Appendix to:
Trade, Domestic Frictions, and Scale Effects
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A Proof of Proposition 1

Using (7), labor market clearing (3) implies that for $m \in \Omega_i$ we have

$$v_m = \left(\frac{t_m}{l_m}\right)^{1/(1+\theta)} \Delta_i, \tag{A1}$$

where

$$\Delta_i^{1+\theta} = \sum_n \sum_{k \in \Omega_n} \frac{\tau_{ni}^{-\theta}}{\sum_j \sum_{l \in \Omega_j} t_l v_l^{-\theta} \tau_{nj}^{-\theta}} v_k l_k. \tag{A2}$$

Plugging ($A1$) back into (A2) and using the definition of T_i in (6), after simplifications , a system of equations in Δ_i for $i = 1, ..., N$,

$$\Delta_i^{1+\theta} = \sum_n \frac{\tau_{ni}^{-\theta}}{\sum_j T_j^{1/(1+\theta)} L_j^{\theta/(1+\theta)} \Delta_j^{-\theta} \tau_{nj}^{-\theta}} T_n^{1/(1+\theta)} L_n^{\theta/(1+\theta)} \Delta_n.$$

Plugging ($A1$) into (1), and using (7), $\tau_{nn} = 1$ for all n, as well as the definition of X_{ni}, yield, after simplifications,

$$X_{ni} = \frac{T_i^{1/(1+\theta)} L_i^{\theta/(1+\theta)} \Delta_i^{-\theta} \tau_{nj}^{-\theta}}{\sum_j T_j^{1/(1+\theta)} L_j^{\theta/(1+\theta)} \Delta_j^{-\theta} \tau_{nj}^{-\theta}} T_n^{1/(1+\theta)} L_n^{\theta/(1+\theta)} \Delta_n. \tag{A3}$$

Also note that

$$\frac{X_i}{L_i} = \frac{\sum_n X_{ni}}{L_i} = \sum_n \frac{T_i^{1/(1+\theta)} L_i^{\theta/(1+\theta)} \Delta_i^{-\theta} \tau_{nj}^{-\theta}}{\sum_j T_j^{1/(1+\theta)} L_j^{\theta/(1+\theta)} \Delta_j^{-\theta} \tau_{nj}^{-\theta}} T_n^{1/(1+\theta)} L_n^{\theta/(1+\theta)} \Delta_n = \left(T_i/L_i\right)^{1/(1+\theta)} \Delta_i.$$

Our definition $w_i \equiv X_i/L_i$ implies that

$$w_i = \left(T_i/L_i\right)^{1/(1+\theta)} \Delta_i. \tag{A4}$$

Plugging this expression for w_i into $(A3)$ yields (4). From $(A1)$, and (2) and (7), with $\tau_{nn} = 1$ for all n, we have for region $m \in \Omega_n$

$$p_m = \gamma^{-1} \left(\sum_{k \in \Omega_i} t_k^{1/(1+\theta)} l_k^{\theta/(1+\theta)} \Delta_i^{-\theta} \tau_{ni}^{-\theta} \right)^{-1/\theta}.$$

Using $(A4)$ and the expression for T_i in (6), we get (5). Finally, combining expressions (4) and (5), and using $\lambda_{nn} \equiv X_{nn}/X_n$ and $\tau_{nn} = 1$, we get the expression in (8) for real wages under frictionless internal trade.

B Proof of Proposition 2

Replacing (1) in the paper into $X_{ni} \equiv \sum_{m \in \Omega_n} \sum_{k \in \Omega_i} x_{mk}$, and using A2, (7), and $t_k = T_i/M_i$ for $k \in \Omega_i$, and $x_m = X_n/M_n$ for $m \in \Omega_n$, we get, for $n \neq l$,

$$X_{ni} = \sum_{m \in \Omega_n} \frac{T_i w_i^{-\theta} \tau_{ni}^{-\theta}}{\sum_{j \neq n} T_j w_j^{-\theta} \tau_{nj}^{-\theta} + (M_n - 1)(T_n/M_n) w_n^{-\theta} \delta_n^{-\theta} + (T_n/M_n) w_n^{-\theta}} \frac{X_n}{M_n},$$

while

$$X_{nn} = \sum_{m \in \Omega_n} \frac{(M_n - 1)(T_n/M_n) w_n^{-\theta} \delta_n^{-\theta} + (T_n/M_n) w_n^{-\theta}}{\sum_{j \neq n} T_j w_j^{-\theta} \tau_{nj}^{-\theta} + (M_n - 1)(T_n/M_n) w_n^{-\theta} \delta_n^{-\theta} + (T_n/M_n) w_n^{-\theta}} \frac{X_n}{M_n}.$$

Turning to the price index, we know that for $m \in \Omega_n$ we have $p_m = P_n$. Hence,

$$P_n = \gamma^{-1} \left(\sum_{j \neq n} T_j w_j^{-\theta} \tau_{nj}^{-\theta} + (M_n - 1) \frac{T_n}{M_n} w_n^{-\theta} \delta_n^{-\theta} + \frac{T_n}{M_n} w_n^{-\theta} \right)^{-1/\theta}.$$

Collecting terms and using (11), we get (4) and (5) as in the paper. Finally, combining expressions (4) and (5), and using $\lambda_{nn} \equiv X_{nn}/X_n$, we get the expression for real wages under symmetry in (12), and under A1.

C Proof of Proposition 3

Assumptions A1 and A2 imply that equilibrium wages are determined by the system

$$w_i L_i = \sum_n \frac{L_i w_i^{-\theta} \tau_{ni}^{-\theta}}{\sum_j L_j w_j^{-\theta} \tau_{nj}^{-\theta}} w_n L_n,$$

with

$$\tau_{nn}^{-\theta} = \frac{1}{M_n} + \frac{M_n - 1}{M_n}\delta_n^{-\theta}.$$

Given A3 and letting $\Phi \equiv \sum_j M_j w_j^{-\theta} \tau^{-\theta}$, we then have

$$\begin{aligned} w_i M_i &= \frac{w_i^{-\theta}\left(1-\delta^{-\theta}\right) + w_i^{-\theta} M_i \delta^{-\theta}}{\Phi + w_i^{-\theta}\left(1-\delta^{-\theta} + M_i\left(\delta^{-\theta} - \tau^{-\theta}\right)\right)} w_i M_i \\ &\quad + \sum_{n \neq i} \frac{M_i w_i^{-\theta} \tau^{-\theta}}{\Phi + w_n^{-\theta}\left(1-\delta^{-\theta} + M_n\left(\delta^{-\theta} - \tau^{-\theta}\right)\right)} w_n M_n, \end{aligned}$$

and hence,

$$\frac{w_i^{1+\theta}}{\Phi + w_i^{-\theta}\left(1-\delta^{-\theta} + M_i\left(\delta^{-\theta} - \tau^{-\theta}\right)\right)} = \frac{\tau^{-\theta}\Gamma}{\Phi}, \tag{A5}$$

where $\Gamma \equiv \sum_n \frac{w_n M_n}{\Phi + w_n^{-\theta}\left[1-\delta^{-\theta}+M_n\left(\delta^{-\theta}-\tau^{-\theta}\right)\right]}$. Since $\tau > \delta$, then $\delta^{-\theta} > \tau^{-\theta}$, so that the left-hand side is decreasing in M_i and increasing in w_i. This implies that if $M_i > M_j$ then necessarily $w_i > w_j$: larger countries have higher wages. In contrast, if $\tau = \delta$, then the left-hand side is invariant to M_i and hence w must be common across countries.

To compare import shares across countries in a given equilibrium, note that domestic trade shares are given by

$$\lambda_{ii} = \frac{1 + \left(M_i - 1\right)\delta^{-\theta}}{\Phi w_i^{\theta} + 1 - \delta^{-\theta} + M_i\left(\delta^{-\theta} - \tau^{-\theta}\right)}.$$

Plugging this expression into ($A5$) and rearranging yields

$$w_i^{1+\theta}\left(1 - \frac{1-\delta^{-\theta} + M_i\left(\delta^{-\theta} - \tau^{-\theta}\right)}{1 + \left(M_i - 1\right)\delta^{-\theta}}\lambda_{ii}\right) = \tau^{-\theta}\Gamma. \tag{A6}$$

Since $w_i > w_j$ when $M_i > M_j$,

$$\frac{1-\delta^{-\theta}+M_i(\delta^{-\theta}-\tau^{-\theta})}{1-\delta^{-\theta}+M_i\delta^{-\theta}}\lambda_{ii} > \frac{1-\delta^{-\theta}+M_j(\delta^{-\theta}-\tau^{-\theta})}{1-\delta^{-\theta}+M_j\delta^{-\theta}}\lambda_{jj}.$$

But since $\frac{1-\delta^{-\theta}+x(\delta^{-\theta}-\tau^{-\theta})}{1-\delta^{-\theta}+x\delta^{-\theta}}$ is decreasing in x, then $M_i > M_j$ also implies that

$$\frac{1-\delta^{-\theta}+M_i(\delta^{-\theta}-\tau^{-\theta})}{1-\delta^{-\theta}+M_i\delta^{-\theta}} < \frac{1-\delta^{-\theta}+M_j(\delta^{-\theta}-\tau^{-\theta})}{1-\delta^{-\theta}+M_j\delta^{-\theta}},$$

and hence $\lambda_{ii} > \lambda_{jj}$.

For price indices, note that

$$(\gamma P_n)^{-\theta} = \sum_j M_j w_j^{-\theta} \tau_{nj}^{-\theta} = \Phi + w_n^{-\theta} \left(1 - \delta^{-\theta} + M_n \left(\delta^{-\theta} - \tau^{-\theta}\right)\right).$$

Hence, ($A5$) implies that

$$w_n^{1+\theta} P_n^{\theta} = \frac{\gamma^{-\theta} \tau^{-\theta} \Gamma}{\Phi}. \tag{A7}$$

Again, since $w_i > w_j$ when $M_i > M_j$, then $P_i < P_j$. Combining the results for wages and price indices, real wages are also increasing in size. Moreover, if $\tau = \delta$, then the result that wages are the same across countries immediately follows from ($A7$), which also implies that the price index is the same across countries.

D Proof of Proposition 4

The result trivially follows from replacing assumptions A4, A4', and A4'', subsequently, into the expressions in the paper for real wages, trade flows, and price indices in (12), (4) and (5), respectively. The nominal wage follows from multiplying the real wage by the price index.

E Equivalence with Melitz (2003) Model

Assume that productivity draws in each region z_m are from a Pareto distribution with shape parameter θ and lower bound b_m. Replacing the expression for regional trade flows in (1) in the paper into $X_{ni} \equiv \sum_{m \in \Omega_n} \sum_{k \in \Omega_i} x_{mk}$, we get

$$X_{nl} = \sum_{m \in \Omega_n} \sum_{k \in \Omega_l} l_k b_k^{\theta} v_k^{-\theta} d_{mk}^{-\theta} \sum_{k'} l_{k'} b_{k'}^{\theta} v_{k'}^{-\theta} d_{mk'}^{-\theta} x_m.$$

The equivalent of A2 here would be $b_m = b_{m'} = b_n$ for all $m, m' \in \Omega_n$. Replacing, we get

$$X_{nl} = \frac{L_l b_l^{\theta} w_l^{-\theta} \tau_{nl}^{-\theta}}{\sum_j L_j b_j^{\theta} w_j^{-\theta} \tau_{nj}^{-\theta}} X_n,$$

for all n, l, and τ_{nn} defined as in (11). Analogously to the results in Melitz (2003)'s, the productivity cut-off for a region $m \in \Omega_n$ is given by:

$$z_{km}^* = C_0 \left(\frac{f_m}{l_m}\right)^{1/(\sigma-1)} \frac{v_k d_{mk}}{p_m},$$

where C_0 is a constant. Turning to the price index, we get

$$\begin{aligned} P_n^{1-\sigma} &= \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \sum_j \sum_{k\in\Omega_j} l_k \left(v_k d_{mk}\right)^{1-\sigma} \int_{z_{km}^*}^{\infty} z^{\sigma-1} b_k^\theta z^{-\theta-1} dz \\ &= C_1 \sum_j \sum_{k\in\Omega_j} l_k b_k^\theta \left(v_k d_{mk}\right)^{1-\sigma} \left(z_{mk}^*\right)^{\sigma-1-\theta} \\ &= C_1 \sum_j \sum_{k\in\Omega_j} l_k b_k^\theta \left(v_k d_{mk}\right)^{1-\sigma} \left(\left(\frac{f_m}{l_m}\right)^{1/(\sigma-1)} \frac{v_k d_{mk}}{p_m}\right)^{\sigma-1-\theta}, \end{aligned}$$

where C_1 is a constant. Further, assumption A2 in this case also implies that $f_m = f_n$. Hence, for $m \in \Omega_n$, $P_n = p_m$. Replacing and after some algebra, we get

$$\begin{aligned} P_n^{-\theta} &= C_2 \sum_{j\neq n} L_j b_j^\theta \left(w_j \tau_{nj}\right)^{-\theta} \left(\frac{f_n}{L_n/M_n}\right)^{1-\frac{\theta}{\sigma-1}} + C_2 (L_n/M_n) b_n^\theta w_n^{-\theta} \left(\frac{f_n}{L_n/M_n}\right)^{1-\frac{\theta}{\sigma-1}} \left(\left(M_n-1\right)\delta_n^{-\theta}+1\right) \\ &= C_2 \left(\frac{f_n}{L_n/M_n}\right)^{1-\frac{\theta}{\sigma-1}} \sum_j L_j b_j^\theta \left(w_j \tau_{nj}\right)^{-\theta}, \end{aligned}$$

where C_2 is a constant. Thus,

$$\sum_j L_j b_j^\theta \left(w_j \tau_{nj}\right)^{-\theta} = C_2^{-1} P_n^{-\theta} \left(\frac{f_n}{L_n/M_n}\right)^{-[1-\theta/(\sigma-1)]},$$

and hence,

$$\lambda_{nn} = \frac{L_n b_n^\theta w_n^{-\theta} \tau_{nn}^{-\theta}}{C_2^{-1} P_n^{-\theta} \left(\frac{f_n}{L_n/M_n}\right)^{-\left(1-\frac{\theta}{\sigma-1}\right)}},$$

so that the real wage for country n

$$U_n = C_2^{-1/\theta} L_n^{1/\theta} b_n \tau_{nn}^{-1} \lambda_{nn}^{-1/\theta} \left(\frac{f_n}{L_n/M_n}\right)^{1/\theta-1/(\sigma-1)},$$

and

$$\begin{aligned} U_n &= C_2^{-1/\theta} L_n^{1/\theta} b_n \tau_{nn}^{-1} \lambda_{nn}^{-1/\theta} \left(\frac{U_n f_n}{L_n/M_n} \right)^{1/\theta - 1/(\sigma-1)} \\ &= C_2^{-1/\theta} M_n^{1/\theta} (L_n/M_n)^{1/(\sigma-1)} b_n \tau_{nn}^{-1} \lambda_{nn}^{-1/\theta} f_n^{1/\theta - 1/(\sigma-1)}. \end{aligned}$$

Thus, if f_n does not vary with L_n/M_n, the growth rate would be $g_L/(\sigma-1)$. To have the growth rate be g_L/θ, we need to assume that f_n scales up with L_n/M_m proportionally, or $\theta \approx \sigma - 1$, in which case

$$U_n \sim b_n \times L_n^{1/\theta} \times \tau_{nn}^{-1} \times \lambda_{nn}^{-1/\theta}.$$

F Additional Table: Decomposition of the General Model

Table A.1: The Role of Domestic Frictions and Real Wages.

	Real Wage					Gains from trade
	Scale effects	Int' trade	Domestic frictions	Full model	Data	Full model
	(1)	(2)	(3)	(4)	(5)	(6)
Australia	0.49	0.49	0.65	0.66	0.97	1.03
Austria	0.47	0.49	0.60	0.72	1.11	1.26
Benelux	0.81	0.82	0.78	0.89	1.16	1.18
Canada	0.57	0.58	0.71	0.76	0.86	1.11
Switzerland	0.57	0.58	0.64	0.78	0.88	1.30
Denmark	0.34	0.37	0.69	0.81	0.94	1.22
Spain	0.53	0.53	0.62	0.69	1.14	1.12
Finland	0.39	0.42	0.79	0.89	0.84	1.12
France	0.73	0.74	0.80	0.88	1.07	1.14
Great Britain	0.82	0.83	0.85	0.92	1.00	1.09
Germany	0.93	0.94	0.89	0.95	0.92	1.10
Greece	0.35	0.36	0.54	0.61	0.90	1.14
Hungary	0.29	0.31	0.58	0.68	0.65	1.19
Ireland	0.27	0.30	0.54	0.68	1.32	.128
Iceland	0.18	0.22	0.36	0.56	1.17	1.46
Italy	0.59	0.60	0.64	0.69	1.20	1.13
Japan	1.09	1.09	1.15	1.14	0.71	1.02
Korea	0.74	0.75	0.86	0.88	0.63	1.05
Mexico	0.37	0.37	0.38	0.40	0.78	1.11
Norway	0.36	0.39	0.73	0.83	1.11	1.15
New Zealand	0.29	0.29	0.58	0.59	0.74	1.04
Poland	0.58	0.59	0.61	0.68	0.50	1.15
Portugal	0.36	0.37	0.52	0.61	0.97	1.16
Sweden	0.50	0.51	0.68	0.78	0.81	1.19
Turkey	0.38	0.39	0.42	0.46	0.61	1.10
United States	1.00	1.00	1.00	1.00	1.00	1.02
Avg all	0.54	0.55	0.68	0.75	0.92	1.15
Avg 6 smallest	0.30	0.33	0.61	0.73	1.02	1.21
Avg 6 largest	0.86	0.86	0.89	0.93	0.98	1.08

Column 1 refers to the model with only scale effects, column 2 to the model with scale effects and international trade, column 3 to the model with scale effects and domestic trade costs, and column 4 to the model with scale effects, international trade, and domestic trade costs. The real wage in the data (column 5) is the real GDP (PPP-adjusted) per unit of equipped labor. Column 6 shows the gains from trade (i.e. change in the real wage from autarky to the one with the observed trade levels) computed using the calibrated model. All variables are calculated relative to the United States. The six smallest countries (with respect to R&D-adjusted size) are Iceland, Ireland, New Zealand, Finland, Norway, and Denmark, while the six largest countries are Italy, France, Great Britain, Germany, Japan, and the United States.

G Additional Table: Summary Statistics. Data and Model.

Table A.2: Calibrated Model and Data: Summary Statistics.

	Average			Size elasticity
	full sample	6 largest countries	6 smallest countries	
Real Wage				
data	0.92	0.95	1.02	-0.01 (0.03)
no dom.fric.	0.49	0.59	0.38	0.20 (0.01)
full dom.fric	0.76	0.85	0.71	0.13 (0.02)
sym. dom.fric.	0.75	0.75	0.78	0.09 (0.02)
Nominal Wage				
data	0.83	0.91	1.01	0.07 (0.06)
no dom.fric.	0.67	0.72	0.63	0.10 (0.01)
full dom.fric	0.85	0.85	0.88	0.07 (0.02)
sym. dom.fric.	0.82	0.80	0.88	0.06 (0.02)
Price Index				
data	0.88	0.97	1.00	0.07 (0.04)
no dom.fric.	1.45	1.25	1.71	-0.09 (0.01)
full dom.fric	1.09	1.02	1.16	-0.05 (0.01)
sym. dom.fric.	1.11	1.10	1.14	-0.03 (0.01)
Import Share				
data	2.6	1.8	3.4	-0.23 (0.06)
no dom.fric.	10.6	6.0	16.0	-0.39 (0.09)
full dom.fric	4.8	3.7	6.0	-0.28 (0.06)
sym. dom.fric.	3.7	3.6	3.7	-0.15 (0.07)

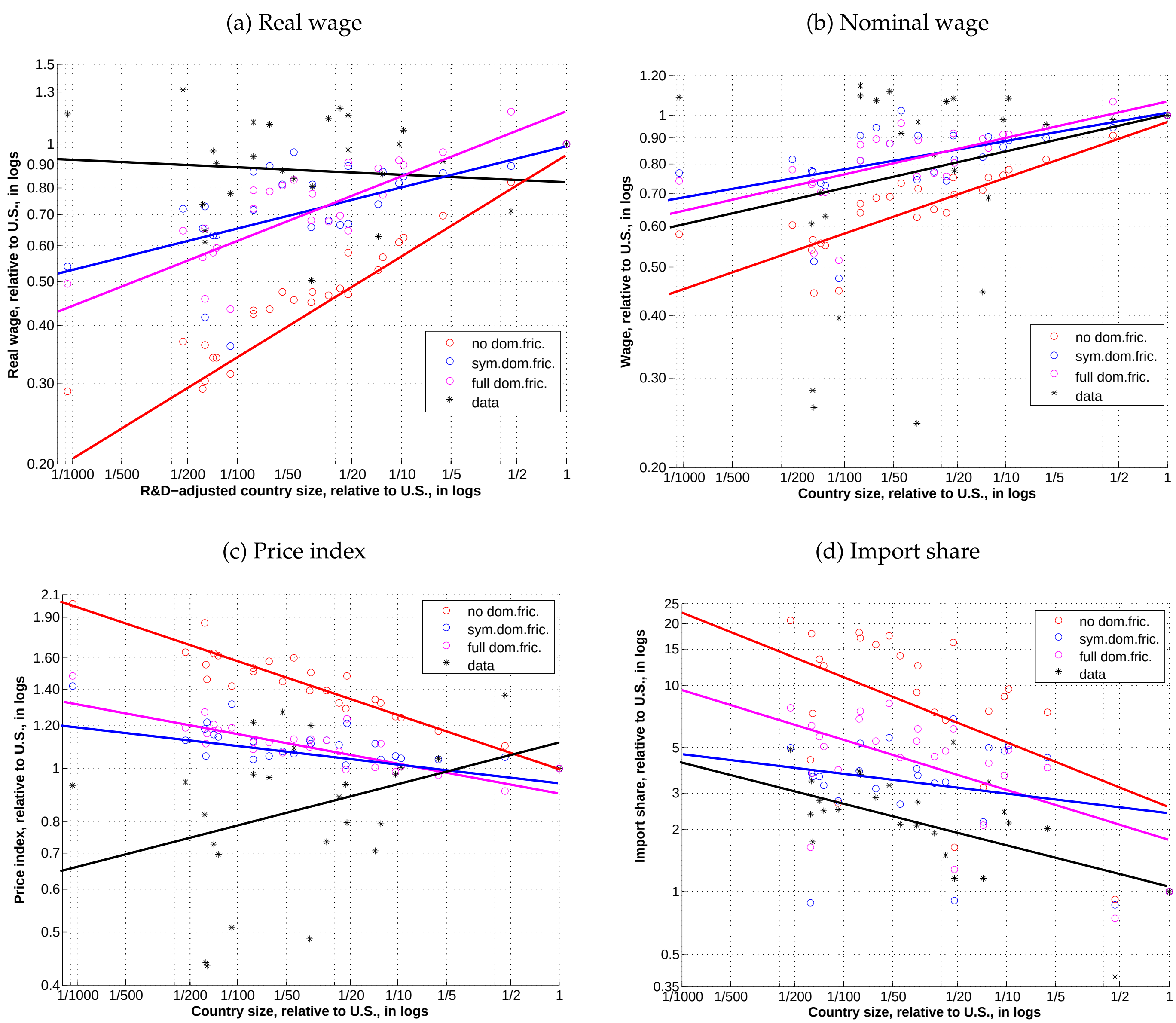
''full dom.fric.", ''sym.dom.fric.", and ''no dom.fric." refer, respectively, to the calibrated full and symmetric model with domestic trade costs, and the model with no domestic trade costs. Variables are calculated relative to the United States. The size elasticity of each variable is from an OLS regressions with a constant and robust standard errors (in parenthesis). The six smallest countries (with respect to R&D-adjusted size) are Iceland, Ireland, New Zealand, Finland, Norway, and Denmark, while the six largest countries are Italy, France, Great Britain, Germany, Japan, and the United States.

H Calibration with Symmetric Regions

As mentioned in the paper, we need to calibrate the matrix of international trade costs to calculate the equilibrium nominal wages, prices, and trade shares, under A2.

We parametrize international trade costs as $\tau_{ni} = \beta_1 dist_{ni}^{\beta_3}$, for $i \neq n$, and $dist_{ni}$ the geographical distance between country i and n (i.e., distance between the most populated cities from *Centre d'Etudes Prospectives et Informations Internationales*). Since the model under A2 delivers country-level gravity, we can directly impose $\beta_3 = 0.27$ and choose β_1 to match the average bilateral international trade share observed in the data, as before. Figure H.1 shows the results for the model with symmetric regions (blue) and compare them with the general model (pink) and the model without domestic trade costs (red); the data are also shown (black).

Figure A.1: Calibrated Model and Data.



‘‘No dom.fric." refers to the model without domestic trade costs; ‘‘sym. dom.fric." refers to the symmetric model with domestic trade costs; ‘‘full dom.fric." refers to the model without A2 but domestic trade costs. In the data: the real wage is computed as real GDP (PPP-adjusted) divided by equipped labor, L_n; the nominal wage is calculated as GDP at current prices divided by equipped labor, L_n; the price index is calculated as the nominal wage divided by the real wage; and import shares refer to total imports, as share of absorption, in the manufacturing sector. R&D-adjusted country size refers to $\phi_n L_n$, where ϕ_n is the share of R&D employment. Solid lines fitted through the dots.