A Multinational Production and Non-Tradable Goods

In the model of Section 2, international trade was the only channel through which countries could gain from openness. But, arguably, the activity of multinational firms could be even more important. We now incorporate multinational production as an extra channel for the gains from openness. To such end, we extend the model of Section 2 by allowing technologies to be used outside of the region where they originate; whenever this happens we say that there is multinational production (MP). To proceed we assume that countries are a collection of symmetric regions.

We follow Ramondo and Rodríguez-Clare (2013) and assume that a technology has a productivity $z_n$ in each country $n = 1, \ldots, N$. To introduce frictions to the “movement of ideas” within countries, analogously to the way we introduced domestic frictions for trade, we assume that each technology has a “home region” in each country. Using a technology originated in country $i$ for production outside of the technology’s home region in country $i$ entails an iceberg-type efficiency loss, or “MP cost,” of $h_{ii} \geq 1$. Moreover, using a technology originated in country $i$ in the technology’s home region in country $l \neq i$ entails an MP cost of $\gamma_{li} \geq 1$. Finally, the total MP cost associated with using a technology from country $i$ outside of the technology’s home region in country $l \neq i$ is $\gamma_{li}h_{ll}$.\footnote{The assumption that technologies have a home region in each country is made to keep the treatment of domestic and foreign technologies consistent. We assume that technologies originated in country $i$ are “born” in a particular region and then face an MP cost $h_{ii}$ to be used in another region of country $i$. The analogous assumption for the use of technologies from $i$ in country $n \neq i$ is that they also have a region in country $n$ where they are “reincarnated” (their home region), and then face an MP cost $h_{nn}$ to be used in another region of country $n$.}

In sum, each technology is characterized by three elements: first, the country $i$ from which it originates; second, a vector that specifies the technology’s productivity parameter in each country, $z = (z_1, \ldots, z_N)$; and third, a vector that specifies the technology’s home region in each country, $m = (m_1, \ldots, m_N)$. The effective productivity of a technology $(i, z, m)$ is $z_i$ if used in region $m_i$, $z_i/h_{ii}$ if used in region $m \in \Omega_i$ with $m \neq m_i$, $z_i/\gamma_{li}$ if used in region $m_l$ for $l \neq i$, and $z_i/\gamma_{li}h_{ll}$ if used in region $m \in \Omega_l$ for $l \neq i$ and $m \neq m_l$. We assume that productivity levels in $z$, for technologies originating in country $i$, are
independently drawn from the Fréchet distribution with parameters $T_i$ and $\theta$, and we assume that $m_n$ is uniformly and independently drawn from the set $\Omega_n$.

In the model with MP, we introduce both tradable and non-tradable goods, since around half of MP flows in the data occur in non-tradable goods. We assume that tradable goods are intermediate goods while non-tradable goods are final goods. There is a continuum of final goods and a continuum of intermediate goods, both in the interval $[0, 1]$. Preferences over final goods are CES with elasticity of substitution $\sigma > 0$. Intermediate goods are used to produce a composite intermediate good with a CES aggregator with elasticity $\sigma > 0$. The composite intermediate together with labor are used, via a Cobb-Douglas production function, to produce final and intermediate goods with labor shares $\alpha$ and $\beta$, respectively.

We assume that MP is possible in both the final and intermediate goods, and that the MP costs are the same in both cases. Further, we assume that $1 \leq d_{nn} = h_{nn}$. Consider a particular intermediate good whose home region is $m_n$. The price of this good in other regions of country $n$ ($m \in \Omega_n, m \neq m_n$) is determined by $z/d_{nn}$ if traded and $z/h_{nn}$ if produced locally via MP. Our assumption that $d_{nn} = h_{nn}$ implies that there is indifference between these two options. We assume that the indifference is broken in favor of trade, which implies that there is no MP across regions within countries for intermediate goods. Summing up, there is "domestic" MP in final, but not intermediate, goods, whereas trade is feasible in intermediate, but not final, goods, within countries. Across countries, MP is feasible in both types of goods, while trade is only possible in intermediate goods.

Our object of interest is the equilibrium real wage in each country $n$, which we compare with the real wage in the data. In the model with trade, MP, and domestic frictions, analogously to the baseline model, equilibrium wages can be written as

$$w_n = \mu^M \times \frac{1}{\phi_n} \times \frac{L_{nn}}{\gamma_{mn}^{-1} \tau_{nn}^{-\eta}} \times \frac{\lambda_{nn}^{-\theta}}{\pi_{nn}^{-\frac{1+\eta}{\theta}}} \times \frac{1}{\pi_{nn}},$$  

(A1)

where $\mu^M$ is a positive constant,

$$\eta \equiv \frac{1 - \alpha}{\beta},$$  

(A2)

and

$$\gamma_{mn} \equiv \left[ \frac{1}{M_n} + \frac{M_n - 1}{M_n} h_{nn}^{-\theta} \right]^{-1/\theta},$$  

(A3)

and $\pi_{nn}$ is the domestic MP share.\(^2\) There are several points to be made about the result

\(^2\)Formally, $\pi_{li} \equiv Y_{li}/Y_l$, where $Y_{li}$ is value of production in country $l$ with technologies originated in country $i$, and $Y_l \equiv \sum_i Y_{li}$.
in (A1). First, the pure scale effect now has elasticity $(1 + \eta)/\theta$ rather than $1/\theta$. The reason is that there are scale effects operating in both the final and intermediate goods sectors. The scale effect elasticity in the final goods sector is $1/\theta$, as in the baseline model, but this elasticity is $\eta/\theta$ in the intermediate goods sector. The term $\eta$ captures the amplification of gains by the factor $1/\beta$ in the intermediate goods sector because of the input-output loop and the weakening of the overall effect due to intermediate goods being only used with share $1 - \alpha$ in the production of final goods. Second, the real wage is now affected by frictions to domestic trade and to “domestic” MP. The impact of domestic trade frictions is $\tau_{nn} - \eta$, while the impact of domestic MP frictions is $\gamma_{nn}^{-1}$. Third, the gains from trade are now captured by $\lambda_{nn}^{-\eta/\theta}$, rather than $\lambda_{nn}^{-1/\theta}$. Finally, the term $\pi_{nn}^{-1+\eta/\theta}$ captures the gains from MP (i.e., the change in the real wage from a situation with no MP to the observed equilibrium), for both final and intermediate goods. The gains from openness are just the product of the gains from trade and the gains from MP.

As the last term in (A1) indicates, the gains from MP can be expressed as a function of observed flows, in the same way the gains from trade are. Data on the gross value of production for multinational affiliates from $i$ in $n$ are used as the empirical counterpart of bilateral MP flows in the model, which in turn are used to compute the MP shares, $\pi_{nn}$, from Ramondo, Rodriguez-Clare, and Tintelnot (2015). The labor shares $\alpha$ and $\beta$ are set to 0.75 and 0.50, respectively, following Alvarez and Lucas (2007), while the parameter $\theta$ is set to a value of 6 following the different approaches described in the paper. It is worth noting here that $(1 + \eta)/\theta = 1/4$ so that the strength of scale effects is the same as in the baseline calibration. Our calibration of domestic frictions for trade in goods is equivalent to the procedure described for the symmetric model in the paper. For $\theta = 6$, we get $d = 1.81$, and we assume that $d = h$.

Columns 2 to 7 in Table A1 show each term in the right-hand side of (A1), relative to the United States. Given our assumption that $h = d$, $\gamma_{nn} = \tau_{nn}$, still, these frictions are different across countries due to differences in $M_n$. Together with $(1 + \eta)/\theta = 1/4$ and the (re)calibration of $d$ to satisfy (11), there is no difference in the role of domestic frictions here with respect to the symmetric model. But the gains from trade are now $\lambda_{nn}^{-\eta/\theta}$, with $\eta/\theta = 1/12$, rather than $\lambda_{nn}^{-1/\theta}$ with $1/\theta = 1/4$. Consequently, the gains from trade have a smaller role now, as shown in column 6 of Table A1, although the gains from openness also include the gains from MP. But as column 3 indicates, MP does not help much to increase real wages, relative to the United States, for small countries because the United States has large gains from MP. While only Japan has lower gains from trade than the United States (column 2), several countries have lower gains from MP than the United States.
The result in the paper still holds: the existence of domestic frictions, rather than openness, remains the dominant channel to bring the calibrated model closer to the data. For instance, for Denmark, adding MP does not help much quantitatively to bring the relative real wage in the calibrated model closer to the one observed in the data: the implied relative real wage is 0.76, against 0.94 in the data, and 0.86 in the baseline model. More generally, looking at the average for the six smallest countries in the sample, trade and MP openness together help to close around three percent of the gap between the standard model with only scale effects and the data on relative real wages, while domestic frictions close almost 50 percent of the gap.

As a final remark, suppose that there is no MP, but we add non tradable final goods to the baseline model of Section 2. This would require setting $\pi_{nn} = 1$ in (A1), and taking a stand about the nature of non-tradable goods. If these goods were local at the region level, then $h_{nn} \to \infty$, and $\gamma_{nn} = (1/M_n)^{-1/\theta}$; if they were local at the country level, then $h_{nn} = 1$ and $\gamma_{nn} = 1$. The question is then: how much would the baseline results change by just adding non tradable goods? In the first case ($h_{nn} \to \infty$), our baseline results would be reinforced: a country like Denmark would reach a real wage (relative to U.S.) of 0.93, and domestic frictions would explain almost 90 percent of the gap between the data and the model with only scale effects. A lower bound would be obtained if, instead, non-tradable goods were national ($h_{nn} = 1$): for Denmark, the real wage would be half the United States’s (versus 0.85 in our baseline calibration). Still, domestic frictions, as opposed to openness to trade, would have the largest role in bringing the model closer to the data.

A.1 Equilibrium Analysis

The following Proposition characterizes trade and MP flows for the model of trade and MP with domestic frictions presented in Section A.

We introduce the following notation: $c_{l}^f \equiv Aw_{l}^{\alpha}(P_{l}^{g})^{1-\alpha}$, $c_{l}^g \equiv Aw_{l}^{\beta}(P_{l}^{g})^{1-\beta}$ and $Y_{s}^{l} \equiv \sum_i Y_{li}^{s}$, where $P_{l}^{g}$ is the price index of intermediate goods and where $Y_{li}^{f}$ and $Y_{li}^{g}$ denote the value of production of final and intermediate goods, respectively. It is easy to show that $Y_{l}^{g} = \eta w_{l} L_{l}$ while $Y_{l}^{f} = w_{l} L_{l}$.

Proposition 1. Country-level trade flows are

$$X_{nl} = \frac{\Gamma_{l}(\tau_{nl}c_{l}^{g})^{-\theta}}{\sum_{\nu} \Gamma_{\nu}(\tau_{n\nu}c_{\nu}^{g})^{-\theta}} X_{n}, \tag{A4}$$
while country-level MP flows in intermediate and final goods are

\[ Y_{li}^s = \frac{T_i \gamma_{li}^{-\theta}}{\Gamma_l} Y_{l}^s \quad \text{and} \quad Y_{li}^s = \frac{T_l}{\Gamma_l} Y_{i}^s \quad \text{for} \quad s = g, f, \quad (A5) \]

and price indices at the country-level are

\[ P^g_n = \mu^{-1} \left( \sum_l \Gamma_l \left( c_l^g \right)^{-\theta} \tau_{nl}^{-\theta} \right)^{-1/\theta}, \quad (A6) \]

and

\[ P^f_n = \mu^{-1} c_n^f \left( \gamma_n^{-\theta} \Gamma_n \right)^{-1/\theta}, \quad (A7) \]

where

\[ \gamma_{ll} \equiv \left( \frac{1}{M_l} + \frac{M_l - 1}{M_l} h_{ll}^{-\theta} \right)^{-1/\theta}. \quad (A8) \]

**Proof:** First, note that no intermediate goods will be produced with technologies outside of their home region. This is because of our assumption that \( h_{nn} = d_{nn} \), with the indifference broken in favor of trade rather than MP. Now, for \( k \in \Omega_l \), we have an analogous result as in (1)m except that now, instead of \( \tilde{T}_k \), we have \( \sum_{i \neq l} \frac{M_i \tilde{T}_i}{M_l} \gamma_{il}^{-\theta} + \tilde{T}_k \). Country-level trade flows are then

\[ X_{nl} = \frac{\left( \sum_{i \neq l} T_i \gamma_{il}^{-\theta} + T_l \right) w_l^{-\theta} \tau_{nl}^{-\theta}}{\sum_j \left( \sum_{i \neq j} T_i \gamma_{ji}^{-\theta} + T_j \right) w_j^{-\theta} \tau_{nj}^{-\theta}} X_i = \frac{\Gamma_l w_l^{-\theta} \tau_{nl}^{-\theta}}{\sum_j \Gamma_j w_j^{-\theta} \tau_{nj}^{-\theta}} X_i. \]

MP shares are simply given by the contribution of each source to \( \Gamma_l \), hence

\[ Y_{li}^s/\Gamma_l = T_i \gamma_{li}^{-\theta}/\Gamma_l \quad \text{and} \quad Y_{li}^s/\Gamma_l = T_l/\Gamma_l \quad \text{for} \quad s = f, g. \]

The price index for intermediate goods is simply \( \gamma^{-1} \left( \sum_j \Gamma_j w_j^{-\theta} \tau_{nj}^{-\theta} \right)^{-1/\theta} \), while for final goods we have

\[ (\mu P_n^f)^{-\theta} = \sum_{i \neq n} \frac{M_i \tilde{T}_i}{M_n} \gamma_{ni}^{-\theta} \left( 1 + (M_n - 1) h_{nn}^{-\theta} \right) + (M_n - 1) \tilde{T}_n h_{nn}^{-\theta} + \tilde{T}_n \]

\[ = \sum_{i \neq n} T_i \gamma_{ni}^{-\theta} \gamma_{nn}^{-\theta} + T_n \gamma_{nn}^{-\theta} = \gamma_{nn}^{-\theta} \Gamma_n. \]

\[ \square \]

The results for trade flows are very similar to those in a model with only trade, except that now technology levels are augmented because of the possibility of using technologies
from other countries, appropriately discounted by the efficiency costs: \( \Gamma_l \equiv \sum_{i \neq l} T_i \gamma_{li}^{-\theta} + T_l \). Note that if MP costs go to infinity, then \( \Gamma_l \to T_l \), as in the model with no MP of Section 2.

We now derive an expression for real wages. First, from (A4) and (A6), we get

\[
\frac{c_n^g}{P_n^g} = \mu \Gamma_n^{1/\theta} \tau_{mn}^{-1} \lambda_{mn}^{-1/\theta}.
\]

Using (A5),

\[
\frac{c_n^g}{P_n^g} = \mu T_n^{1/\theta} \gamma_{nn}^{-1/\theta} \left( \frac{Y_{gn}^g}{Y_n^g} \right)^{-1/\theta}.
\]

Using \( c_n^g = B w_n^\beta (P_n^g)^{1-\beta} \),

\[
\frac{w_n}{P_n^g} = B^{-1/\beta} \mu^{1/\beta} T_n^{1/\beta} \tau_{mn}^{-1/\beta} \lambda_{mn}^{-1/\beta} \left( \frac{Y_{mn}^g}{Y_n^g} \right)^{-1/\beta \theta}.
\] (A9)

From (A7) and (A5), we get

\[
P_n^f = c_n^f \mu^{1-\gamma_{nn}} T_n^{-1/\theta} \left( \frac{Y_{mn}^f}{Y_n^f} \right)^{1/\theta}.
\]

Using \( c_n^f = A w_n^\alpha (P_n^g)^{1-\alpha} \) and (A9) yield

\[
P_n^f = A B^n w_n \mu^{-(1+\eta)} T_n^{1+\eta} \gamma_{nn}^{-1/\theta} \lambda_{mn}^{\eta/\theta} \left( \frac{Y_{mn}^g}{Y_n^g} \right)^{\eta/\theta} \left( \frac{Y_{mn}^f}{Y_n^f} \right)^{1/\theta}.
\]

Further rearranging yields

\[
\frac{w_n}{P_n^f} = A^{-1} B^{-\eta} \mu^{(1+\eta)} T_n^{1+\eta} \gamma_{nn}^{-\eta} \lambda_{mn}^{-\eta/\theta} \left( \frac{Y_{mn}^g}{Y_n^g} \right)^{-\eta/\theta} \left( \frac{Y_{mn}^f}{Y_n^f} \right)^{-1/\theta}.
\]

Using \( Y_{nn}^g / Y_n^g = Y_{nn}^f / Y_n^f = T_l / \Gamma_l = \pi_l \) and \( T_n = \phi_n L_n \), and setting \( \mu^M \equiv A^{-1} B^{-\eta} \mu^{(1+\eta)} \) yields (A1).

References

### Table A1: The Symmetric Model with Multinational Production.

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Column 1 refers to the first term (size), column 2 to the third term (gains from trade), column 3 to the fourth term (gains from MP), and column 5 (domestic frictions) to the second term, respectively, on the right-hand side of \( (A1) \). Column 4 are the gains from openness, \( GO_n = GT_n \times GMP_n \). The real wage in the data is the real GDP (PPP-adjusted) per unit of equipped labor. All variables are calculated relative to the United States. The six smallest countries (with respect to R&D-adjusted size) are Iceland, Ireland, New Zealand, Finland, Norway, and Denmark, while the six largest countries are Italy, France, Great Britain, Germany, Japan, and the United States.