Marshallian Externalities, Comparative Advantage, and International Trade*

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Abstract

There is strong evidence for the existence of external economies of scale that are limited in their industrial and geographical scope. What are the implications of these Marshallian externalities for the patterns of international trade, the welfare gains from trade, and industrial policy? The standard model in the literature assumes that firms engage in perfect competition and ignore the effect of their actions on industry output and productivity. This has the unfortunate implication that any assignment of industries across countries is consistent with equilibrium. To avoid this predicament, we follow Grossman and Rossi-Hansberg (2010) and assume that firms in each industry engage in Bertrand competition and understand the implications of their decisions on industry output and productivity. We develop three main results. First, we show that the indeterminacy of international trade patterns still persists for some industries when trade costs are low. Second, we apply these results in a full general equilibrium analysis and reexamine the implications of Marshallian externalities for industrial policy. Our results indicate that the additional welfare gains from moving to the Pareto-superior equilibrium depend positively on the strength of Marshallian externalities and negatively on the strength of comparative advantage, and – using reasonable parameter estimates – are at most about 2%. Finally, our framework allows us to ask whether Marshallian externalities lead to additional gains from trade. Our quantitative analysis indicates that this is indeed the case, and that Marshallian externalities increase overall gains from trade by around 50%.

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1 Introduction

There is strong evidence for the existence of external economies of scale that are limited in their industrial and geographical scope.[1] Such external economies of scale are commonly known as Marshallian or agglomeration externalities. The central idea is that the concentration of production in a particular location generates external benefits for firms in that location through knowledge spillovers, labor pooling, and close proximity of specialized suppliers. Classic examples include Silicon Valley software industry, Detroit car manufacturing, and Dalton carpets. More recent examples point to the potential significance of these externalities for international trade. For instance, Qiaotou, Wenzhou and Yanbu are all relatively small regions in China that account for 60% of world button production, 95% of world cigarette lighter production, and dominate global underwear production, respectively.[2]

What are the implications of these Marshallian externalities for the patterns of international trade, the welfare gains from trade, and industrial policy? The standard model in the literature assumes that firms engage in perfect competition and ignore the effect of their actions on industry output and productivity. This has the unfortunate implication that any assignment of industries across countries is consistent with equilibrium. To avoid this predicament, we follow Grossman and Rossi-Hansberg (2010) and assume that firms in each industry engage in Bertrand competition and understand the implications of their decisions on industry output and productivity. We develop three main results. First, we show that the indeterminacy of international trade patterns still persists for some industries when trade costs are low, and points to a potential role for industrial policy.[3] Second, we follow up by applying these results in a full general equilibrium analysis, and reexamine the implications of Marshallian externalities for industrial policy. To assess the welfare importance of industrial policy, we construct a quantitative general equilibrium Ricardian trade model with Marshallian externalities. Our results indicate that the additional welfare gains from moving to the Pareto-superior equilibrium depend positively on the strength of Marshallian externalities and negatively on the strength of comparative advantage, and – using reasonable parameter estimates – are at most about 2%. Finally, our framework allows us to ask whether Marshallian externalities lead to additional gains from trade. Our quantitative analysis indicates that this is indeed the case, and that Marshallian externalities increase overall gains from trade by around 50%.

The standard approach to incorporate Marshallian externalities in an international trade model has been to assume that perfectly competitive firms take productivity as given, even though it depends positively on aggregate industry output (see Chipman, 1969). Typical results are the existence of multiple Pareto-rankable equilibria, the possibility that trade patterns may run counter to "natural" comparative advantage,[4] and the possibility that some countries may lose from trade.[5] An important implication was that Marshallian externalities provided a theoretical basis for infant-industry protection. For instance, Ethier (1982, henceforth Ethier) using a standard two country, two sector (one with constant returns to scale and the other with increasing returns to scale) Ricardian model formally confirmed the infant-industry argument.

GRH refer to the results that trade patterns can be inconsistent with "natural" comparative advantage and that a country can potentially lose from trade as "pathologies". To avoid such "pathologies", GRH propose an alternative equilibrium analysis in a Ricardian model of trade with Marshallian externalities. In particular, they follow Dornbush, Fischer and Samuelson (1977) and assume that there is a continuum of industries (instead of the two industries of the standard trade model), and they abandon perfect competition and assume instead that firms engage in Bertrand competition. Consistent with Bertrand competition, GRH assume that firms recognize the effect that they have on the scale of production and, through external economies of scale, the effect on their own productivity. They show that under frictionless trade the multiplicity of equilibria disappears, and the unique equilibrium entails specialization according to "natural" comparative advantage, as would have occurred in a standard constant returns to scale framework. Moreover, they argue

[1] See for example Caballero and Lyons (1989, 1990, 1992); Chan, Chen and Cheung (1995); Segoura (1996); Henriksen, and Steen and Ulltveit-Moe (2001)).

[2] See Krugman (2009) for a nice discussion of this.

[3] Note that the indeterminacy arises because of multiple equilibria. As such, we think of industrial policy as a policy in which a country attempts to select an equilibrium that leads to higher national welfare.

[4] In a Ricardian context, this means that the pattern of specialization can run counter to the ranking of relative (exogenous) productivities when measured at a common scale of production.

[5] See early work by Graham (1923), Ohlin (1933), Matthews (1949-50), Kemp (1964), Melvin (1969), and Markusen and Melvin (1982).

that if trade costs are low, the equilibrium is unique and entails complete specialization. For intermediate trade costs GRH show that there is no equilibrium in pure strategies, and they propose a mixed strategy equilibrium in which there is a probability that one country supplies the global market or sell only in the domestic market. As expected, for large enough trade costs a good becomes non-traded. In essence, GRH argues that, even with trade costs, the pattern of trade is consistent with "natural" comparative advantage, and there are always gains from trade. An important implication is that external economies of scale no longer seem to provide a theoretical foundation for industrial policy, calling into question the robustness of the argument for protection.

Consistent with GRH, we find that there are always gains from trade, but our results differ in that we find that trade patterns can indeed be "pathological" when trade costs are low. In Section 2 we provide an analysis of the equilibrium configurations under different trade costs for a single industry that exhibits Marshallian externalities using a simplified version of GRH. We find that GRH's uniqueness result for the case of low trade costs relies on an implicit assumption of industry specific trade costs which are inversely related to the strength of comparative advantage, with basically zero trade costs for the "no comparative" advantage industry. Once we allow for more general "low" trade costs, multiple complete specialization equilibria arises for a set of industries with "weak" comparative advantage implying that trade patterns need not be consistent with "natural" comparative advantage. More importantly, the multiplicity hints at a potential role for industrial policy, by which we mean a policy where a country tries to select an equilibrium associated with a higher real income for itself.

As in GRH, the equilibrium under intermediate trade costs has firms in the country with a comparative advantage mixing over a local and global-pricing strategy, so there may or may not be trade. However, the equilibrium mixed strategy is not the one proposed by GRH. In fact, we illustrate that there is a profitable deviation from the mixed strategy proposed by GRH and then propose an alternative mixed strategy and establish that it is indeed an equilibrium. Interestingly, unlike the multiple mixed strategy equlibrium initially proposed by GRH, we find that the common mixing probabilities are unique suggesting a unique mixed strategy equilibrium among the particular class of equilibria we consider. An important caveat is that this mixed strategy only applies to industries for which a particular country has a "strong" comparative advantage (defined below). At this point, we have not characterized the equilibrium for the case of a "weak" comparative advantage industry. Of course, as in GRH, for large enough trade cost there is no trade.

In section 3 we we apply the results under partial equilibrium to examine the welfare implications of Marshallian externalities in general equilibrium. We characterize a set of complete specialization equilibria for the case of symmetric countries and low trade costs which are common across industries.[6] In particular, we characterize a set of "disputed" industries for which each country has a relatively "weak" comparative advantage. Importantly, note that an equilibrium in which a country produces a larger set of these industries is associated with a higher national welfare. Also, note that the trade cost at which this set is the largest is also the one for which the potential welfare gains from industrial policy is the biggest. We show that the set of "disputed" industries first monotonically increases with trade costs, and then shrinks as we approach the maximum trade cost under which there is complete specialization equilibrium. In that regard, our analysis highlights the trade cost for which the largest set of disputed industries occurs. This is the focus of our quantitative analyses exploring both the potential scope for industrial policy, as well as a first look at the quantitative importance of Marshallian externalities for the gains from trade.

To assess the importance of industrial policy, we move to a more general setting with potentially asymmetric countries, and focus on the trade cost associated with the largest set of disputed industries. We do so in order to get a sense of the maximum possible scope for industrial policy. We build a quantitative general equilibrium Ricardian trade model with Marshallian externalities using a probabilistic framework similar to that in Eaton and Kortum (2002, henceforth EK) and quantify the welfare implications of two extreme equilibria: one in which a country is arbitrarily assigned the production of all the disputed industries and one in which all those industries are assigned to the other country. In particular, we compute real wages for each equilibria which we then use to calculate the additional welfare gains from producing the entire set of disputed industries. We use three independent estimates for the two key parameters of our model which govern the strength of Marshallian externalities and that of comparative advantage. Our results indicate

[6] The assumption of trade costs which are constant across industries is a useful simplification for quantitative exercises and is commonly used in the trade literature. See, for instance, Dornbush, Fischer and Samuelson (1977), Eaton and Kortum (2002), and Alvarez and Lucas (2007).

that the additional welfare gains from moving to the Pareto-superior equilibrium depend positively on the strength of Marshallian externalities and negatively on the strength of comparative advantage, and – using reasonable parameter estimates – are at most about 2%.

By highlighting a theoretical basis for industrial policy, our analysis is broadly consistent with that of Ethier and the standard textbook approach to modelling static externalities in international trade. However, its implications are distinct in five main ways. First, unlike in Ethier, trade is always welfare improving. In that sense, the potential scope for industrial policy does not arise because of an attempt to avoid a bad equilibrium where a country loses from trade. Instead, in our framework, the incentive is to move to an equilibrium that entails *additional* welfare gains from trade. Second, Ethier argues that the case for protection applies to the smaller of two relatively similar sized economies, and is "less likely the greater the degree of increasing returns". In our analysis, a potential scope for industrial policy applies equally to the case of two similar sized countries as it does to the case of a relatively small country and a large one. Moreover, the scope for industrial policy increases with the degree of increasing returns by raising the maximum trade cost for which multiple complete specialization equilibria applies, and thereby expanding the set of disputed industries for which industrial policy potentially applies. Third, in Ethier, a potential role for industrial policy arises in a world with frictionless trade. This is, however, not the case in our framework. In a world with frictionless trade there is no potential role for industrial policy. A potential role arises when trade costs between both countries are positive but not too high. Finally, we move beyond theory and embed our model in a quantitative framework to get a sense of the potential importance of industrial policy.

We go further by also investigating the potential importance of Marshallian externalities for the gains from trade. In particular, our framework allows us to ask and provide insights to new questions: do Marshallian externalities imply additional gains from trade? If so, how important are these externalities for the overall gains from trade? Insights from the case of low trade costs with two symmetric countries suggests that Marshallian externalities do, in fact, imply larger gains from trade over and above those predicted by a traditional constant returns to scale framework. More importantly, a decomposition of these gains suggests the contribution can be substantial. The median parameter estimates indicate Marshallian externalities can account for approximately 35% of the overall gains from trade.

By highlighting Marshallian externalities as a potentially important channel of gains from trade, our work is also related to the international trade literature that focuses on quantifying the contribution of a particular margin to the overall gains from trade; see for instance recent work by Broda and Weinstein (2006), and Feenstra and Kee (2008), Goldberg et al (2009), and Feenstra and Weinstein (2009). While this literature has made significant progress in highlighting new margins of gains, the implications for the size of the total gains from trade has not changed (see Arkolakis et. al., 2010). In contrast, our analysis indicates that Marshallian externalities may not only be a significant margin of gains, but also one which has important implications for measuring the overall gains from trade. In this regard, our preliminary results seem to provide some support for the widespread perception among trade economists that the gains from trade are larger than those predicted by traditional quantitative trade models.[7]

Finally, we demonstrate that in the absence of trade costs the model readily extends to a multicountry setting, and yields a simple intuitive expression for the gains from trade. In particular, the welfare gains from trade depend only on the expenditure share on domestically produced goods and the two key parameters governing the strength of both comparative advantage and Marshallian externalities.

2 The Model

Our model is a simplified version of GRH. In particular, we assume Cobb-Douglas preferences (as was done in the earlier literature) instead of the more general constant elasticity of substitution (CES) preferences used in GRH. Also, for expositional simplicity, we assume a particular functional form for the channel through which Marshallian externalities operate. We start first by outlining the general environment. Next we proceed with a partial equilibrium analysis in which we focus on a particular industry and characterize equilibria for different levels of trade costs. Finally, in a full general equilibrium analysis, we characterize a set of complete specialization equilibria for the case of low trade costs in order to gain insights for our quantitative analysis in the following section.

[7] See Arkolakis et al (2008) for a discussion about this.

2.1 The General Environment

There are two countries, Home (H) and Foreign (F), and a continuum of industries/goods indexed $v \in [0,1]$. Preferences in each country $i = H, F$ are identical and uniform Cobb-Douglas with associated industry demand $x_i(p(v)) = D_i / p_i(v)$ where D_i is the aggregate expenditure in country i, and $p_i(v)$ is the price in country i and industry v.

Labor is the only factor of production and is inelastically supplied. Labor can move freely across industries within a country, but is immobile across countries. We denote by L_i and w_i the labor supply and wage in country i. The production technology has constant or increasing returns to scale due to external economies at the local industry level. In country i it takes $a_i(v) / (X_i(v))^{\phi}$ units of labor to produce a unit of output, where $a_i(v) > 0$ is an exogenous productivity parameter, $X_H(v)$ is the total production of the good in country i, and ϕ is the parameter which governs the strength of Marshallian externalities. In each industry and in each country there are two producers in Home and Foreign, $m_H(v) = m_F(v) = 2$.[8] Markets are segmented, so that firms can set arbitrarily different prices across the two markets.[9] Firms in each industry engage in Bertrand (price) competition in each market: setting a price above the minimum price leads to no sales; setting a price below all other prices allows the firm to capture the entire market; and setting a price equal to the minimum price implies that the market is shared among all those firms that set this same price.

Trade costs are of the "iceberg" type, so that delivering a unit of the good from one country to the other requires shipping $\tau(v) \geq 1$ units.

We make the following restriction on the the parameter which governs the strength of Marshallian externalities:

Assumption 1: $0 \leq \phi < 1/2$.

Note that if $\phi = 0$ then the technology exhibits constant returns to scale, and the standard results obtain. In particular, the equilibrium entails complete specialization in Home for $w_H a_H(v) \tau(v) \leq w_F a_F(v)$, complete specialization in Foreign for $w_H a_H(v) \geq w_F a_F(v) \tau(v)$, and the equilibrium entails no trade otherwise.

Also, for reasons explained below in the subsection on partial equilibrium, we also restrict the range of trade costs as follows:

Assumption 2: *Trade costs $\tau(v)$ are bounded above by $1/\phi$* for every v*.*

2.2 Partial Equilibrium

In this subsection we focus on a particular industry, and treat wages as exogenous and fixed at w_H and w_F. For simplicity we restrict the analysis to the case in which demand is symmetric in the two countries, namely $D_H = D_F$. Formally,

Assumption 3: *Demand is symmetric in the two countries: $D_H = D_F$. Without loss of generality we set D_H and D_F to one.*

2.2.1 Autarky

Consider the autarky equilibrium in Home. Market equilibrium under autarky requires that production be equal to demand $X_H = x_H(p_H)$, while Betrand competition leads to average cost pricing, $p_H = w_H a_H / (X_H)^{\phi}$. These two equations imply

$$p_H^A = w_H a_H / (x_H(p_H^A))^{\phi}. \tag{1}$$

A sufficient condition for existence is that the demand curve be steeper than the supply curve, that is,

$$\phi < 1. \tag{2}$$

[8] As in GRH, we can accomodate a finite number of firms in each industry and country. However, doing so complicates the intermediate trade costs analysis without adding any interesting insights. Hence, we simply assume two firms from the onset.

[9] In principle, one could also consider the case of integrated markets. However, we follow GRH and assume the simplier case of segmented markets.

This condition is guaranteed by Assumption 1. In principle, there could be an additional equilibrium at $p_H^A = 0$ if $(X_H)^\phi$ is not bounded above. In what follows we ignore this possibility.

Consider the allocation with p_H^A as determined by (1) and imagine that a firm deviates by charging a price p_H slightly lower than p_H^A. The deviant would capture the whole market and make profits of

$$\pi(p_H) \equiv \left(p_H - \frac{w_H a_H}{(x_H(p_H))^\phi} \right) x_H(p_H).$$

But since $p_H^A = w_H a_H / \left(x_H(p_H^A)\right)^\phi$, then it is easy to show that (2) implies $\pi(p_H) < 0$ for all $p_H < p_H^A$. We henceforth use p_H^A to denote the solution to (1). Under assumption 3 this is simply $p_H^A = (w_H a_H)^{1/(1-\phi)}$.

2.2.2 Frictionless Trade

GRH show that Bertrand competition gives rise to a unique equilibrium in which the pattern of trade is governed by Ricardian comparative advantage. In particular, Home will export the good if $w_H a_H < w_F a_F$, and the opposite will occur if $w_H a_H > w_F a_F$. Without loss of generality, we henceforth assume that $w_H a_H < w_F a_F$, so that Home has a comparative advantage in the good under consideration. In equilibrium, Home firms will sell at average cost in both the Home and Foreign markets, so the equilibrium price in both markets is determined by:

$$p_H^{FT} = \frac{w_H a_H}{(x_H(p_H^{FT}) + x_F(p_H^{FT}))^\phi}.$$

Firms make zero profits, and no firm can profitably deviate – in particular, the assumption that $\phi < 1$ implies that any firm in Home or Foreign would make losses by charging a price lower than p_H^{FT}. Moreover, the assumption that $\phi < 1$ also implies that the equilibrium is unique.

Can there be an equilibrium in which Foreign firms dominate the industry, selling in both markets? The answer is no. To see this, note that such an equilibrium would have a price p_F^{FT} in both markets given implicitly by

$$p_F^{FT} = \frac{w_F a_F}{(x_H(p_F^{FT}) + x_F(p_F^{FT}))^\phi}.$$

A Home firm could shave this price, capture both markets, achieve economies of scale that lead to productivity $(x_H(p_F^{FT}) + x_F(p_F^{FT}))^\phi / a_H$, and achieve cost

$$\frac{w_H a_H}{(x_H(p_F^{FT}) + x_F(p_F^{FT}))^\phi}.$$

This is lower than p_F^{FT} by the starting assumption that $w_H a_H < w_F a_F$, so the deviation is profitable.

2.2.3 Costly Trade

We study three types of equilibria: complete specialization (i.e., firms from one country supply both markets); mixed strategy equilibria (i.e., firms from one country randomize over which markets to serve and the price to charge, while firms from the other country offer to serve their own market at the autarky price); no trade (i.e., firms from each country serve only their own market). We start by considering the possibility of an equilibrium with complete specialization.

Complete specialization equilibrium If Home serves both markets, the equilibrium entails prices p_H and $p_F = \tau p_H$, with p_H determined by

$$p_H = \frac{w_H a_H}{(x_H(p_H) + x_F(\tau p_H))^\phi}. \tag{3}$$

To establish that this is an equilibrium, we need to consider the possible deviations by Home and Foreign firms.

A preliminary result is that the best that any firm can do is to shave prices p_H and p_F, i.e., it is never optimal to charge strictly lower prices than p_H and p_F. Recall that in autarky and under frictionless trade this is guaranteed by $\phi < 1$. But in the presence of trade costs, we need a more stringent condition. This is because there is an additional gain to a firm in lowering the domestic price, because now the economies of scale lead to lower costs that can also be exploited in exports. We can show that $\phi < 1$ and $\tau < 1/\phi$ (guaranteed by Assumptions 1 and 2, respectively) imply that the best possible deviation for a Home or Foreign firm entails shaving prices p_H and p_F (see the Appendix for a formal statement and proof of this result).

Consider now a deviation by a Home firm. Since prices p_H and p_F with p_H given by (3) imply that Home firms make zero profits in both markets, then a Home firm cannot make positive profits with any alternative set of prices. So this establishes that there is no profitable deviation for a Home firm.

Turning to Foreign firms, we need to study two possible deviations: first, no firm from Foreign should find it optimal to take over the world market by undercutting Home firms in both markets, and second, no firm from Foreign should find it optimal to displace Home firms from the Foreign market. Writing x_H and x_F as shorthand for $x_H(p_H)$ and $x_F(\tau p_H)$, with p_H implicitly defined by (3) , a sufficient condition for it to be unprofitable for Foreign firms to take over both markets is given by

$$\left[\frac{w_H a_H}{(x_H + x_F)^\phi} - \frac{w_F a_F \tau}{(x_H + x_F)^\phi}\right] x_H + \left[\frac{w_H a_H \tau}{(x_H + x_F)^\phi} - \frac{w_F a_F}{(x_H + x_F)^\phi}\right] x_F \leq 0.$$

This is equivalent to

$$\frac{a_H}{a_F} \leq \frac{w_F}{w_H} \frac{\tau x_H + x_F}{x_H + \tau x_F}. \tag{4}$$

In turn, a sufficient condition for it to be unprofitable for Foreign firms to displace Home firms from the Foreign market (only) is

$$\left[\frac{w_H a_H \tau}{(x_H + x_F)^\phi} - \frac{w_F a_F}{(x_F)^\phi}\right] x_F \leq 0. \tag{5}$$

This is equivalent to

$$\frac{a_H}{a_F} \leq \frac{w_F}{w_H} \frac{(x_H + x_F)^\phi / (x_F)^\phi}{\tau}. \tag{6}$$

The second term on the RHS of (6), $\frac{(x_H+x_F)^\phi/(x_F)^\phi}{\tau}$, captures the *trade cost-scale effect* trade-off: the larger the benefits of economies of scale from capturing both markets, the larger the trade cost needs to be to effectively protect Foreign firms in their domestic market.

The previous arguments establish that if conditions (4) and (6) are satisfied for p_H given by the solution of (3), then there is an equilibrium with complete specialization in which Home firms serve both markets. Similarly, an equilibrium with complete specialization where Foreign serves both markets would have prices p_H and p_F, where $p_H = \tau p_F$ and p_F is implicitly defined by

$$p_F = \frac{w_F a_F}{(x_H(\tau p_F) + x_F(p_F))^\phi}. \tag{7}$$

This is an equilibrium if the following two conditions are satisfied:

$$\frac{a_H}{a_F} \geq \frac{w_F}{w_H} \frac{\tau x_H(\tau p_F) + x_F(p_F)}{x_H(\tau p_F) + \tau x_F(p_F)}, \tag{8}$$

$$\frac{a_H}{a_F} \geq \frac{w_F}{w_H} \frac{\tau}{(x_H(\tau p_F) + x_F(p_F))^\phi / (x_H(\tau p_F))^\phi}. \tag{9}$$

If trade costs are low, i.e., if τ close to 1, condition (4) implies condition (6) and condition (8) implies (9). In other words, a deviation to serve the domestic market only is never profitable when trade costs are low, since such a deviation implies high costs due to small scale but small benefits from being able to save on trade costs.

Let $\omega = w_H/w_F$ and $\beta = a_H/a_F$. Our assumption above that $a_H w_H < a_F w_F$ (so that Home has a comparative advantage) implies that $\beta\omega < 1$. Using Assumption 3, so that $x_H(p_H) = 1/p_H$ and $x_F(p_F) = 1/p_F$, then conditions (4) and (6) with $p_F = \tau p_H$, can be written as

$$\beta\omega \leq \frac{\tau x_H(p_H) + x_F(\tau p_H)}{x_H(p_H) + \tau x_F(\tau p_H)} = \frac{\tau + 1/\tau}{2} \equiv g_H(\tau) \tag{10}$$

and

$$\beta\omega \leq \frac{\left(x_H(p_H) + x_F(\tau p_H)\right)^{\phi} / \left(x_F(\tau p_H)\right)^{\phi}}{\tau} = \tau^{-1}\left(1+\tau\right)^{\phi} \equiv h_H(\tau). \tag{11}$$

Similarly, conditions (8) and (9) with $p_H = \tau p_F$ can be written as

$$\beta\omega \geq \frac{2}{1/\tau + \tau} \equiv g_F(\tau) \tag{12}$$

and

$$\beta\omega \geq \tau\left(1+\tau\right)^{-\phi} \equiv h_F(\tau). \tag{13}$$

Note that $g_F(\tau) = 1/g_H(\tau)$ and $h_F(\tau) = 1/h_H(\tau)$. For future reference we refer to this as symmetry. Moreover, note that $g_F(1) = 1$ and $g_F(\tau) < 1$ for $\tau > 1$.

To proceed, we need some additional notation. Let $\tau_{MAX} = 1/\phi$. Let $\widetilde{\tau}$ be implicitly defined by $g_F(\tau) = h_F(\tau)$, let $\tau_H^{CS}(\beta\omega)$ and $\tau_F^{CS}(\beta\omega)$ be implicitly defined by $\beta\omega = h_H(\tau)$ and $\beta\omega = h_F(\tau)$, respectively, and let $\tau_F^0(\beta\omega)$ be implicitly defined by $\beta\omega = g_F(\tau)$. We need to consider two cases: strong and weak comparative advantage. We say that Home has a strong comparative if $\beta\omega < g_F(\widetilde{\tau})$, as in Figure 1a. We say that Home has a weak comparative advantage if $g_F(\widetilde{\tau}) \leq \beta\omega < 1$, as in Figure 1b.

Proposition 1 *Assume that $\beta\omega < 1$. There are two cases. Case a: Home has a strong comparative advantage, i.e., $\beta\omega < g_F(\widetilde{\tau})$. Then for $\tau \in [1, \min\left\{\tau_H^{CS}(\beta\omega), \tau_{MAX}\right\}]$ there is a unique equilibrium with complete specialization, and this equilibrium has Home serving both markets. Case b: Home has a weak comparative advantage, i.e., $g_F(\widetilde{\tau}) \leq \beta\omega < 1$. Then for $\tau \in [1, \tau_F^0(\beta\omega)] \cup [\tau_F^{CS}(\beta\omega), \tau_H^{CS}(\beta\omega)]$ there is a unique equilibrium with complete specialization, and this equilibrium has Home serving both markets, whereas for $\tau \in [\tau_F^0(\beta\omega), \tau_F^{CS}(\beta\omega)]$ there are two complete specialization equilibria, one with Home serving both markets, and another with Foreign serving both markets.*

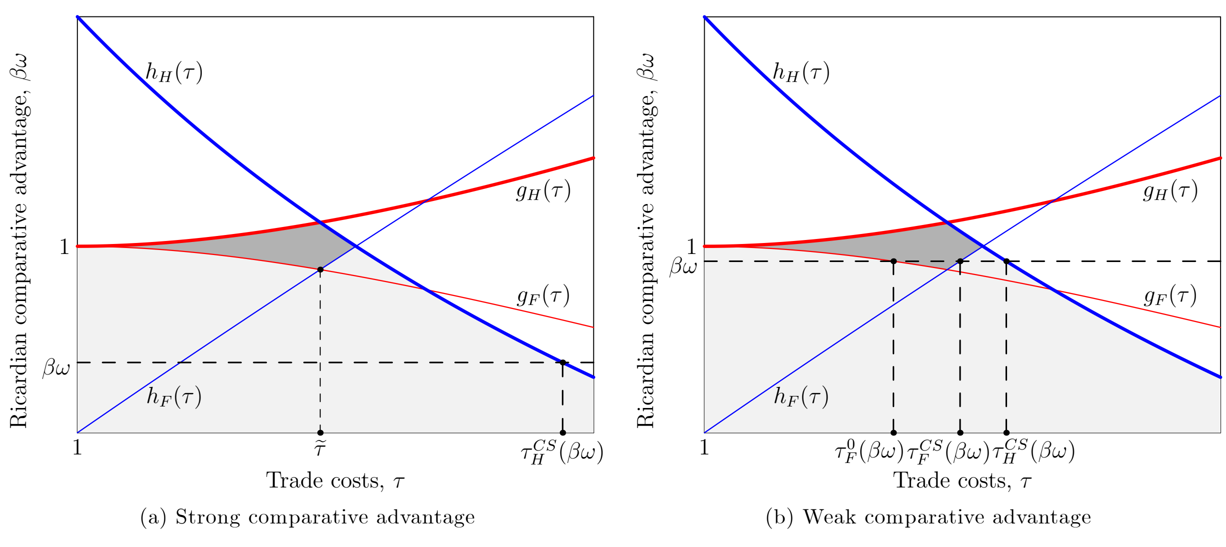

Figure 1: Complete specialization equilibria with low trade costs, $\phi = 0.3$

Equilibrium with no trade Let's now consider the conditions for there to be an equilibrium with no trade. This equilibrium would have the Home price p_H^A determined as in (1), while p_F^A is determined analogously, i.e., $p_F^A = \frac{w_F a_F}{(x_F(p_F^A))^{\phi}}$. The condition necessary for this to be an equilibrium is that neither Home nor Foreign firms find it profitable to sell in both markets. As explained above, Assumption 2 implies that the best deviation would be to charge the highest possible price while serving both markets. Thus, writing x_H^A and

x_F^A as shorthand for $x_H(p_H^A)$ and $x_F(p_F^A)$, respectively, the condition that Home firms do not make profits from this deviation is

$$\left[\frac{w_H a_H}{\left(x_H^A\right)^\phi} - \frac{w_H a_H}{\left(x_H^A + x_F^A\right)^\phi}\right] x_H^A + \left[\frac{w_F a_F}{\left(x_F^A\right)^\phi} - \frac{w_H a_H \tau}{\left(x_H^A + x_F^A\right)^\phi}\right] x_F^A \leq 0. \tag{14}$$

The first term on the LHS of (14) is the profit that a Home firm selling in both markets could attain by undercutting slightly the local price in a potential equilibrium with no trade, while the second term represents the loss the firm incurrs by selling in the foreign country in spite of the high trade cost. If (14) is satisfied, an equilibrium with no trade would be immune to a deviation by a Home firm targeting both markets. Using Assumption 3 this can be rewritten as

$$\tau \geq \frac{2\left(1+(\beta\omega)^{1/(1-\phi)}\right)^\phi - 1}{(\beta\omega)^{1/(1-\phi)}} \equiv \tau_H^{NT}(\beta\omega).$$

Similarly, the condition necessary for Foreign firms not to make profits from a deviation to sell in both markets is given by

$$\tau \geq \frac{2\left(1+(\beta\omega)^{-1/(1-\phi)}\right)^\phi - 1}{(\beta\omega)^{-1/(1-\phi)}} \equiv \tau_F^{NT}(\beta\omega).$$

It is easy to show that $\beta\omega < 1$ implies $\tau_H^{NT}(\beta\omega) > \tau_F^{NT}(\beta\omega)$, and hence both conditions for non-tradability are satisfied if and only if $\tau \geq \tau_H^{NT}(\beta\omega)$. This establishes the following result:

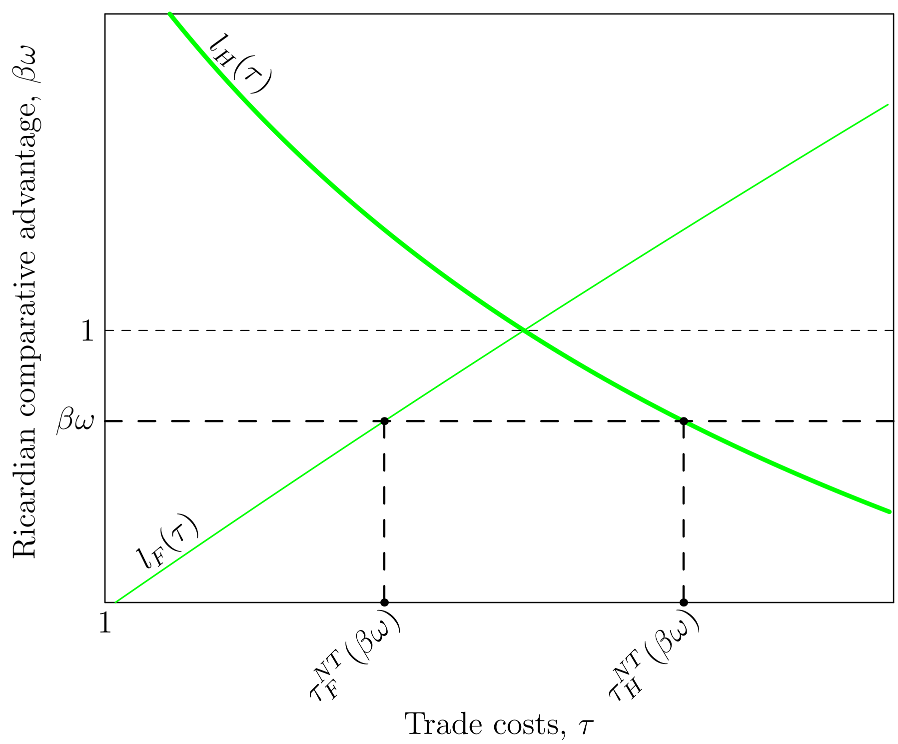

Figure 2: Equilibrium with no trade, $\phi = 0.3$

Proposition 2 *Assume that $\beta\omega < 1$. An equilibrium with no trade exists if and only if*

$$\tau \geq \tau_H^{NT}(\beta\omega). \tag{15}$$

If $\phi = 0$, so that production exhibits constant returns to scale, then $\tau_H^{NT}(\beta\omega) = 1/\beta\omega$, implying that there is no trade if and only if $\tau > 1/\beta\omega$ or equivalently $w_H a_H \tau > w_F a_F$, which is the standard result in the Ricardian model of trade. As ϕ increases from zero, it can be verified that $\tau_H^{NT}(\beta\omega)$ increases, implying that a higher trade cost is needed for no trade to be an equilibrium. It is easier to see this for the case of no comparative advantage, i.e., $\beta\omega = 1$. In this case, with $\phi = 0$ there would be no trade as long as $\tau > 1$. In contrast, with $\phi > 0$, a no-trade equilibrium requires a non-negligible trade cost – in particular, it requires $\tau > \tau_H^{NT}(1) = 2^{1+\phi} - 1$. Note that Assumption 1 implies that $2^{1+\phi} - 1 > 1$.

Since $\tau_H^{NT}(.)$ and $\tau_F^{NT}(.)$ are monotonic, their inverse is well defined. Letting $l_H(\tau) \equiv \left(\tau_H^{NT}(\tau)\right)^{-1}$ and $l_F(\tau) \equiv \left(\tau_F^{NT}(\tau)\right)^{-1}$, the conditions $\tau \geq \tau_H^{NT}(\beta\omega)$ and $\tau \geq \tau_F^{NT}(\beta\omega)$ are equivalent to $\beta\omega \geq l_H(\tau)$ and $\beta\omega \geq l_F(\tau)$. Figure 2 illustrates.

Equilibrium with mixed strategies GRH argue that for intermediate trade costs there is no equilibrium in pure strategies. Our analysis confirms that this is indeed the case. The curve $l_H(\tau)$ is decreasing and intersects the horizontal line with $\beta\omega = 1$ at point $\tau_H^{NT}(1)$. It is readily verified that $\tau_H^{CS}(1) < \tau_H^{NT}(1)$. Moreover, as shown in the Appendix, the curve $h_H(\tau)$ is always below the curve $l_H(\tau)$, so $\tau_H^{CS}(\beta\omega) < \tau_H^{NT}(\beta\omega)$. This implies that, given $\beta\omega \leq 1$, there is no pure strategy equilibrium for $\tau \in]\tau_H^{CS}(\beta\omega), \tau_H^{NT}(\beta\omega)[$. In other words, condition (4) is satisfied but conditions (6) and (14) are not – the violation of (6) implies that complete specialization in Home is not an equilibrium because Foreign firms would deviate to displace Home firms from their local market, and the violation of (14) implies that no trade is not an equilibrium because Home firms would deviate and seize both markets.

GRH argue that for intermediate trade costs there exists an equilibrium in which Home firms randomize between a strategy that leads to only sales in Home (the local strategy) and a strategy that ensures sales in both markets (the global strategy). The challenge in constructing such an equilibrium is that Home sales entail a profit while sales in Foreign entail a loss, so Home firms would be tempted to shave the Home price and charge a high price in Foreign, in that way capturing all the profits associated with local sales and avoiding the losses in the Foreign market. In fact, the equilibrium proposed by GRH can be shown to allow for a profitable deviation where a Home firm slightly shaves the Home price in the global strategy thereby appropriating all the profits in Home and making positive expected profits (see the Appendix for the formal argument).

We now propose an alternative mixed strategy equilibrium that holds when Home has a "superior comparative advantage," where we use "superior" rather than "strong" (used before) because the two concepts are different. We say that Home has a superior comparative advantage if $\beta\omega < l_F(\hat{\tau})$, where $\hat{\tau}$ is defined implicitly by $h_H(\hat{\tau}) = l_F(\hat{\tau})$ (see Figure 3).

Assume again that (4) is satisfied, whereas (6) and (14) are both violated. Let $\Phi_H(p)$ and $\Phi_F(p)$ be the profits made in Home and in Foreign by a Home firm that captures both markets selling at prices p_H in Home and p_F^A in Foreign, i.e.,

$$\Phi_H(p_H) \equiv \left[p_H - \frac{w_H a_H}{(x_H(p_H) + x_F(p_F^A))^{\phi}}\right] x_H(p_H)$$

and

$$\Phi_F(p_H) \equiv \left[p_F^A - \frac{w_H a_H \tau}{\left(x_H(p_H) + x_F(p_F^A)\right)^{\phi}}\right] x_F(p_F^A).$$

For this case, we propose the following equilibrium. Foreign firms price so as to compete only for their domestic market – in particular, they set a prohibitively high price for exports and a local price of p_F^A. Home firms pursue a mixed strategy: with probability q they charge a prohibitively high price for sales in Foreign and a local price of p_H^A and with probability $1-q$, they contest both markets by shaving price p_F^A to capture the Foreign market, while setting a domestic price p_H that is drawn from the distribution

$$F(p_H) = \frac{1}{M(p_H)} \int_s^{p_H} \zeta(y) M(y) dy \Big/ \int_s^{p_H^A} \zeta(y) M(y) dy + \frac{M(p_H) - 1}{M(p_H)}, \tag{16}$$

with support $p_H \in [s, p_H^A]$, where

$$\zeta(y) \equiv \frac{\Phi'_H(y) + \Phi'_F(y)}{\Phi_H(y)},$$

and

$$M(y) \equiv \exp\left(\int_s^y \frac{\Phi'_H(t) + \Phi'_F(t)/2}{\Phi_H(t)} dt\right).$$

It is easy to verify that $F(s) = 0$, $F(p_H^A) = 1$ and $F'(p_H) > 0$. The mixing probability q is given by,

$$q_H = \left(1 + \int_s^{p_H^A} \zeta(y) M(y) dy\right)^{-1}. \tag{17}$$

Finally, s is determined implicitly by (17) and

$$\Phi_H(s) + \left(q_H + \frac{1 - q_H}{2}\right) \Phi_F(s) = 0 \tag{18}$$

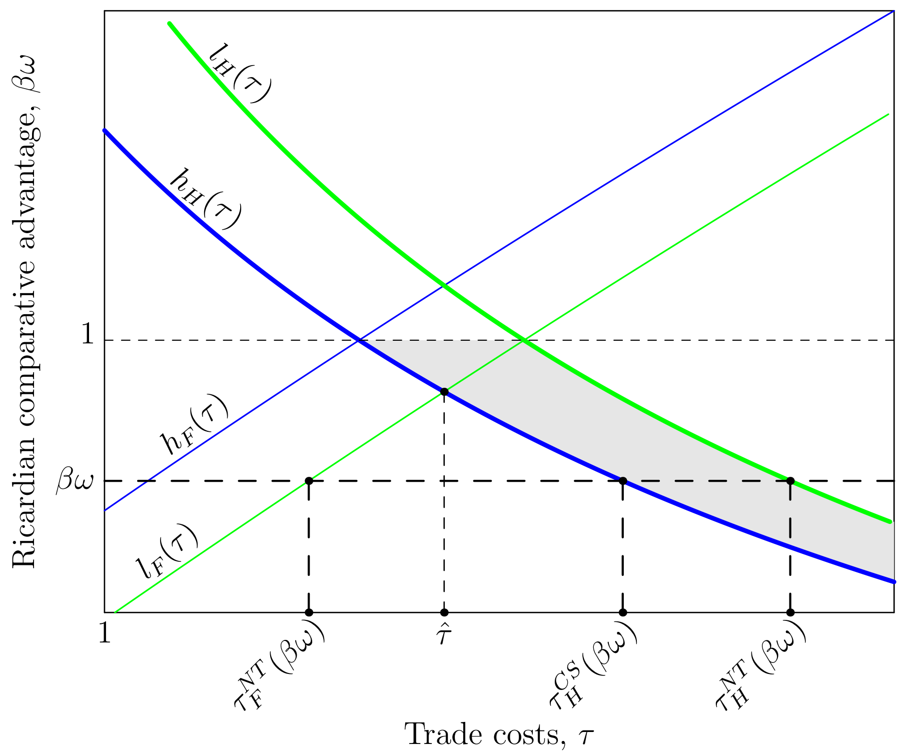

Figure 3: Equilibrium with mixed stategies, $\phi = 0.3$

Formally,

Proposition 3 *Assume that Home has a superior comparative advantage, i.e., $\beta\omega < l_F(\hat{\tau})$, where $\hat{\tau}$ is defined implicitly by $h_H(\hat{\tau}) = l_F(\hat{\tau})$. For $\tau \in]\tau_H^{CS}(\beta\omega), \tau_H^{NT}(\beta\omega)[$ the equilibrium entails Foreign firms charging p_F^A in Foreign and making no sales in Home, and Home firms following a mixed strategy where with probability q_H they follow the "local strategy" according to which they charge p_H^A in Home and make no sales in Foreign and with probability $1 - q_H$ they follow the "global strategy" according to which they shave p_F^A in Foreign and charge a price $p_H \in [s, p_H^A]$ in Home according to the distribution $F(p_H)$ in (16), with q_H and s satisfying (17) and (18).*

Proof. We begin by deriving $F(p_H)$. Home firms earn zero profits when they pursue their local strategy in all states of nature. Thus, a Home firm pursuing the global strategy should also expect zero profits. Moreover, for a Home firm to be willing to set prices p_H according to $F(p_H)$, the expected profits for any $p_H \in \left[s, p_H^A\right]$ should also be zero. To derive this expected profit given p_H in the global strategy, suppose first that the other Home firm pursues its local strategy. The profits are then $\Phi_H(p_H) + \Phi_F(p_H)$. If the other firm pursues its global strategy, expected profits associated with a Home price of p_H are

$$\left[\Phi_H(p_H) + \frac{\Phi_F(p_H)}{2}\right](1 - F(p_H)) + \int_s^{p_H} \frac{\Phi_F(y)}{2} dF(y).$$

Thus, expected profits for a Home firm setting prices p_H and p_H^A when the other Home firm pursues the proposed mixed strategy are

$$\begin{aligned} \Pi\left(p_H\right) \quad \equiv \quad & q_H\left(\Phi_H(p_H) + \Phi_F(p_H)\right) \\ & + (1 - q_H)\left\{\left[\Phi_H(p_H) + \frac{\Phi_F(p_H)}{2}\right](1 - F(p_H)) + \int_s^{p_H} \frac{\Phi_F(y)}{2} dF(y)\right\} \end{aligned}$$

Our mixed strategy requires $\Pi(p_H) = 0$ for all $p_H \in [s, p_H^A]$. Differentiating $\Pi\left(p_H\right)$ with respect to p_H, setting $\Pi'(p_H) = 0$ and solving for $F'(p_H)$ yields

$$\left(1 - q_H\right) F'(p_H) = q_H \frac{\Phi_H'(p_H) + \Phi_F'(p)}{\Phi_H(p)} + \left(1 - q_H\right)\left[\frac{\Phi_H'(p_H) + \Phi_F'(p_H)/2}{\Phi_H(p_H)}\right]\left(1 - F(p_H)\right).$$

The solution to this differential equation is

$$F(p_H) = \frac{q_H}{1 - q_H} \int_s^{p_H} \zeta(y) \frac{M(y)}{M(p_H)} dy + 1 - \frac{M(s)}{M(p_H)}. \tag{19}$$

Noting that $M(s) = 1$, setting $F(p_H^A) = 1$ and solving for q_H yields (17). Plugging this back into (19) yields (16). Finally, we also need that $\Pi(s) = 0$. This implies that

$$\Phi_H(s) + \left(q_H + \frac{1 - q_H}{2}\right)\Phi_F(s) = 0.$$

This equation together with (17) can then be solved to yield the equilibrium value of s.

We need to study all possible deviations by Home and Foreign firms. A Foreign firm could deviate by going global, shaving prices p_H^A and p_F^A. If both Home firms pursue their local strategy, which happens with probability q_H^2 , the Foreign firm would capture both markets and make profits of

$$\Upsilon \equiv \left[p_H^A - \frac{w_F a_F \tau}{\left(x_H(p_H^A) + x_F(p_F^A)\right)^\phi}\right] x_H(p_H^A) + \left[p_F^A - \frac{w_F a_F}{\left(x_H(p_H^A) + x_F(p_F^A)\right)^\phi}\right] x_F(p_F^A).$$

Otherwise, the Foreign firm would simply sell in the local market and make zero profits. So we need to establish that $\Upsilon < 0$. One can readily verify that there exists a unique $\hat{\tau}$ such that for $\beta\omega < l_F(\hat{\tau})$, we have $\tau_F^{NT}(\beta\omega) < \tau_H^{CS}$ (Figure 3 illustrates this). Since our mixed strategy applies for $\tau \in]\tau_H^{CS}(\beta\omega), \tau_H^{NT}(\beta\omega)[$, then we have $\tau > \tau_F^{NT}(\beta\omega)$ implies $\Upsilon < 0$.

To describe the possible deviations by Home firms, we use notation $p_F \lessapprox p_F^A$ to mean a firm shaves p_F^A (since a Home firm always makes losses in the Foreign market, firms will never want to charge a price lower than they need to capture this market). There are four possible types of pricing strategies by Home firms: (i) $p_H > p_H^A$ and $p_F > p_F^A$ (no entry- yields zero profits); (ii) $p_H \leq p_H^A$ and $p_F \lessapprox p_F^A$ (competing for the global market); (iii) $p_H > p_H^A$ and $p_F \lessapprox p_F^A$ (competing for foreign market only); and (iv) $p_H \leq p_H^A$ and $p_F > p_F^A$ (competing for domestic market only). But pricing strategy (i) strictly dominates pricing strategy (iii) since Home firms make losses on export sales. This implies that we can rule out strategy (iii). The conjectured equilibium above essentially considers mixing across a version of (iv) with $p_H = p_H^A$ and $p_F > p_F^A$, and (ii) as well as mixing within strategy (ii). Moreover, Home firms are indifferent between strategy $p_H = p_H^A$ and $p_F > p_F^A$ and (i) since in both strategies yield zero expected profits. Hence, our final step entails explicitly ruling out the version of strategy (iv) with $p_H < p_H^A$ and $p_F > p_F^A$ as a possible deviation.

First, note that if $p_H < s$, the expected profits are

$$q_H \left[p_H - \frac{w_H a_H}{(x_H(p_H))^{\phi}} \right] x_H(p_H) + (1 - q_H)\,\Phi_H(p_H).$$

We need this expression to be non-positive. But since this is increasing in p_H (recall that $p_H x_H(p_H) = 1$ and that $x_H(p_H)/(x_H(p_H))^{\phi}$ is decreasing by the assumption that $\phi < 1/2$), it is enough to check that the expected profits of this type of deviation are non-positive for $p_H \geq s$. For this case, the expected profits are

$$\widetilde{\Pi}(p_H) \equiv q_H \Gamma_H(p_H) + (1 - q_H)(1 - F(p_H))\,\Phi_H(p_H),$$

where $\Gamma_H(p_H) \equiv \left[p_H - \frac{w_H a_H}{(x_H(p_H))^{\phi}} \right] x_H(p_H)$. Since $\Gamma_H(p_H^A) = 0$ and $F(p_H^A) = 1$ then $\widetilde{\Pi}\left(p_H^A\right) = 0$. We now show that $\widetilde{\Pi}'(p_H) \geq 0$, implying that $\widetilde{\Pi}(p_H) \leq 0$ for all p_H. First, $\Pi'(p_H) = 0$ implies

$$(1 - q_H)\,\Phi_H(p_H) F'(p_H) = q_H\left[\Phi_H'(p_H) + \Phi_F'(p_H)\right] + (1 - q_H)\left[\Phi_H'(p_H) + \Phi_F'(p_H)/2\right](1 - F(p_H)).$$

Second,

$$\widetilde{\Pi}'(p_H) = q_H \Gamma_H'(p_H) + (1 - q_H)\,\Phi_H'(p_H)(1 - F(p_H)) - (1 - q_H) F'(p_H)\Phi_H(p_H).$$

Combining these two expressions yields

$$\widetilde{\Pi}'(p_H) = q_H\left[\Gamma_H'(p_H) - \Phi_H'(p_H) - \Phi_F'(p_H)\right] - (1 - q_H)\left(\Phi_F'(p_H)/2\right)(1 - F(p_H)).$$

One can easily verify that $\Phi_H'(p_H) > 0$, $\Phi_F'(p_H) < 0$ and $\Gamma_H'(p_H) > 0$. Hence, a sufficient condition for $\widetilde{\Pi}'(p_H) \geq 0$ is that

$$\Gamma_H'(p_H) - \Phi_H'(p_H) - \Phi_F'(p_H) \geq 0.$$

Simple differentiation reveals that $\Gamma_H'(p_H) - \Phi_H'(p_H) - \Phi_F'(p) \geq 0$ if and only if

$$(1 - \phi)\left[\left(1 + p_H/p_F^A\right)^{1+\phi} - 1 \right] \geq (1 - \phi\tau)\, p_H/p_F^A.$$

But $\tau > 1$ and $\phi < 1/\tau$ so $0 < \phi < \phi\tau < 1$, hence the previous inequality is satisfied under our assumptions. We conclude that $\widetilde{\Pi}(p_H) \leq 0$ for any p_H.[10] ■

We have not been able to find the equilibrium for the case with $\tau \in]\tau_H^{CS}(\beta\omega), \tau_H^{NT}(\beta\omega)[$ and in which Home does not have a superior comparative advantage. The problem is that the strategies presented in Proposition 3 are not an equilibrium because Foreign firms would profit by a global deviation. Our conjecture is that for this case, we require mixing by firms within both countries. This is left for future research.

2.3 General Equilibrium

In this section, we consider the entire set of industries and characterize, formally, a set of complete specialization equilibria for symmetric countries when trade costs are low. The results imply that the multiplicity applies to a set of "disputed" industries (industries for which both countries have a relatively weak comparative advantage), and hints at a possible role for industrial policy. We now proceed to characterize the full general equilibrium. For any industry v, define $z_i \equiv 1/a_i^{\alpha}$ where $\alpha \equiv 1/(1 - \phi)$ and z_i is the productivity

[10] What happens if the condition for the good to be non-traded is almost satisfied, i.e., $\tau \lesssim \tau_H^{NT}(\beta\omega)$ so that $\Phi_H(p_H^A) + \Phi_F(p_H^A) \lesssim 0$? This implies that s and q_H satisfy $s \lesssim p_H^A$ and $q_H \lesssim 1$. So the equilibrium transitions smoothly from the mixed strategy equilibrium in Proposition 3 to the pure strategy equilibrium with no trade in Proposition 2. What happens if the condition for complete specialization in Home to be an equilibrium is almost satisfied, i.e., $\tau \gtrsim \tau_H^{CS}(\beta\omega)$ so that $p_F^A = \tau p_H^0$ where p_H^0 solves $\Phi(p_H^0) = 0$? Our conjecture is that s and q satisfy $s \gtrsim p_0$ and $q \gtrsim 0$ because $\zeta(y) \approx \infty$ for $y \gtrsim p_0$.

of firms in country i.[11] Note that $\alpha \geq 1$ ($\phi \geq 0$) with $\alpha = 1$ ($\phi = 0$) resulting in the special case of constant returns to scale. For convenience we use α instead of ϕ in what follows. Let T_i represent country i's state of technology. For expositional simplicity, we make the following additional assumptions which we use throughout this section.

Assumption 3': *Countries are symmetric:* $T_H = T_F$ and $L_H = L_F$. *Without loss of generality we set* T_H, T_F, L_H *and* L_F *to one.*

Notice that symmetry here implies $L_H = L_F$, and is distinct from our assumption of symmetric demand, $D_H = D_F$, in our partial analysis. For our general equilibrium analysis, we essentially dispense with Assumption 3 (demand is symmetric) in favor of Assumption 3'. We do this, for the most part, because in general equilibrium, we allow relative wages to be endogeneously determined. We also assume a country specific parametric distributional structure for industry productivity similar to that used in EK. In particular,

Assumption 5: *In any country* i, *the productivity for each industry is independently drawn from a country specific Frechet distribution*

$$F_i(z) = e^{-T_i z^{-\theta}}. \tag{20}$$

As in EK, T_i represents country i's absolute advantage across the continuum of industries, whereas θ determines comparative advantage within the continuum. While in principle one does not need a probabilistic framework for the theoretical analysis on two symmetric countries, we present it here for convenience since it becomes relevant to our quantitative exercises later on.

2.3.1 Complete Specialization Equilibria

Define $\beta(v) \equiv a_H(v)/a_F(v)$. Order industries such that $\beta(v)$ is continuous and strictly increasing in v. Importantly, note that the analog of $g_H(.)$, $g_F(.)$, $h_H(.)$ and $h_F(.)$ from our partial analysis are: $g_H(\omega,\tau) \equiv \left[\frac{\tau+\omega^{-1}/\tau}{1+\omega^{-1}}\right]$; $g_F(\omega,\tau) \equiv \left[\frac{1+\omega^{-1}}{1/\tau+\omega^{-1}\tau}\right]$; $h_H(\omega,\tau) \equiv \frac{1}{\tau}\left[\frac{\tau+\omega^{-1}}{\omega^{-1}}\right]^{\phi}$; and $h_F(\omega,\tau) \equiv \tau\left[\frac{1}{1+\omega^{-1}\tau}\right]^{\phi}$. Note also that our partial equilibrium analysis focuses on a particular industry v. The difference here is that we consider the entire set of industries. Hence, our conditions for complete specialization by the Home and Foreign country modified by the appropriate $g_i(.)$ and $h_i(.)$ functions (along with the fact that β is a function of v) still apply here, namely, (10) and (11) for the Home country, and (12) and (13) for the Foreign one.

In Proposition 1, we showed that there is multiple complete specialization equilibria when trade costs are low and comparative advantage is weak. Here we characterize the set of "disputed" industries for which multiple complete specialization equilibria applies. In order to give a rough sketch of the idea behind the multiplicity of complete specialization equilibria when considering all industries, imagine for a moment relative wages are fixed. Also, let v_H^g and v_F^g solve (10) and (12) when it holds with equality. Importantly, we argue that for trade costs low enough, (11) and (13) are also satisfied for v_H^g and v_F^g respectively. Moreover, $v_H^g > v_F^g$ along with the fact that $\beta(v)$ increasing implies an overlapping range of industries for which the

[11] To see why we define $z_i \equiv 1/a_i^{\alpha} \Leftrightarrow a_i \equiv 1/z_i^{1/\alpha}$, first recall there are $n_i \geq 2$ producers/firms in each industry and country. Firms (indexed by m) in a particular industry v and country, say Home, have access to identical production technology

$$X_H^m = \left[(z_H)^{1/\alpha}(X_H)^{(\alpha-1)/\alpha}\right] L_H^m$$

where $X_H = \sum_m X_H^m$ and $L_H = \sum_m L_H^m$. Assuming firms take wages as given, unit cost is given by

$$\frac{w_H}{(z_H)^{1/\alpha}(X_H)^{(\alpha-1)/\alpha}}$$

But $a_H \equiv 1/z_H^{1/\alpha}$ and $\phi \equiv (\alpha-1)/\alpha$, so we have

$$\frac{w_H a_H}{(X_H)^{\phi}}.$$

In particular, production exhibits national increasing returns to scale at the local industry level, and is given by

$$X_H = z_H L_H^{\alpha}.$$

conditions for complete specialization in each country are simultaneously satisfied (we establish later that this is in fact the case). Before proceeding further, we first define the concept of a complete specialization allocation.

Definition 1 *A complete specialization allocation (CSA), $\widetilde{v}$, is an allocation where all goods $v \leq \widetilde{v}$ are produced only by the Home country and all goods $v > \widetilde{v}$ are produced by the Foreign country.*

As such, we consider an allocation $\widetilde{v} \in [v_F^g, v_H^g]$ such that goods $v \leq \widetilde{v}$ production is concentrated in the Home country, and goods $v > \widetilde{v}$ production is concentrated in the Foreign one. In that regard, we consider equlibria in which each country gets a share of the disputed industries (and in the extreme case all or none of the disputed industries) such that increasing $\widetilde{v}$ entails expanding the partitioned set of industries which the Home country produces and vice-versa.[12] Hence, Home country's domestic sales and export sales are $\widetilde{v}w_H L_H$ and $\widetilde{v}w_F L_F$ respectively (note that, in equilibrium, income and expenditure in the Home and Foreign country are $w_H L_H$ and $w_F L_F$ respectively- we formally define a complete specialization equilibrium in what follows). Full employment in Home requires

$$w_H L_H = \widetilde{v} w_H L_H + \widetilde{v} w_F L_F. \tag{21}$$

Assumption 4, so that $L_H = L_F = 1$ implies

$$\omega = \omega(\widetilde{v}) \equiv \frac{\widetilde{v}}{1-\widetilde{v}}. \tag{22}$$

Using assumption 4 ($T_H = T_F = 1$), assumption 5 (the productivity distribution is Fretchet), and bearing the ordering of goods in mind, one can derive

$$\beta\left(v\right) = \left(\frac{v}{1-v}\right)^{1/\alpha\theta}. \tag{23}$$

Clearly $\beta\left(.\right)$ is strictly increasing in v. We now define formally a complete specialization equilibrium.

Definition 2 *A complete specialization equilibrium (CSE) is a CSA which satisfies the appropriately modified versions of* (10)-(13) *for all goods i.e. a CSA in which firms in neither country have an incentive to deviate and take over their home markets or both markets; and labor markets clear so that $\omega = \omega(\widetilde{v})$.*

Let $v_H^g(\tau)$ be the solution to

$$\beta\left(v\right)\omega\left(v\right) = g_H\left(\omega\left(v\right), \tau\right) \tag{24}$$

and $v_H^h(\tau)$ be the solution to

$$\beta\left(v\right)\omega\left(v\right) = h_H\left(\omega\left(v\right), \tau\right) \tag{25}$$

Similarly, let $v_F^g(\tau)$ be the solution to

$$\beta\left(v\right)\omega\left(v\right) = g_F\left(\omega\left(v\right), \tau\right) \tag{26}$$

and $v_F^h(\tau)$ be the solution to

$$\beta\left(v\right)\omega\left(v\right) = h_F\left(\omega\left(v\right), \tau\right) \tag{27}$$

Importantly, note that symmetry implies $v_H^g(\tau) = v(\tau)$ and $v_F^g(\tau) = 1 - v(\tau)$, and $v_H^h(\tau) = v(\tau)$ and $v_F^h(\tau) = 1 - v(\tau)$. We exploit this property throughout the proof of Proposition 4 in the appendix.

Proposition 4 *There exists a unique $\widetilde{\tau}$ that satisfies $v_H^g\left(\widetilde{\tau}\right) = v_H^h\left(\widetilde{\tau}\right)$ (and also $v_F^g\left(\widetilde{\tau}\right) = v_F^h\left(\widetilde{\tau}\right)$) and there exists a unique $\widehat{\tau}$ satisfying $v_H^h\left(\tau\right) = v_F^h(\tau) = 1/2$. For $\tau \leq \widetilde{\tau}$, any CSA $\widetilde{v} \in [v_F^g(\tau), v_H^g(\tau)]$ is a CSE, and for $\tau \in (\widetilde{\tau}, \widehat{\tau}]$, any CSA $\widetilde{v} \in \left[v_F^h\left(\tau\right), v_H^h\left(\tau\right)\right]$ is a CSE.*

[12] Since the industries are ordered with decreasing comparative advantage for Home country, any other non-partitioned assignment is strictly dominated in terms of efficiency. Hence, we confine ourselves to connected sets so that complete specialization equilibria can be characterized by a single $\widetilde{v}$.

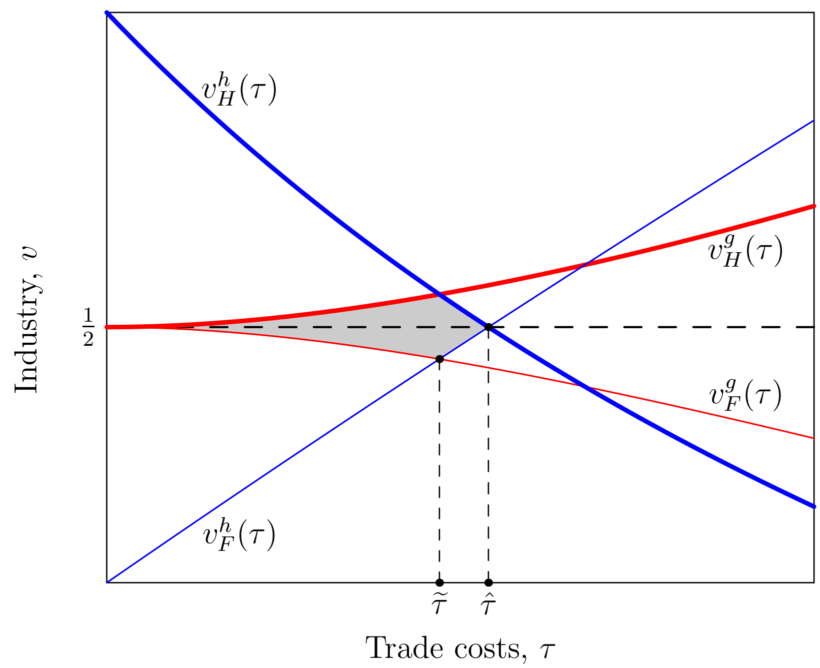

Figure 4: Set of complete specialization equilibria with low trade costs

A interesting implication of Proposition 4 is that as we initially increase trade costs, the set of disputed industries for which multiple complete specialization equilibria applies expands. Beyond $\widetilde{\tau}$, the set shrinks. Figure 4 illustrates. More importantly, multiple equilibria with respect to this disputed set suggests a potential role for industrial policy. We explore this further in a more general setting in what follows, fixing our attention on complete specialization equilibria for $\tau = \widetilde{\tau}$. As illustrated in Figure 4, the largest possible set of disputed industries occurs at $\widetilde{\tau}$, and since our objective is to guage the maximum possible role for industrial policy, our quantitative exercise hones in on the case $\tau = \widetilde{\tau}$. Given this, it seems sufficient to restrict our analysis to the case $\tau \leq \widetilde{\tau}$ in which the global-deviation conditions are the binding constraints.

In the section on welfare that immediately follows we explore this potential for industrial policy by considering the welfare implications of two extreme equilibria: one in which all the disputed industries are produced by the Home country, and the other in which the converse occurs. More importantly, we also explore quantitatively how important external economies of scale are for the overall gains from trade.

3 Welfare

Recall from the previous subsection that for $\tau \leq \widetilde{\tau}$, satisfying the global-deviation conditions for both Home and Foreign implied the respective local-deviation conditions were also satisfied. From Proposition 4 we know that the set of equilibria is given by $\widetilde{v} \in [v_F^g(\tau), v_H^g(\tau)]$. For the subsection on industrial policy we restrict the analysis to the two extreme equilibria in this set, namely $\widetilde{v} = v_F^g(\tau)$ and $\widetilde{v} = v_H^g(\tau)$. We refer to the equilibrium with $\widetilde{v} = v_i^g(\tau)$ as the i equilibrium. Here we relax the assumptions that the level of technology, T_i, and the size of the labor force, L_i, are the same across countries. In our quantitative exercise we check to verify that the local-deviation conditions are satisfied.

Let π_{HH} and π_{FF} be the the share of expenditure devoted to local production in Home and Foreign, respectively. Our assumption on preferences implies $\pi_{HH} = \widetilde{v}$ and $\pi_{FF} = 1 - \widetilde{v}$. The following proposition outlines key objects of our welfare analysis, namely, real wages in each country. Let P_i be the appropriate price index in country i.

Proposition 5 *Real wage in country i is given by*

$$\frac{w_i}{P_i} = \left(T_i/\eta^{\theta}\right)^{1/\theta} L_i^{\alpha-1} \pi_{ii}^{-1/\theta} \left(\frac{\Upsilon_i}{w_i L_i}\right)^{\alpha-1} \tag{28}$$

where

$$\Upsilon_H \equiv \left(w_H L_H + w_F L_F/\tau\right)^{\pi_{HH}} \left(\left(\frac{\left(\beta\left(\widetilde{v}\right)\omega\left(\widetilde{v}\right)\right)^{\alpha}}{\tau}\right)^{1/(\alpha-1)} \left(w_H L_H/\tau + w_F L_F\right)\right)^{1-\pi_{HH}} \tag{29}$$

and

$$\Upsilon_F \equiv \left(w_H L_H/\tau + w_{FF} L_F\right)^{\pi_{FF}} \left(\left(\frac{1}{\tau\left(\beta\left(\widetilde{v}\right)\omega\left(\widetilde{v}\right)\right)^{\alpha}}\right)^{1/(\alpha-1)} \left(w_H L_H + w_F L_F/\tau\right)\right)^{1-\pi_{FF}}. \tag{30}$$

The first two terms in (28) capture the effect of technology and size on real wages, respectively. The third term is the gains from trade through comparative advantage, and as we explain below, the last term captures the gains from trade arising through economies of scale.

Let w_i^A/P_i^A be the autarky real wage in country i.

Corollary 1 *The gains from trade in country i are*

$$\frac{w_i/P_i}{w_i^A/P_i^A} = \pi_{ii}^{-1/\theta} \left(\frac{\Upsilon_i}{w_i L_i}\right)^{\alpha-1} \tag{31}$$

where Υ_i is given by (29) *and* (30) *for $i = H$ and F, respectively.*

The expression in (31) highlights the two channels for gains from trade in our model: the gains from comparative advantage ($\pi_{ii}^{-1/\theta}$), and the gains from economies of scale ($(\Upsilon_i/w_i L_i)^{\alpha-1}$). The first term requires no explanation (see EK and Arkolakis et al (2010)). The second term represents the additional gains through economies of scale associated with concentrating global production in a single location. In turn, this term can be decomposed into two parts. Focusing on Home, the first part, $\left(\frac{w_H L_H + w_F L_F/\tau}{w_H L_H}\right)^{(\alpha-1)\pi_{HH}}$, captures the gains from economies of scale associated with the expansion of industries $\frac{w_H L_H + w_F L_F/\tau}{w_H L_H}$ at Home for $v \leq \widetilde{v}$, while the second part, $\left[\left(\frac{(\beta(\widetilde{v})\omega(\widetilde{v}))^{\alpha}}{\tau}\right)\left(\frac{w_H L_H/\tau + L_F}{w_H L_H}\right)^{\alpha-1}\right]^{1-\pi_{HH}}$, captures those gains associated with larger scale of industries at Foreign relative to Home in autarky by $\frac{w_H L_H/\tau + w_F L_F}{w_H L_H}$ for $v \geq \widetilde{v}$, with the adjustment term $\left(\frac{(\beta(\widetilde{v})\omega(\widetilde{v}))^{\alpha}}{\tau}\right)$ arising because of the difference in unit costs at the cut-off $v = \widetilde{v}$. We exploit these expressions in our welare analyses on industrial policy and gains from trade below.

3.1 Industrial Policy

We think of industrial policy as that which moves the economy to a superior equilibrium. In particular, industrial policy in Home would be aimed at switching from the F to the H equilibrium, and the opposite would be the case for industrial policy in Foreign. The question we are interested in here is the following: How large is the increase in the real wage for Home (Foreign) associated with a switch from the F (H) to the H (F) equilibrium?[13]

We now present a quantitative exercise to shed some light on this question. We need to set values for parameters governing the strength of Marshallian externalities (α) and the strength of comparative advantage (θ). For the external economies of scale parameter, we use implied estimates from three independent studies:

[13] Without loss of generality, consider the Home country. Let w_H^i/P_H^i be the real wage in Home country for equilibrium $i = H, F$. Then the potential welfare gains for Home from producing all the disputed industries relative to producing none is simply

$$\frac{w_H^H/P_H^H - w_H^F/P_H^F}{w_H^F/P_H^F} \times 100.$$

Antweiler and Trefler (2002) general equilibrium approach using data on 71 countries ($\alpha = 1.054$); Fuss and Gupta (1981) analysis using Canadian data ($\alpha \equiv 1/(1-\phi) = 1.15$); and Paul and Siegel (1999) partial equlibrium approach using industry level US manufacturing data ($\alpha \equiv 1/(1-\phi) = 1.3$). For the comparative advantage parameter we use three estimates for θ coming from EK, namely 3.6, 8.28 and 12.86.

The results are reported in the tables below. Not surprisingly, the set of disputed industries increases with trade costs, and the trade cost associated with the largest set of disputed industries increases with the scale parameter (recall that we restrict our attention to $\tau = \widetilde{\tau}$, refer to Figure 4). In all cases, we compute the trade cost associated with the largest set of disputed industries, $\widetilde{\tau}$, and analyze the welfare implications of the two extreme equilibria. For symmetric countries, we need only examine the welfare implications of giving all the disputed industries to the Home country. In all other cases we use H and F to indicate whether we we are considering the equilibrium in which all the disputed goods production go to either Home or Foreign respectively. Importantly, note that a lower θ implies greater variability of productvity across the entire set of industries, and thus stronger forces of comparative advantage. In contrast a higher α implies stronger external economies of scale.

Table 1 Gains from Disputed Industries (Symmetric Countries)

α		1.05			1.15			1.30	
θ	3.60	8.28	12.86	3.60	8.28	12.86	3.60	8.28	12.86
$\widetilde{\tau}$		1.04			1.10			1.19	
Scope Indust. Pol.	**0.06%**	**0.07%**	**0.07%**	**0.40%**	**0.45%**	**0.46%**	**1.46%**	**1.64%**	**1.69%**

Table 1 indicates that there are additional gains from trade associated with producing the entire set of disputed industries and as a result gives us a measure of the potential scope for industrial policy. The magnitudes of these additional gains range from a negligble 0.06% to at most about 2%, with the potential importance increasing strongly with the strength of Marshallian externalities (α), and decreasing weakly with the strength of comparative advantage ($1/\theta$). Essentially, a higher α or a higher θ increases the scope for industrial policy by expanding the set of disputed industries. The former does so by expanding the range of low trade costs for which industrial policy applies and as a result raises the low trade cost associated with the largest set of disputed industries ($\widetilde{\tau}$).

Table 2 Home has Superior Technology ($T_H = 2$, $T_F = 1$)

	$\theta = 12.86,\ \alpha = 1.30$	
	H	F
Gains from disputed industries	**1.48%**	**1.55%**
Share of disputed industries	1.36%	1.36%

Might asymmetries alter the basic result above? In Table 2, we explore technology asymmetry by assuming the Home country has on average superior technology, and analyze the welfare implications using only the highest implied external economies of scale and comparative advantage parameter estimates so as to focus on the maximum possible scope for industrial policy. The results indicate that the additional welfare gains for both countries do not diverge much from the case of symmetric countries, that is, additional welfare gains of 1.48% for the Home country and 1.55% for the Foreign one, with the disputed industries accounting for 1.36% of all industries.

Table 3 Home has a Larger Labor Force ($L_H = 2$, $L_F = 1$))

	$\theta = 12.86,\ \alpha = 1.30$	
	H	F
Gains from disputed industries	**0.70%**	**1.39%**
Share of disputed industries	0.66%	0.66%

In Table 3, we explore the possibility when one country has a larger labor force. The additional welfare gains from producing the set of disputed industries are approximately two times higher for the small country, 1.39% versus 0.70% for the large one (here again we use only the highest Marshallian externalities and comparative advantage parameter estimates). However, the maximum potential gains are still within the range implied by our benchmark case of symmetric countries.

While, in principle, there appears to be a potential role for industrial policy that results from the indeterminancy of trade patterns for a set of weak comparative advantage industries in the presence of low trade costs, quantitatively the scope for such a role appears to be modest.

3.2 The Gains from Trade

In this subsection, we ask: how do Marshallian externalities affect the overall gains from trade? We do a decomposition of gains from trade implied by the expression (31) using the parameter estimates from the previous section for the case of symmetric countries with low trade costs. Next, we show that the model readily extends to a multicountry setting when there are no barriers to trade, and yields interesting insights regarding the gains from trade.

3.2.1 Decomposition of the Gains from Trade: A First Look

In the previous subsection, we identified two sources of gains from trade: the gains from comparative advantage ($\pi_{ii}^{-1/\theta}$); and the gains from external economies of scale ($(\Upsilon_i/w_i L_i)^{\alpha-1}$). Also, note that, given trade shares, accounting for Marshallian externalities imply larger gains from trade over and above those of a traditional constant returns framework, which captures only the gains from comparative advantage. A decomposition of these gains for the case of symmetric countries is reported in Table 4 below. The third row reports the overall gains from trade in percentage terms[14], whereas the last two rows report the contribution of comparative advantage and Marshallian externalities to the overall gains from trade for the case of low trade costs, $\tau = \widetilde{\tau}$.[15] We focus on the "natural" equilibrium in which each country produces exactly one half of the entire set disputed industries, that is, $\widetilde{v} = 1/2$.

Table 4 Gains from Trade (Symmetric Countries with Low Trade Costs)

α		1.05			1.15			1.30	
θ	3.60	8.28	12.86	3.60	8.28	12.86	3.60	8.28	12.86
$\widetilde{\tau}$		1.04			1.12			1.18	
Total	22.95%	10.27%	7.03%	26.13%	13.68%	10.52%	31.81%	19.29%	16.12%
Comp. Adv.	93.19%	85.61%	79.29%	82.95%	65.30%	53.91%	69.72%	47.46%	36.06%
Marsh. Ext.	**6.81%**	**14.39%**	**20.71%**	**17.05%**	**34.70%**	**46.09%**	**30.28%**	**52.54%**	**63.94%**

In Table 4 we see that the contribution of external economies of scale to the overall gains from trade ranges from approximately 7% (strongest comparative advantage, weakest Marshallian externalities) to 64% (weakest comparative advantage, strongest Marshallian externalities). Interestingly, our middle range estimates of the two key parameters ($\theta = 8.28$, $\alpha = 1.18$), imply the contribution can be substantial, with Marshallian externalities accounting for roughly 35% of the overall gains from trade of 14%.

3.2.2 Gains from Marshallian Externalities: A Multicountry Framework

In our partial analysis, we have already established that in the absense of trade costs there exists a unique equilibrium in which the patterns of specialization are consistent with comparative advantage. In the Appendix we demonstrate that the case of costless trade can be readily generalized to multiple countries in

[14] In particular, we compute $\frac{w_i/P_i - w_i^A/P_i^A}{w_i^A/P_i^A} \times 100$, where w_i/P_i is the real wage in the equilibrium with trade and w_i^A/P_i^A is the autarky real wage for country i.

[15] We calculate the contribution of Marshalllian externalities to the overall gains from trade by computing $\ln\left[\Upsilon_i/w_i L_i\right]^{(\alpha-1)\theta} / \ln\left[(w_i/P_i) / \left(w_i^A/P_i^A\right)\right]$.

which the gains from trade for any country n can be calculated using a simple formula depending only on the expenditure share on domestically produced goods and the two key parameters governing the strength of comparative advantage and Marshallian externalities.[16] Formally,

Proposition 6 *Under frictionless trade, the gains from trade for any country n are*

$$\frac{w_n/P_n}{w_n^A/P_n^A} = \pi_{nn}^{-1/\theta}\pi_{nn}^{-(\alpha-1)}. \tag{32}$$

Here again the first term captures the gains from comparative advantage, while the second that from Marshallian externalities. Equation (32) has an interesting implication. As is consistent with a the standard EK-type model with no economies of scale, the overall gains from trade depend primarily on a country's expenditure share on its own goods, and as such can vary across countries. However, a simple decomposition of the gains illustrate that, given trade shares, the contribution of each channel to the total gains from trade is constant across countries. Formally,

Corollary 2 *For any country n, comparative advantage and Marshallian externalities account for shares of the overall gains from trade $\frac{1}{1+(\alpha-1)\theta}$ and $\frac{(\alpha-1)\theta}{1+(\alpha-1)\theta}$, respectively.*

Table 5 Gains from Trade (Frictionless Trade)

α	1.05			1.15			1.30		
θ	3.60	8.28	12.86	3.60	8.28	12.86	3.60	8.28	12.86
Comp. Adv.	84.75%	70.72%	60.86%	64.94%	44.60%	34.14%	48.08%	28.70%	20.59%
Marsh. Ext.	**15.25%**	**29.28%**	**39.14%**	**35.06%**	**55.40%**	**65.86%**	**51.92%**	**71.30%**	**79.41%**

Table 5 reports a decomposition of the gains from trade using corollary along with the parameter estimates from the previous subsection. Here the results indicate that Marshallian externalities account for at least about 15% and at most approximately 79% of the total gains from trade. Interestingly, the median parameter estimates suggests a contribution of more than a half of the overall gains from trade.

4 Concluding Remarks

In this paper we provide insights to longstanding questions regarding external economies of scale and its implications for the patterns of international trade, the gains from trade, and a role for industrial policy. Our paper contributes to the literature by revisiting the implications of trade costs in a new game theoretic framework by GRH designed mainly to overturn the indeterminancy of trade patterns associated with the prevalence of multiple equilibria in the early literature.

In the main, we make three points. First, as is consistent with the early literature, we show that trade patterns are indeterminate for a set of "weak" comparative advantage industries in the presence of Marshallian externalities and low trade costs. More importantly, we demonstrate that the multiple equilibria associated with this set of "disputed" industries implies trade patterns need not be consistent with "natural" comparative advantage. Second, we show that the multiple Pareto-rankable equilibria associated with the set of "disputed" industries also provides a motive for industrial policy. We follow up with a quantitative exploration of its potential importance. The quantitative evidence suggests modest welfare gains of at most about 2%. Finally, our framework allows us to ask whether Marshallian externalities lead to additional gains from trade. Our analysis indicates that this is indeed the case. In particular, using the median parameter estimates, our quantitative results imply that Marshallian externalities can account for approximately 35% of the overall gains from trade.

[16] One can verify that this case can also be readily extended to more general CES preferences and its associated demand with constant price elasticity $\sigma > 1$. The only additional restriction required in this case is $\sigma < 1 + \lambda\theta$, where $\lambda \equiv \alpha(1-\sigma) + \sigma$.

A Appendix

A.1 Partial Equilibrium

Profits are increasing in prices

We have already established that $\phi < 1$ is sufficient to imply that profits are increasing in the price for a firm that sells in a single market. Now consider the case of a firm that sells in both markets. A Home firm that sells at prices p_H and p_F in Home and Foreign makes profits of

$$\pi(p_H, p_F) \equiv \left[p_H - \frac{w_H a_H}{A\left(x_H(p_H) + x_F(p_F)\right)}\right] x_H(p_H) + \left[p_F - \frac{w_H a_H \tau}{A\left(x_H(p_H) + x_F(p_F)\right)}\right] x_F(p_F).$$

Simple differentiation reveals that, $\pi_2(p_H, p_F) > 0$ if $\phi < 1$, but $\pi_1(p_H, p_F) > 0$ requires a more stringent condition, namely $\phi < \tau \frac{x_H(p_H)+x_F(p_F)}{x_H(p_H)+\tau x_F(p_F)}$. A sufficient condition here is that $\tau < 1/\phi$, which is stated above as Assumption 3.

Proof of Proposition 1. Define $\tau_{MAX} \equiv 1/\phi$ and $\phi_{MAX} \equiv 1/2$ (recall Assumption 1 requires $\phi \leq 1/2$). To prove the existence of $\widetilde{\tau}$ and τ_H^{CS} we use two results: first, that $g_H(1) = 1$ and $g_H(\tau)$ is increasing for $\tau > 1$, and second that $h_H(1) > 1$ and $h_H(\tau)$ is decreasing for $\tau \geq 1$ with $h_H(\tau_{MAX}) < 1$. To prove the first result, note that $2g_H'(\tau) = 1 - 1/\tau^2 > 0$ implies $g_H'(\tau) > 0$. To prove the second result, note that $\frac{\partial \ln h_H(\tau)}{\partial \ln \tau} = \phi \frac{\tau}{1+\tau} - 1$. Since $\phi < 1$ (Assumption 1), then this is negative. Also, note that $h_H(\tau_{MAX}) = \tau_{MAX}^{-1}\left(1 + \tau_{MAX}\right)^{\phi} = \phi\left(1 + 1/\phi\right)^{\phi}$. Since $\phi\left(1 + 1/\phi\right)^{\phi}$ is increasing in ϕ, to prove that $h_H(\tau_{MAX}) < 1$ for any $\phi \in (0, \phi_{MAX}]$ it is sufficient to show that $\phi_{MAX}\left(1 + 1/\phi_{MAX}\right)^{\phi_{MAX}} < 1$. But this is clearly satisfied. These two results along with the continuity of both $g_H(.)$ and $h_H(.)$ imply that there exists a unique $\widetilde{\tau}$ which is higher than 1, with $g_H(\widetilde{\tau}) > 1$. Moreover, for any $\beta\omega \in [h_H(\tau_{MAX}), 1)$ there exists a unique τ_H^{CS}. Symmetry implies that $\widetilde{\tau}$ uniquely satisfies $g_F(\tau) = h_F(\tau)$ and $g_F(\widetilde{\tau}) < 1$. Symmetry also implies that $g_F(1) = 1$ and $g_F(\tau)$ is decreasing for $\tau > 1$, and second that $h_F(1) < 1$ and $h_F(\tau)$ is increasing for $\tau \geq 1$ with $h_F(\tau_{MAX}) > 1$. One can also verify that there exists a unique τ such that $h_H(\tau) = h_F(\tau) = 1$ and $\tau > \widetilde{\tau}$.

We now need to establish that for every ϕ the ranges for both cases a and b exist, i.e., $h_H(\tau_{MAX}) < g_F(\widetilde{\tau}) < 1$. We have already shown above that the second inequality holds. Hence, we need to show that for every ϕ we have $h_H(\tau_{MAX}) < g_F(\widetilde{\tau})$. Recall that $h_H(\tau_{MAX}) = \tau_{MAX}^{-1}\left(1 + \tau_{MAX}\right)^{\phi} = \phi\left(1 + 1/\phi\right)^{\phi}$ and $\widetilde{\tau}$ is implicitly defined by $g_F(\tau) = h_F(\tau)$ or equivalently $\frac{2}{1/\tau+\tau} = \tau\left(1+\tau\right)^{-\phi}$. So we need to show that for all $\phi \in [(0, \phi_{MAX}]$ we have $h_H(\tau_{MAX}) < g_F(\widetilde{\tau})$ or $\phi\left(1 + 1/\phi\right)^{\phi} < g_F(\widetilde{\tau})$. Define $\widetilde{\tau}_{MAX}$ as that which implicitly solves $\frac{2}{1/\tau+\tau} = \tau\left(1+\tau\right)^{-\phi_{MAX}}$. Note that $\phi\left(1+1/\phi\right)^{\phi}$ is increasing in ϕ. Also, note that $\widetilde{\tau}$ is increasing in ϕ implies $g_F(\widetilde{\tau})$ is decreasing in ϕ. Hence, it is sufficient to show $\phi_{MAX}\left(1 + 1/\phi_{MAX}\right)^{\phi_{MAX}} < g_F(\widetilde{\tau}_{MAX})$. One can then readily verify that this is satisfied. The result then follows, that is, for any $\phi \in [0, 1/2]$ there exists a range for which both case (a), $g_F(\widetilde{\tau}) > \beta\omega \geq h_H(\tau_{MAX})$, and (b), $1 > \beta\omega \geq g_F(\widetilde{\tau})$, apply.

The results for cases a and b then follow ■

Intermediate Trade Costs

We now establish that for any good $\beta\omega \leq 1$ there exists a range of trade costs for which no pure strategy in which production is either concentrated in a single country nor one in which there is domestic only production can be sustained as an equilibrium. Recall first that by Proposition 2 we know that an equilibrium with no trade exists if and only if $\beta\omega \geq l_H(\tau)$. As established above, $l_H(\tau)$ is decreasing and intersects the horizontal line with $\beta\omega = 1$ at point $\tau_H^{NT}(1) = 2^{1+\phi} - 1$. It is readily verified that $\tau_H^{CS}(1) < \tau_H^{NT}(1)$. To see this recall that $\tau_H^{CS}(1)$ is defined implicitly by $1 = h_H(\tau) \equiv \frac{(1+\tau)^{\phi}}{\tau}$. Since $h_H(.)$ is strictly decreasing, to show that $\tau_H^{NT}(1) > \tau_H^{CS}(1)$, it is sufficient to show that $1 > \frac{\left(1+\tau_H^{NT}(1)\right)^{\phi}}{\tau_H^{NT}(1)} \iff \tau_H^{NT}(1) > \left(1 + \tau_H^{NT}(1)\right)^{\phi}$, or $2^{1+\phi} - 1 > \left(2^{1+\phi}\right)^{\phi}$. But this is satisfied for all $\phi < 1/2$, a restriction satisfied by Assumption 1.

We now establish that the curve $h_H(\tau)$ is always below the curve $l_H(\tau)$, so that $\tau_H^{CS}(\beta\omega) < \tau_H^{NT}(\beta\omega)$. This further implies that for any relevant $\beta\omega \leq 1$ there is no pure strategy equilibrium for $\tau \in]\tau_H^{CS}(\beta\omega), \tau_H^{NT}(\beta\omega)[$. As mentioned above our analysis is restricted to the range of τ that satisfies Assumption 3, i.e., $\tau < 1/\phi$.

Define $\tau_{MAX} \equiv 1/\phi$. Assumption 2 implies our analysis is relevant for any $\tau \in [1, \tau_{MAX}]$. We now proceed to establish that $l_H(\tau) > h_H(\tau)$ for all $\tau \in \left[\tau_H^{NT}, \tau_{MAX}\right]$. We do this in three steps: first, we first show that $l_H(\tau_{MAX}) > h_H(\tau_{MAX})$, second, we establish that $l'_H(\tau) \le h'_H(\tau) \le 0$ for all $\tau \in (1, \tau_{MAX})$, and third, we establish the final result using steps one and two.

Step 1: Since $l_H(.)$ is decreasing then $l_H(\tau_{MAX}) > h_H(\tau_{MAX})$ is equivalent to $1 < \phi l_H\left(h_H(1/\phi)\right)$, which in turn is equivalent to

$$1 < \frac{2\left(1 + \left(\phi\left(1+1/\phi\right)^{\phi}\right)^{1/(1-\phi)}\right)^{\phi} - 1}{\left(\phi\left(1+1/\phi\right)^{\phi}\right)^{1/(1-\phi)}/\phi}.$$

It can be verified that this inequality is satisfied for $0 \le \phi \le 1/2$.

Step 2: Now we proceed to show that $l'_H(\tau) \le h'_H(\tau) \iff |h'_H(\tau)| \le |l'_H(\tau)|$ for all $\tau \in (1, \tau_{MAX})$. Totally differentiating $\frac{2\left(1+y^{1/(1-\phi)}\right)^{\phi}-1}{y^{1/(1-\phi)}} = \tau$, we have

$$l'_H(\tau) = -\frac{(1-\phi)\,y}{\left(\tau - \phi\left(\frac{1+\tau y^{1/(1-\phi)}}{1+y^{1/(1-\phi)}}\right)\right)}.$$

Similary, we have $h'_H(\tau) = -\left(\frac{1}{\tau} - \frac{\phi}{1+\tau}\right) y$. Hence, $|h'_H(\tau)| \le |l'_H(\tau)|$ if

$$\frac{1}{\tau} - \frac{\phi}{1+\tau} \le \frac{1-\phi}{\left(\tau - \phi\left(\frac{1+\tau y^{1/(1-\phi)}}{1+y^{1/(1-\phi)}}\right)\right)}$$

A sufficient condition for this is

$$\phi \le \frac{1}{\tau}$$

which is clearly satisfied for $\tau = \tau_{MAX}$. The result then follows.

Step 3: We now establish that $l_H(\tau) > h_H(\tau)$ for all $\tau \in \left[\tau_H^{NT}, \tau_{MAX}\right]$. From the analysis above along with Step 1, we already know that $l_H(\tau_H^{NT}) > h_H(\tau_H^{NT})$ and $l_H(\tau_{MAX}) > h_H(\tau_{MAX})$. Suppose by contradiction there exists $\tau' \in \left(\tau_H^{NT}, \tau_{MAX}\right)$ such that $l_H(\tau') = h_H(\tau')$. Then $l'_H(\tau) \le h'_H(\tau)$ (by Step 2) along with $l_H(\tau_H^{NT}) > h_H(\tau_H^{NT})$ implies $l(\tau_{MAX}) < h(\tau_{MAX})$. A contradiction. Hence, the result follows.

Mixed Strategy proposed by GRH

GRH propose an equilibrium in which Foreign firms do not export and charge a price p_F^A while Home firms mix between a local strategy (no export) with p_H^A and a global pricing strategy, where firms charge price p_F^A in Foreign and a price p_H^G in Home that satisfies $\Phi_H(p_H^G) + \Phi_F(p_H^G) = 0$, where $\Phi_H(p_H^G)$ and $\Phi_F(p_H^G)$ are defined in the text. As a first step, we show that $\Phi_F(p_H^G) < 0$, implying that Home firms make losses in Foreign. To see this, let

$$\pi(p_H, p_F) \equiv \left[p_H - \frac{w_H a_H}{(x_H(p_H) + x_F(p_F))^{\phi}}\right] x_H(p_H) + \left[p_F - \frac{w_H a_H \tau}{(x_H(p_H) + x_F(p_F))^{\phi}}\right] x_F(p_F),$$

and note that $\Phi_H(p_H^G) + \Phi_F(p_H^G) = 0$ can be written as $\pi(p_H^G, p_F^A) = 0$. Let's imagine for a second that $p_H^G = p_H^0$, where p_H^0 is defined implicitly by

$$p_H^0 = \frac{w_H a_H}{(x_H(p_H^0) + x_F(\tau p_H^0))^{\phi}}.$$

In this case we would have $\pi(p_H^0, \tau p_H^0) = 0$ – if Home firms charged prices p_H^0 and τp_H^0 then they would indeed make zero profits. But the violation of condition (5) implies that $\tau p_H^0 > p_F^A$, so charging τp_H^0 in Foreign cannot be part of an equilibrium. Instead, the proposed strategy is to charge p_H^G in Home and p_F^A in Foreign – with $p_H^G = p_H^0$, this means prices p_H^G in Home and p_F^A in Home, leading to profits $\pi(p_H^0, p_F^A)$. Our result that profits are increasing in prices (i.e., the best that a deviating firm can do is to shave current prices) implies that $\pi_2 > 0$, so $\pi(p_H^0, \tau p_H^0) = 0$ implies that $\pi(p_H^0, p_F^A) < 0$. It is easy to see that

$\pi(p_H^0, p_F^A) = \Phi_H(p_H^0) + \Phi_F(p_H^0)$, hence we can conclude that $\Phi_H(p_H^0) + \Phi_F(p_H^0) < 0$. But $p_F^A < \tau p_H^0$ implies that

$$\Phi(p_H^0) \equiv \left[p_H^0 - \frac{w_H a_H}{\left(x_H(p_H^0) + x_F(p_F^A)\right)^\phi}\right] x_H(p_H^0) > \left[p_H^0 - \frac{w_H a_H}{\left(x_H(p_H^0) + x_F(\tau p_H^0)\right)^\phi}\right] x_H(p_H^0) = 0,$$

hence $\Phi_H(p_H^0) > 0$. Combined with $\Phi_H(p_H^0) + \Phi_F(p_H^0) < 0$, we then conclude that $\Phi_F(p_H^0) < 0$. Since p_H^G is defined by $\pi(p_H^G, p_F^A) = 0$ then the fact that $\pi_1 > 0$ implies that $p_H^G > p_H^0$. But since $\Phi_F' < 0$, we finally conclude that $\Phi_F(p_H^G) < 0$.

As a second step, we show that $\Phi_F(p_F^G) < 0$ implies that there exists a profitable deviation to the proposed strategy. If the probability of choosing the local strategy is q_H, the expected profits made by a Home firm under the global strategy are $\left(\Phi_H(p_H^G) + \Phi_F(p_H^G)\right)\left(q_H + \frac{1-q_H}{2}\right) = 0$. Now consider a deviation to a pure strategy with price in Foreign equal to p_F^A and the local price just below p_H^G, say at $p_H' = p_H^G - \varepsilon'$. The profits under the deviation are $q_H\left[\Phi_H(p_H') + \Phi_F(p_H')\right] + (1 - q_H)\left[\Phi_H(p_H') + \Phi_F(p_H')/2\right]$. Since $p_H' \approx p_H^G$ then $\Phi_H(p_H') + \Phi_F(p_H') \approx 0$ and $\Phi_F(p_H') \approx -\Phi_H(p_H')$, hence profits under this deviation are close to $(1 - q_H)\,\Phi_H(p_H')/2$, and this is positive. Intuitively, by charging a slightly lower price in the domestic market, a Home firm secures all the profits from Home sales while not incurring more losses in Foreign.

A.2 General Equilibrium

Lemma 1 *The functions $v_H^g(\tau)$ and $v_F^g(\tau)$ exist (this entails existence and uniqueness of a solution in v to (24) and (26) respectively), $v_H^g(\tau)$ is increasing and $v_F^g(\tau)$ is decreasing, and for any $\tau > 1$ we have $v_H^g(\tau) > 1/2 > v_F^g(\tau)$.*

Proof of Lemma 1. Recall that $v_H^g(\tau)$ and $v_F^g(\tau)$ are implicitly defined by $\beta(v)/\omega(v) = g_H(\omega(v), \tau)$ and $\beta\left(v\right)\omega\left(v\right) = g_F\left(\omega\left(v\right), \tau\right)$. Consider first $v_H^g(\tau)$. We have $g_H(\omega(v), \tau) = \frac{\tau + (1/v-1)/\tau}{1+1/v-1} = \tau v + (1-v)/\tau = v(\tau - 1/\tau) + 1/\tau$, hence $\beta(v)\omega(v) = g_H(\omega(v), \tau)$ is equivalent to

$$(1/v - 1)^{-1/\xi} = v(\tau - 1/\tau) + 1/\tau,$$

where $\xi \equiv \alpha\theta/\left(1 + \alpha\theta\right)$. Both the LHS and RHS are increasing in v (since $\tau > 1/\tau$). To show that there exists a solution, note that for $v = 1/2$ we have the $RHS > LHS$, whereas for $v = 1$ the LHS is infinite while the RHS is τ, so $LHS > RHS$. This along with the continuity of the LHS and the RHS guarantees existence. For uniqueness, note that the derivative of the LHS is

$$\frac{1}{\xi}\left(1/v - 1\right)^{-1/\xi - 1}\frac{1}{v^2} = \frac{1}{\xi}\left(1/v - 1\right)^{-1/\xi}\frac{1}{v\left(1 - v\right)}$$

while the derivative of the RHS is $\tau - 1/\tau$. Then we require

$$\frac{1}{\xi}\left(1/v - 1\right)^{-1/\xi}\frac{1}{v\left(1 - v\right)} > \tau - 1/\tau$$

Since both the LHS and the RHS are increasing in v, we know that any point of intersection occurs at $v \in (1/2, 1)$. In fact, any intersection satisfies $(1/v - 1)^{-1/\xi} = v(\tau - 1/\tau) + 1/\tau$. Evaluating the LHS derivative at such an intersection yields

$$\frac{1}{\xi}\left[v(\tau - 1/\tau) + 1/\tau\right]\frac{1}{v\left(1 - v\right)} = \frac{1}{\xi\left(1 - v\right)}(\tau - 1/\tau) + \frac{1/\tau}{v\left(1 - v\right)}$$

Clearly,

$$\frac{1}{\xi\left(1 - v\right)}(\tau - 1/\tau) + \frac{1/\tau}{v\left(1 - v\right)} > \tau - 1/\tau$$

establishing uniqueness.

Now we proceed to show that $v_H^g(\tau)$ is increasing. Note that the RHS, $v(\tau - 1/\tau) + 1/\tau$, is increasing in τ. This is obvious because the derivative w.r.t. τ is $v - \left(1/\tau^2\right)\left(1 - v\right)$, and since $v > 1/2$ and $\tau > 1$ then this is positive. The result then follows from the implicit function theorem.

The fact that $v_F^g(\tau)$ exists, is unique, and is strictly decreasing follows directly from symmetry. In particular, symmetry implies $v_H^g(\tau) = 1 - v_F^g(\tau)$. The result then follows.

Finally, for a given τ, it clearly follows that $v_H^g(\tau) > 1/2 > v_F^g(\tau)$ ■

Let $\widetilde{\tau}$ be defined implicitly by $v_H^g(\widetilde{\tau}) = v_H^h(\widetilde{\tau})$. Note that if $v_H^g(\widetilde{\tau}) = v_H^h(\widetilde{\tau})$ then by symmetry we have $v_F^g(\widetilde{\tau}) = v_F^h(\widetilde{\tau})$. Then it is easy to see that for $\tau \leq \widetilde{\tau}$ any CSA $\widetilde{v} \in [v_F^g(\tau), v_H^g(\tau)]$ is also a CSE. This follows because for $\tau \leq \widetilde{\tau}$ we have $v_H^g(\tau) < v_H^h(\tau)$ and $v_F^g(\tau) > v_F^h(\tau)$, hence conditions (10) and (12) imply conditions (11) and (13). Before establishing this, we first establish the existence of $v_H^h(\tau)$ and $v_F^h(\tau)$.

Lemma 2 *The functions $v_H^h(\tau)$ and $v_H^h(\tau)$ exist (this entails existence and uniqueness of a solution in v to* (25) *and* (27)*), $v_H^h(\tau)$ is decreasing and $v_F^h(\tau)$ is increasing in τ.*

Proof of Lemma 2. Consider first $v_H^h(\tau)$. Since $h_H(\omega(v), \tau) \equiv \frac{1}{\tau}\left[\frac{\tau+(1/v-1)}{(1/v-1)}\right]^{(\alpha-1)/\alpha} = \frac{1}{\tau}\left[1+\frac{\tau v}{1-v}\right]^{\phi}$, the fact that $v_H^h(\tau)$ is implicitly defined by $\beta(v)\omega(v) = h_H(\omega(v), \tau)$ is equivalent to $v_H^h(\tau)$ being implicitly defined by

$$(1/v-1)^{-1/\xi} = \frac{1}{\tau}\left[1+\frac{\tau v}{1-v}\right]^{\phi}$$

or

$$\left(\frac{v}{(1-v)^{1-\xi\zeta}}\right)^{1/\xi} = \frac{1}{\tau}\left[(\tau-1)v+1\right]^{\phi}$$

Both the LHS and RHS are increasing in v (since $\tau > 1$ and $\xi\phi < 1$). To show that there exists a solution, note that for $v = 0$ we have the $RHS > LHS$, whereas for $v = 1$ the LHS is infinite while the RHS is $1/\tau^{1-\phi}$, so $LHS > RHS$. This along with the continuity of both the LHS and the RHS guarantees existence. For uniqueness, note that the derivative of the LHS is

$$\frac{1}{\xi}\left(\frac{v}{(1-v)^{1-\xi\phi}}\right)^{1/\xi}\frac{1}{v}\frac{1-\xi\phi v}{1-v}$$

while the derivative of the RHS is

$$\phi\frac{1}{\tau}\left[(\tau-1)v+1\right]^{\phi}\frac{\tau-1}{(\tau-1)v+1}$$

Then we require

$$\frac{1}{\xi}\left(\frac{v}{(1-v)^{1-\xi\phi}}\right)^{1/\xi}\frac{1}{v}\frac{1-\xi\phi v}{1-v} > \phi\frac{1}{\tau}\left[(\tau-1)v+1\right]^{\phi}\frac{\tau-1}{(\tau-1)v+1}$$

Any intersection satisfies $\left(\frac{v}{(1-v)^{1-\xi\phi}}\right)^{1/\xi} = \frac{1}{\tau}\left[(\tau-1)v+1\right]^{\phi}$. Evaluating the LHS derivative at such an intersection implies we require

$$\frac{1}{\xi}\frac{1}{\tau}\left[(\tau-1)v+1\right]^{\phi}\frac{1}{v}\frac{1-\xi\phi v}{1-v} > \phi\frac{1}{\tau}\left[(\tau-1)v+1\right]^{\phi}\frac{\tau-1}{(\tau-1)v+1}$$

or

$$\frac{1}{\xi}\frac{1}{\tau}\frac{1}{v}\frac{1-\xi\phi v}{1-v} > \phi\frac{\tau-1}{\tau}\frac{1}{(\tau-1)v+1}$$

which is clearly satisfied. Uniqueness then follows.

Now we proceed to show that $v_H^h(\tau)$ is decreasing. Note that the RHS, $\frac{1}{\tau}\left[(\tau-1)v+1\right]^{\phi}$, is decreasing in τ. The result then follows from the implicit function theorem.

The fact that $v_F^h(\tau)$ exists, is unique, and is increasing follows directly from symmetry. In particular, symmetry implies $v_H^h(\tau) = 1 - v_F^h(\tau)$. The result then follows ■

Proof of Proposition 4. Note that the implicit function $v_H^g(\tau)$ and $v_F^g(\tau)$ characterize the limiting goods for which the global-deviation condition for both Home and Foreign are satisfied respectively, while (25) and

(27) capture the limiting goods for which the local-deviation condition is also satisfied. Consider first Home country. Recall that $v_H^h(\tau)$ is the implicit function which solves

$$(1/v-1)^{-1/\xi} = \frac{1}{\tau}\left[\frac{\tau v}{1-v}+1\right]^{\phi}$$

where $\xi = \alpha\theta/(1+\alpha\theta)$ and $\phi = (\alpha-1)/\alpha$.

Also, recall that $v_H^g(\tau)$ is the implicit function which solves

$$(1/v-1)^{-1/\xi} = \tau v + \frac{1-v}{\tau}.$$

We proceed as follows. First, we establish the existence of a unique $\widehat{\tau}$ satisfying $v_H^h(\widehat{\tau}) = 1/2$. Second, we show that $\tau_{MAX} > \widehat{\tau}$. Third, we show there exists a unique $\widetilde{\tau}$ satisfying $v_H^g(\widetilde{\tau}) = v_H^h(\widetilde{\tau})$ and $\widetilde{\tau} < \widehat{\tau}$. Finally, we establish that for the relevant range of trade costs, any CSA within the two extreme cases is also a CSE.

Note that to show there exists a unique $\widehat{\tau}$ satisfying $v_H^h(\widehat{\tau}) = 1/2$ is equivalent to establishing there exists a unique $\widehat{\tau}$ satisfying $\tau = [\tau+1]^{\phi}$. But this follows from Assumption 1 ($\phi \leq 1/2$). To show that $\tau_{MAX} > \widehat{\tau}$, it sufficient to show that $\tau_{MAX} > \widehat{\tau}_{MAX}$ where τ_{MAX} implicitly solves $\tau = [\tau+1]^{\phi_{MAX}}$. But this is clearly satisfied. The fact that $\widehat{\tau}$ also satisfies $v_H^h(\widehat{\tau}) = v_F^h(\widehat{\tau}) = 1/2$ follows directly from symmetry. In particular, symmetry implies $v_H^g(\tau) = 1 - v_F^g(\tau)$. The result then follows.

Evaluating the first and second equations above at $\tau = 1$ yields $\left(v_H^h/\left(1-v_H^h\right)\right)^{1/\xi} = \left[v_H^h/\left(1-v_H^h\right)+1\right]^{\phi}$ and $\left(v_H^g/\left(1-v_H^g\right)\right)^{1/\xi} = 1$ respectively. Since $\left[v_H^h/\left(1-v_H^h\right)+1\right]^{\phi} > 1$, we have $v_H^h(1) > v_H^g(1) = 1/2$. Also, $v_H^g(1) = 1/2$ and $v_H^g(.)$ strictly increasing (by lemma 1) imply $v_H^g(\widehat{\tau}) > 1/2$. So we have $v_H^g(\widehat{\tau}) > v_H^h(\widehat{\tau}) = 1/2$. Since, both $v_H^h(.)$ and $v_H^g(.)$ are continuous there exists $\widetilde{\tau}$ and $\widetilde{\tau} < \widehat{\tau}$. Uniqueness follows from $\frac{dv_H^g(\tau)}{d\tau} > 0$ (by lemma 1) and $\frac{dv_H^h(\tau)}{d\tau} < 0$ (by lemma 2). The fact that $\widetilde{\tau}$ also satisfies $v_F^g(\widetilde{\tau}) = v_F^h(\widetilde{\tau})$ and $v_F^h(\tau) < v_F^g(\tau)$ for any $\tau < \widetilde{\tau}$ follows directly from symmetry. In particular, symmetry implies $v_H^g(\tau) = 1 - v_F^g(\tau)$. The result then follows.

We now proceed to show that for any $\tau \leq \widetilde{\tau}$, any allocation $\widetilde{v} \in [v_F^g(\tau), v_H^g(\tau)]$ with $\widetilde{\omega} \equiv \omega(\widetilde{v})$ is a CSE, i.e., it satisfies (10)-(13). In particular, $\widetilde{v}$ satisfies $\beta(\widetilde{v})\widetilde{\omega} \leq g_H(\widetilde{\omega},\tau), \beta(\widetilde{v})\widetilde{\omega} \leq h_H(\widetilde{\omega},\tau), \beta(\widetilde{v})\widetilde{\omega} \geq g_F(\widetilde{\omega},\tau)$ and $\beta(\widetilde{v})\widetilde{\omega} \geq h_F(\widetilde{\omega},\tau)$. Consider the first of these conditions: $\beta(\widetilde{v})\widetilde{\omega} \leq g_H(\widetilde{\omega},\tau)$. In lemma 1, we showed that at $v = 1/2$ we have $RHS > LHS$ and there exists a unique intersection at $v = v_H^g(\tau)$. So for $v \leq v_H^g(\tau)$ we must have $RHS \geq LHS$, i.e., $\beta(v)\omega(v) \leq g_H(\omega(v),\tau)$ for any $1/2 \leq v \leq v_H^g(\tau)$. Moreover, the fact that both the LHS and RHS are strictly increasing and continuous on $[0,1)$ implies for any $0 \leq v \leq v^g(\tau)$ it must also be the case that $\beta(v)\omega(v) \leq g_H(\omega(v),\tau)$. Similarly, symmetry implies for any $1/2 \geq v \geq v_F^g(\tau)$ we have $\beta(v)\omega(v) \geq g_F(\omega(v),\tau)$ and by extension this is also the case for any $0 \geq v \geq v_F^g(\tau)$. Hence, for any $\widetilde{v} \in [v_F^g(\tau), v_H^g(\tau)]$, we have $\widetilde{\omega}^{-1}g_F(\widetilde{\omega},\tau) \leq \beta(\widetilde{v}) \leq \widetilde{\omega}^{-1}g_H(\widetilde{\omega},\tau)$.

Consider now, the first local-deviation condition: $\beta(v)\omega(v) \leq h_H(\omega(v),\tau)$. We know that $v_H^h(\tau)$ is implicitly defined by

$$(1/v-1)^{-1/\xi} = \frac{1}{\tau}\left[1+\frac{\tau v}{1-v}\right]^{\phi}$$

For $v = 1/2$ we have LHS is 1, while the RHS is $\frac{1}{\tau}[1+\tau]^{\phi}$. So for the relevant range of low trade costs along with the restriction $\alpha < 2$, we have $RHS > LHS$. Moreover, from lemma 2 there exists a unique intersection at $v = v_H^h(\tau)$. Hence, for any $v \leq v_H^h(\tau)$ we have $RHS \geq LHS$ implies that $\beta(v)\omega(v) \leq h_H(\omega(v),\tau)$ for any $1/2 \leq v \leq v_H^h(\tau)$. Similarly, symmetry implies for any $1/2 \geq v \geq v_F^h(\tau)$ we have $\beta(v)\omega(v) \geq h_F(\omega(v),\tau)$. Furthermore, for any $\tau \leq \widetilde{\tau}$, we have already established that $v_H^h(\tau) \geq v_H^g(\tau) \geq 1/2 \geq v_F^g(\tau) \geq v_F^h(\tau)$. Thus, for any $\widetilde{v} \in \left[v_H^h(\tau), v_H^g(\tau)\right]$, we have $\widetilde{\omega}^{-1}h_F(\widetilde{\omega},\tau) \leq \widetilde{\omega}^{-1}g_F(\widetilde{\omega},\tau) \leq \beta(\widetilde{v}) \leq \widetilde{\omega}^{-1}g_H(\widetilde{\omega},\tau) \leq \widetilde{\omega}^{-1}h_H(\widetilde{\omega},\tau)$. So by definition 2, $\widetilde{v}$ is a CSE.

Finally, we have already established that there exists a unique solution to $v_H^h(\widehat{\tau}) = 1/2$. Moreover, since $v_H^g(\tau)$ is increasing and $v_H^g(1) = 1/2$ then with $\widetilde{\tau}$ defined by $v_H^g(\tau) = v_H^h(\tau)$ we must have that $\widehat{\tau} > \widetilde{\tau}$, and $v_H^g(\tau) > v_H^h(\tau)$ for $\tau \in (\widetilde{\tau},\widehat{\tau}]$. Symmetry implies the corresponding result, i.e., for any $\tau \in (\widetilde{\tau},\widehat{\tau}]$, $v_F^g(\tau) < v_F^h(\tau)$. Also note that we have already established above that for $\tau \leq \widetilde{\tau}$ and $v_H^g(\tau) \geq \widetilde{v} \geq v_F^g(\tau)$ we have $\widetilde{\omega}^{-1}g_F(\widetilde{\omega},\tau) \leq \beta(\widetilde{v}) \leq \widetilde{\omega}^{-1}g_H(\widetilde{\omega},\tau)$. By lemma 1, we see that this also applies for any $\tau > 1$.

Similarly, for any $\tau \le \widetilde{\tau}$ and $v_H^h(\tau) \ge \widetilde{v} \ge v_F^h(\tau)$ we already know that $\widetilde{\omega}^{-1} h_F(\widetilde{\omega}, \tau) \le \beta(\widetilde{v}) \le \widetilde{\omega}^{-1} h_H(\widetilde{\omega}, \tau)$. That this also applies for any $\tau \le \widehat{\tau}$ follows from lemma 2 and the fact that for any $\tau \le \widehat{\tau}$ we have $v_H^h(\tau) \ge 1/2 \ge v_F^h(\tau)$. Moreover, for any $\tau \in (\widetilde{\tau}, \widehat{\tau}]$, we have $v_H^g(\tau) \ge v_H^h(\tau) \ge 1/2 \ge v_F^h(\tau) \ge v_F^g(\tau)$. Hence, for any $\tau \in (\widetilde{\tau}, \widehat{\tau}]$ and $\widetilde{v} \in \left[v_F^h(\tau), v_H^h(\tau)\right]$, we have $v_H^g(\tau) \ge v_H^h(\tau) \ge \widetilde{v} \ge v_F^h(\tau) \ge v_F^g(\tau)$ implies $\widetilde{\omega}^{-1} g_F(\widetilde{\omega}, \tau) \le \widetilde{\omega}^{-1} h_F(\widetilde{\omega}, \tau) \le \beta(\widetilde{v}) \le \widetilde{\omega}^{-1} h_H(\widetilde{\omega}, \tau) \le \widetilde{\omega}^{-1} g_H(\widetilde{\omega}, \tau)$. The fact that $\widetilde{v}$ is a CSE then follows from definition 2 ■

A.3 Welfare

A.3.1 Real Wages and the Gains from Trade

Proof of Proposition 5. Allowing for asymmetries in technology and labor force across countries, we can derive a more general version of the relative productivity function, i.e.

$$\beta(v) = \left(\frac{T_F}{T_H}\right)^{1/\alpha\theta} \left(\frac{v}{1-v}\right)^{1/\alpha\theta}. \tag{33}$$

Let $l \equiv L_H/L_F$. Note also that the more general versions of the global-deviation conditions imply: $g_H(\omega, \tau) \equiv \frac{\tau + (\omega l)^{-1}/\tau}{1 + (\omega l)^{-1}}$; and $g_F(\omega, \tau) \equiv \frac{1 + (\omega l)^{-1}}{1/\tau + (\omega l)^{-1}\tau}$. Using (33), we can rewrite (24) and (26) so that each is implicitly solved by v_H and v_F, respectively. Importantly, note that v_H and v_F are also the share of goods that Home firms produce in the extreme cases in which all the disputed industries are allocated to the Home and Foreign country respectively. Mote that (33) implies

$$\widetilde{v} = \frac{T_H}{T_H + T_F \beta^{-\alpha\theta}}. \tag{34}$$

From Proposition 4 we know that the set of equilibria is given by $\widetilde{v} \in [v_F^g(\tau), v_H^g(\tau)]$. In particular, the neccessary global-deviation conditions for complete specialization are satisfied, that is, $\omega(\widetilde{v})^{-1} g_F(\omega(\widetilde{v}), \tau) \le \beta(\widetilde{v}) \le \omega(\widetilde{v})^{-1} g_H(\omega(\widetilde{v}), \tau)$.[17] Consider first the Home country. Then (34) along with $\pi_{HH} = \widetilde{v}$ implies

$$\pi_{HH} = \frac{T_H w_H^{-\alpha\theta}}{\Phi_H} \tag{35}$$

where $\Phi_H = \left[T_H w_H^{-\alpha\theta} + T_F w_F^{-\alpha\theta} (\beta(\widetilde{v})\, \omega(\widetilde{v}))^{-\alpha\theta}\right]$.

Let $p_{ni}(v)$ be the price of good v if i sells to n $(i, n = H, F)$. Note that since we consider only complete specialization, any country which sells the good supplies the world market. Hence, for any industry v we have either price pairs[18]

$$p_{HH} = \frac{w_H a_H}{[x_H(p_{HH}) + x_F(\tau p_{HH})]^{\phi}} \text{ and } p_{FH} = p_{HH}\tau$$

or

$$p_{FF} = \frac{w_F a_F}{[x_H(\tau p_{FF}) + x_F(p_{FF})]^{\phi}} \text{ and } p_{HF} = p_{FF}\tau$$

where $x_H(p) = \frac{w_H L_H}{p}$ and $x_F(p) = \frac{w_F L_F}{p}$.

[17] In our quantitative exercise we also verify that no Home or Foreign firms have an incentive to target their domestic market only, i.e. the local deviation conditions for complete specialization are also satisfied. Both provide a sufficient condition for the existence of a complete specialization equilibrium.

[18] In particular, the relevant prices are

$$p_{HH}(v) = \frac{(w_H a_H(v))^{\alpha}}{(w_H L_H + w_F L_F/\tau)^{(\alpha-1)}} = \frac{1}{z_H(v)} \frac{(w_H)^{\alpha}}{(w_H L_H + w_F L_F/\tau)^{(\alpha-1)}}$$

and

$$p_{FF}(v) = \frac{(w_F a_F(v))^{\alpha}}{(w_H L_H/\tau + w_F L_F)^{(\alpha-1)}} = \frac{1}{z_F(v)} \frac{(w_F)^{\alpha}}{(w_H L_H/\tau + w_F L_F)^{(\alpha-1)}}.$$

The associated price index for Home is then given by[19]

$$P_H = \eta \left(\Upsilon_H\right)^{-(\alpha-1)} \left(\Phi_H\right)^{-1/\theta} \tag{36}$$

where $\eta = e^{-\gamma/\theta}$, γ is Euler's constant, and

$$\Upsilon_H \equiv \left(w_H L_H + w_F L_F/\tau\right)^{\pi_{HH}} \left(\left(\frac{\left(\beta\left(\widetilde{v}\right)\omega\left(\widetilde{v}\right)\right)^{\alpha}}{\tau} \right)^{1/(\alpha-1)} \left(w_H L_H/\tau + w_F L_F\right) \right)^{1-\pi_{HH}}$$

(36) implies $\Phi_H = \left(P_H/\eta\right)^{-\theta} \left(\Upsilon_H\right)^{-(\alpha-1)\theta}$. Substituting into (35) yields real wages

$$\frac{w_H}{P_H} = \left(T_H/\eta^{\theta}\right)^{1/\theta} \pi_{HH}^{-1/\theta} \left(\frac{\Upsilon_H}{w_H}\right)^{(\alpha-1)}.$$

Analagously, we can derive real wages in Foreign ■

Proof of Corollary 1. In autarky,

$$\frac{w_H^A}{P_H^A} = \left(T_H/\eta^{\theta}\right)^{1/\theta} \left(L_H\right)^{(\alpha-1)}.$$

Therefore, gains from trade are

$$\frac{w_H/P_H}{w_H^A/P_H^A} = \pi_{HH}^{-1/\theta} \left(\frac{\Upsilon_H}{w_H L_H}\right)^{(\alpha-1)}.$$

Similarly, we can also derive the gains from trade in Foreign ■

A.3.2 Multiple Countries and Frictionless Trade

We already know that with frictionless trade there exists a unique equilibrium in which the patterns of trade are consistent with "natural" comparative advantage. The difference here is that we consider $K \geq 2$ countries. Here K can be large, but finite. As noted in our general environment, preferences are uniform Cobb-Douglas with its associated demand

$$x_i(v) = \frac{D_i}{p_i\left(v\right)} \tag{37}$$

where D_i is aggregate expenditure by country i.

The stability condition in (2) holds, namely

$$\left(\frac{\alpha-1}{\alpha}\right) < 1$$

and firms in each industry engage in Bertrand competition on the world market. Before proceeding to the proof we first formally restate the definition of an equilibrium.

Definition 3 *Given country size L_i and country specific productivity distribution $F_i\left(z\right)$, an equilibrium with frictionless trade consists of prices $p_{ni}\left(v\right)$, w_i, and quantities $x_i\left(v\right)$ such that: (a) industry markets clear; (b) firms in each industry engage price competition in all markets simultaneously; and (c) labor market clears.*

[19] To see this note first that

$$P_H = \exp\left\{ \int_0^{\widetilde{v}} \ln p_{HH}\left(v\right) dv + \int_{\widetilde{v}}^{1} \ln p_{HF}\left(v\right) dv \right\}.$$

Using

$$p_{HH}\left(v\right) = \frac{\left(w_H a_H\left(v\right)\right)^{\alpha}}{\left(w_H L_H + w_F L_F/\tau\right)^{(\alpha-1)}} = \frac{1}{z_H\left(v\right)} \frac{\left(w_H\right)^{\alpha}}{\left(w_H L_H + w_F L_F/\tau\right)^{(\alpha-1)}}$$

and

$$p_{FF}\left(v\right) = \frac{\left(w_F a_F\left(v\right)\right)^{\alpha}}{\left(w_H L_H/\tau + w_F L_F\right)^{(\alpha-1)}} = \frac{1}{z_F\left(v\right)} \frac{\left(w_F\right)^{\alpha}}{\left(w_H L_H/\tau + w_F L_F\right)^{(\alpha-1)}}$$

along with (20), we can derive the price index as noted in (36).

Proof of Proposition 6. Price competition amongst firms in any domestic industry implies average cost pricing, that is

$$p_{ni}(v) = \frac{w_i}{z_i(v)^{1/\alpha} X_i(v)^{(\alpha-1)/\alpha}} \tag{38}$$

Note that if country i supplies the world market, then $X_i(v) = \sum_{n=1}^{K} x_n(p_{ni}(v)) = \sum_{n=1}^{K} \frac{D_n}{p_{ni}(v)}$. Moreover, the actual prices paid for good v is the lowest from all sources i

$$p_n(v) = \min\{p_{ni}(v); i = 1, ..., K\} \tag{39}$$

Complete specialization along with industry market clearing imply

$$p_{ni}(v) = \frac{w_i^{\alpha}}{z_i(v)\left(\sum_{k=1}^{K} \frac{w_k L_k}{w_i}\right)^{\alpha-1}}.$$

Using the assumption that the productivity distrbution is Fretchet, we can derive the distribution of prices that i presents to[20] n

$$G_{ni}(p) = 1 - e^{-T_i w_i^{-\alpha\theta}\left(\sum_{k=1}^{K} w_k L_k\right)^{(\alpha-1)\theta} p^{\theta}}. \tag{40}$$

From (40) we can derive the price distribution for which country n actually buys some good j, that is, the lowest price of a good in country n will be less than p unless each source's price is greater than p

$$G_n(p) = 1 - \Pi_{i=1}^{K}[1 - G_{ni}(p)]$$

So

$$G_n(p) = 1 - e^{-\Phi_n p^{\theta}}$$

where $\Phi_n = \sum_{i=1}^{K} T_i w_i^{-\alpha\theta}\left(\sum_{k=1}^{K} w_k L_k\right)^{(\alpha-1)\theta}$.

Hence, the probability n buys from i is given by

$$\begin{aligned} \pi_{ni} &= \Pr[p_{ni} \leq \min\{p_{ns} : s \neq i\}] \\ &= \int_0^{\infty} \Pi_{s\neq i}[1 - G_{ns}(p)]\, dG_{ni}(p) \end{aligned}$$

So

$$\pi_{ni} = \frac{T_i w_i^{-\alpha\theta}\left(\sum_{k=1}^{K} w_k L_k\right)^{(\alpha-1)\theta}}{\Phi_n} = \frac{T_i w_i^{-\alpha\theta}}{\sum_{i=1}^{K} T_i w_i^{-\alpha\theta}}$$

With costless trade π_{ni} is the same for all n, and is also independent of v. Hence, π_{ni} is also the share of goods that any country n buys from country i. Moreover, the price of a good country n actually buys from i has the distribution $G_n(p)$ so that π_{ni} is also the share of expenditure by n on goods produced in i, that is

$$\pi_{ni} = \frac{X_{ni}}{X_n} = \frac{T_i w_i^{-\alpha\theta}\left(\sum_{k=1}^{K} w_k L_k\right)^{(\alpha-1)\theta}}{\Phi_n} = \frac{T_i w_i{}^{-\alpha\theta}}{\sum_{k\in\Gamma} T_k w_k{}^{-\alpha\theta}} \tag{41}$$

[20] In autarky the distribution of prices is given by $G_n(p;\{n\}) = 1 - e^{-T_n w_n{}^{-\alpha\theta}(w_n L_n)^{(\alpha-1)\theta} p^{\theta}}$. Hence, it depends not only on the state of country n's technology discounted by its input cost as in EK, but also, because of external economies, on the market size of n.

Also, the associated price index for is

$$P_n = \eta \Phi_n^{-1/\theta} \tag{42}$$

where $\eta = e^{-\gamma/\theta}$ and γ is Euler's constant.

In equilibrium, labor market clearing/trade balance implies

$$w_i L_i = \sum_{n=1}^{K} \pi_{ni} X_n \tag{43}$$

Equation (41) along with (43) yield an expression for relative wages

$$\frac{w_i}{w_n} = \left(\frac{T_i/L_i}{T_n/L_n}\right)^{1/(1+\alpha\theta)} \tag{44}$$

Using (41) and (42) we can derive real wages

$$\frac{w_n}{P_n} = \left(T_n/\eta^{\theta}\right)^{1/\theta} L_n^{\alpha-1} \pi_{nn}^{-1/\theta} \left(\sum_{k=1}^{K} \frac{w_k L_k}{w_n L_n}\right)^{\alpha-1}$$

In autarky real wage is given by

$$\omega_n^A \equiv \frac{w_n^A}{P_n^A} = \left(T_n/\eta^{\theta}\right)^{1/\theta} L_n^{\alpha-1}$$

Hence, gains from trade are

$$\frac{w_n/P_n}{w_n^A/P_n^A} = \pi_{nn}^{-1/\theta} \left(\sum_{k=1}^{K} \frac{w_k L_k}{w_n L_n}\right)^{\alpha-1}.$$

Finally, both (41) and (44) imply

$$\pi_{nn} = \sum_{k=1}^{K} \frac{w_n L_n}{w_k L_k}.$$

The result then follows ■

Proof of Corollary 2. From (32) we can then simply derive the contribution of comparative advantage and Marshallian externalities using $\frac{\ln\left(\pi_{nn}^{-1/\theta}\right)}{\ln\left(\pi_{nn}^{-1/\theta}\pi_{nn}^{-(\alpha-1)}\right)}$ and $\frac{\ln\left(\pi_{nn}^{-(\alpha-1)}\right)}{\ln\left(\pi_{nn}^{-1/\theta}\pi_{nn}^{-(\alpha-1)}\right)}$ respectively. The result then follows ■

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