$ through $5+2*ncf$: cash flows.

Note that when you load the *bonds_weekly.txt* file into Matlab it will be a matrix. The matrix will have many zeros in the columns after the last cash flow, for all but the longest maturity bonds.

- (a) Use the qualitative similarity in the first three principal components (commonly referred to as *shift*, *twist*, and *butterfly* in the context of fixed income markets), to define stylized factors for the U.S. market:
 - (i) parallel shift,
 - (ii) piecewise linear twist,
 - (iii) piecewise linear “parabolic” butterfly.

The twist and butterfly factors should be constructed as follows:

$$T(t) = \begin{cases} -1, & t < t_1 \\ \frac{t-t_m}{t_m-t_1}, & t_1 \leq t < t_m \\ 0, & t = t_m \\ \frac{t-t_m}{t_n-t_m}, & t_m < t \leq t_n \\ 1, & t > t_n \end{cases} \quad B(t) = \begin{cases} -1, & t < t_1 \\ \frac{t-t_\ell}{t_\ell-t_1}, & t_1 \leq t < t_\ell \\ \frac{t-t_\ell}{t_m-t_\ell}, & t_\ell \leq t < t_m \\ 1, & t = t_m \\ \frac{t_h-t}{t_h-t_m}, & t_m < t \leq t_h \\ \frac{t_h-t}{t_n-t_h}, & t_h < t \leq t_n \\ -1, & t > t_n \end{cases}$$

where t_1 is the first vertex, t_n is the last vertex, t_m is a middle vertex, t_ℓ is a vertex between t_1 and t_m , and t_h is a vertex between t_m and t_n . You will need to choose t_ℓ , t_m , and t_h (use your results from part 1 as a guide).

Choose three dates and for each date, plot the yield curve and your stylized factors applied to that yield curve (use a nominal shock value between 25 and 100 basis points).

- (b) Compute time series of weekly returns to shift, twist, and butterfly as linear combinations of returns to key rates. Plot the cumulative weekly returns to shift, twist, and butterfly over the sample period. Do these returns make sense in light of the following observations?
 - A *decrease* in all rates (across the term structure) should result in a positive return to the shift factor.
 - A *flattening* of the term structure should result in a positive return to the twist factor.
 - An *increase* in the *curvature* of the term structure should result in a positive return to the butterfly factor.
- (c) Forecast the risk of different bond portfolios built from the bonds in our estimation universe. Consider:
 - Longer and shorter duration bonds.
 - Pure discount bonds with different maturities.

- Factor-mimicking portfolios.
- (d) Up to now we have been modelling bond returns using a three-factor model (i.e., shift, twist, and butterfly). Consider the following two-factor model:

$$r_i = a + d_i \cdot ds + \frac{1}{2}c_i \cdot (ds)^2 + \epsilon,$$

where r_i is the total return of bond i , d_i is the *modified duration* of bond i , and c_i is the *convexity* of bond i . Both duration and convexity exposures are calculated with respect to a parallel shift in spot rates (note that with respect to a parallel shift in spot rates *modified duration* and *shift exposure* are the same thing). In this case, we are going to *infer* the returns to the duration and convexity factors by regressing (cross-sectionally) the bond returns on the duration and convexity of the bonds, at each date in the sample period. Analogously to part (c), the factor covariance matrix can then be built from this time series of factor returns.

Choose your favorite bond portfolio from part (c) and forecast the risk of this portfolio using the two-factor model. Plot the risk forecasts (over time) of this portfolio using the three-factor model and the two-factor model. Evaluate your risk forecasts (over the entire *out-of-sample* period) using RMSE, QLIKE,⁵ and bias tests.⁶ Do the tests reveal a dominant forecasting methodology?

⁵See Patton and Sheppard (2009).

⁶See Connor, Goldberg, and Korajczyk, Section 14.3.

(4) Linear Factor Models of Currency Risk

(a) We will first investigate whether or not currency returns are normally distributed. Go to bSpace and download the files *readme.txt* and *fxdata.txt*. The *fxdata.txt* file contains daily exchange rate data for the Brazilian Real, Japanese Yen, Australian Dollar, Euro, and British Pound. The time period covered is January 4, 1999 through August 19, 2011. The *readme.txt* file will give you the details on the rates contained in *fxdata.txt*.

(a) Plot the cumulative US dollar returns of each currency over the sample period (on the same plot).

(b) Evaluate the returns for normality using the following tests:

(i) Q-Q Plots

(ii) Jarque-Bera Test

(iii) Anderson-Darling Test

(iv) Lilliefors-Kolmogorov-Smirnov Test

To make sure you understand how the tests work, implement your own version of each test and compare your results to the respective built-in Matlab functions. Why do you need to use the Lilliefors version of the Kolmogorov-Smirnov test? Based on your results, do you think currency returns are normally distributed?

(b) Estimate a covariance and correlation matrices (using the entire sample period) for these currencies using equal weights and exponential weights with a half-life of 21 days.

(a) Create bar plots of the annualized volatilities under equal and exponential weighting.

(b) Create *heat maps* of the correlations under equal and exponential weighting.

Discuss your plots. Do you find anything surprising?

(c) We now consider risk forecasting. All of the forecasts below should be estimated in three ways: expanding window, 63-day rolling window, and exponential weights with a 21-day half-life.

(a) Set up an out-of-sample test to decide which estimation method leads to more accurate volatility forecasts. Evaluate your model on equally-weighted and GDP weighted portfolios, using RMSE, QLIKE,⁷ and bias tests.⁸ Do the tests reveal a dominant forecasting methodology?

(d) This question is based on the *covered interest arbitrage* relationship discussed

⁷See Patton and Sheppard (2009).

⁸See Connor, Goldberg, and Korajczyk, Section 14.3.

in lecture. Consider the following information:

$$r_d = 4\%; \quad r_f = 7\%$$

$$S_0 = \$2.00/\mathcal{L}$$

$$F_0 = \$1.98/\mathcal{L} \quad (1\text{-year delivery}),$$

where the interest rates are annual yields on U.S. (r_d) and U.K. (r_f) bills. Given the above information:

- (a) Where would you lend?
- (b) Where would you borrow?
- (c) Outline an arbitrage strategy and determine the profit of your strategy.

(5) Integrating Linear Factor Models

In this exercise we investigate the construction of a cross-asset class covariance matrix that is consistent with single asset class covariance matrices. Go to bSpace and download the file *integration.zip*. This file contains daily factor return and exposure data for the 1877 largest stocks in the U.S. equity market, and daily factor return and exposure data for non-callable U.S. Treasury bills, notes and bonds. The data files of interest are:

- *eqfret.txt*, this is an ascii file of dimension 1449×5 containing the factor returns for the equity market. The first column is the estimation date of the factor return, the second column is the factor return of the market factor (in this case, the market is represented by the S&P 500), the third column is the factor return of the Fama-French SMB factor, the fourth column is the factor return of the Fama-French HML factor, and the last column is the factor return of a “global” equity factor (estimated by assuming that the exposure to this factor is one for all stocks).
- *eqfexp.txt*, this is an ascii file of dimension 1449×5633 containing the factor exposures for the equity market. The first column is the estimation date of the factor exposure, the second column indicates how many non-zero columns are to follow, the third through the last non-zero columns contain the factor exposures. For each date, this exposure vector can be reshaped to a matrix of dimension 1877×3 . Each row in the reshaped matrix gives the market, SMB, and HML exposures for a particular stock.
- *eqfres.txt*, this is an ascii file of dimension 1449×3756 containing the factor return estimation residuals for the equity market. The first column is the estimation date of the factor return, the second column indicates how many non-zero columns are to follow, the third through the last non-zero columns contain the residuals. For each date, this residual vector can be reshaped to a matrix of dimension 1877×2 . The first column of the reshaped matrix contains the residuals from the market, SMB, and HML factor return estimation. The second column of the reshaped matrix contains the residuals from the “global” equity factor return estimation.
- *fifret.txt*, this is an ascii file of dimension 2432×5 containing the factor returns for the fixed-income market. The first column is the estimation date of the factor return, the second column is the factor return of the shift factor, the third column is the factor return of the twist factor, the fourth column is the factor return of the butterfly factor, and the last column is the factor return of a “global” fixed-income factor (estimated by assuming that the exposure to this factor is one for all bonds).
- *fifexp.txt*, this is an ascii file of dimension 2432×623 containing the factor exposures for the fixed-income market. The first column is the estimation date of the factor exposure, the second column indicates how many non-zero columns are to follow, the third through the last non-zero columns contain the factor exposures. For each date, this exposure vector can be reshaped to a matrix of

dimension $n \times 3$ (where n is the number of bonds for which there are exposures). Each row in the reshaped matrix gives the shift, twist, and butterfly exposures for a particular bond.

- *fifres.txt*, this is an ascii file of dimension 2432×416 containing the factor return estimation residuals for the fixed-income market. The first column is the estimation date of the factor return, the second column indicates how many non-zero columns are to follow, the third through the last non-zero columns contain the residuals. For each date, this residual vector can be reshaped to a matrix of dimension $n \times 2$. The first column of the reshaped matrix contains the residuals from the shift, twist, and butterfly factor return estimation. The second column of the reshaped matrix contains the residuals from the “global” fixed-income factor return estimation.
 - *comdates.txt*, this is an ascii file of dimension 1438×1 containing a list of the estimation dates that are common to both markets.
- (a) Pick an estimation date and estimate single asset class covariance matrices for the equity and fixed-income markets, based on “local” factor returns (i.e., market, SMB, and HML factor returns for the equity market, and shift, twist, and butterfly factor returns for the fixed-income market). Use an exponential weighting with a 252 day half-life. Choose a random portfolio of 25 stocks and make a risk forecast for the portfolio using the provided factor exposures – assume the stocks are equally weighted in the portfolio. Choose a random portfolio of 25 bonds and make a risk forecast for the portfolio using the provided factor exposures – assume the bonds are equally weighted in the portfolio.
 - (b) Construct a cross-asset class covariance matrix following the procedure outlined in section 5 of Shepard (2008).⁹ Based on this covariance matrix, the provided factor exposures, and assuming a factor exposure of 1 to the global factors, make risk forecasts for the portfolios constructed in the previous step. Do the risk forecasts for the portfolios match those from step 1?
 - (c) Adjust the factor exposures for each portfolio as suggested by equation (21) in Shepard and make risk forecasts for your portfolios. Do the risk forecasts for the portfolios match the forecasts from step 1? Are they closer to the step 1 forecasts than those from step 2? If the forecasts do not match, what assumption can you point to that explains the differences?
 - (d) As suggested by equation (23) in Shepard, include an adjustment for the global/local covariance terms and make risk forecasts for your portfolios. Do the risk forecasts for the portfolios match the forecasts from step 1? (*Hint*: at this point your risk forecasts should match those from step 1).
 - (e) Repeat steps 1-4 for at least 2 other estimation dates and/or half-lives.

⁹See Shepard (2008).

(6) Attributing and Budgeting in Linear Factor Models

- (a) To first order, return to a portfolio can be expressed as a value-weighted sum of asset returns, but the risk of a portfolio is not the weighted sum of the asset risks. However, the Euler formula:

$$f(x_1, x_2, \dots, x_n) = \sum_{i=1}^n x_i \frac{\partial f}{\partial x_i}$$

for differentiable, positive linearly homogenous functions can be applied to all coherent risk measures including volatility and expected shortfall.

- (a) Prove the Euler formula holds for a differentiable function f if and only if it is positive linearly homogenous.
- (b) Show that when risk is volatility $\sigma(R_P)$ of a portfolio P , then the marginal contribution of asset i to $\sigma(R_P)$ can be expressed as the product of the volatility of the return of the asset R_i and the linear correlation of the return of the asset R_i with the return of the portfolio R_P , i.e., show that

$$\frac{\partial \sigma(R_P)}{\partial x_i} = \sigma(R_i) \cdot \rho(R_P, R_i),$$

where x_i is the portfolio weight of asset i .

- (c) Show that when risk is expected shortfall $S(R_P) = \mathbb{E}[L_P | L_P > \text{VaR}_P]$ of a portfolio P , where $L_P = -R_P$, then the marginal contribution of asset i to $S(R_P)$ can be expressed as the product of the expected shortfall of the return of the asset R_i and the *shortfall implied correlation* of the return of the asset R_i with the return of the portfolio R_P , i.e., show that

$$\frac{\partial S(R_P)}{\partial x_i} = S(R_i) \cdot \rho^S(R_P, R_i),$$

where

$$\rho^S(R_P, R_i) = \frac{\mathbb{E}[L_i | L_P > \text{VaR}_P]}{\mathbb{E}[L_i | L_i > \text{VaR}_i]},$$

where $L_i = -R_i$.

- (b) Go to bSpace and download the file *equity.zip*. This file contains data on the 1877 largest stocks in the U.S. equity market. The time period covered is January 2, 2004 through December 31, 2009 (1511 trading days). The data files of interest are:
- *secdata.txt*, this is an ascii file of dimension $(1511 * 1877) \times 3$. The first column is an integer corresponding to the tickers that can be found in the *ticker.txt* file, i.e., 1 corresponds to the first ticker in *ticker.txt*, 2 corresponds to the second ticker, and so on. The second column contains the stock returns, and the third column contains the market capitalizations.
 - *ticker.txt*, this is an ascii file containing a list of all the stock tickers in the data set.

- *retdate.txt*, this is an ascii file containing a list of all the dates in the data set.
- (a) Choose a portfolio of 100 stocks from the 1877 stocks in *secdata.txt*. Choose portfolio weights x_i for each stock such that $\sum_{i=1}^{100} x_i = 1$. Using volatility as the risk measure, and the *X-Sigma-Rho* formula ¹⁰, perform a *factor-based* attribution of the *total* risk of the portfolio (you can think of total risk as active risk where the benchmark is cash). Use the return of the market (as represented by the capitalization weighted return of the 1877 stocks in our data set) and the returns to the Fama-French size (SMB) and value (HML) portfolios as the factors. Report the following items for each factor: portfolio exposure, factor return volatility, correlation of factor returns and portfolio returns, marginal contribution to portfolio risk, and contribution to portfolio risk. Report factor totals, specific, and overall totals for the following items: return volatility, correlation of returns and portfolio returns, marginal contribution to portfolio risk, contribution to portfolio risk, and percent contribution to portfolio risk.
 - (b) Perform a *security-based* attribution of the *total* risk of the portfolio. Report the following items for each stock: portfolio weight, stock return volatility, correlation of stock returns and portfolio returns, marginal contribution to portfolio risk, and contribution to portfolio risk. Report totals for the following items: stock return volatility, correlation of stock returns and portfolio returns, marginal contribution to portfolio risk, contribution to portfolio risk, and percent contribution to portfolio risk.
 - (c) Choose a second set of portfolio weights y_i for each stock such that $\sum_{i=1}^{100} y_i = 1$. Call the portfolio formed based on these weights the *benchmark* portfolio. Using volatility as the risk measure, and the *X-Sigma-Rho* formula, perform a *factor-based* attribution of the *active* risk of the portfolio. Report the following items for each factor: portfolio exposure, benchmark exposure, active exposure, factor return volatility, correlation of factor returns and portfolio active returns, marginal contribution to portfolio active risk, and contribution to portfolio active risk. Report factor totals, specific, and overall totals for the following items: return volatility, correlation of returns and portfolio active returns, marginal contribution to portfolio active risk, contribution to portfolio active risk, and percent contribution to portfolio active risk.
 - (d) Perform a *security-based* attribution of the *active* risk of the portfolio. Report the following items for each stock: portfolio weight, benchmark weight, active weight, stock active return volatility, correlation of stock active returns and portfolio active returns, marginal contribution to portfolio active risk, and contribution to portfolio active risk. Report totals for the following items: stock active return volatility, correlation of stock active returns and portfolio active returns, marginal contribution to portfolio active risk, contribution to portfolio active risk, and percent contribution to portfolio

¹⁰See Menchero and Davis (2011).

active risk.

- (e) Repeat parts (a)-(d) using expected shortfall (with $\alpha = 0.05$) as the risk measure.¹¹
- (f) For total risk, find the factor and stock for which the differences between volatility and expected shortfall risk contributions are the greatest. What can you say about the distribution of the returns for this factor and stock versus the distribution of returns for the other factors and stocks? Repeat for the active risk case.

¹¹See Goldberg et al (2010).