# The Simple Analytics of the Melitz Model in a Small Economy*


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**Abstract**

In this paper we present a version of the Melitz (2003) model for the case of a small economy and summarize its key relationships with the aid of a simple figure. We then use this figure to provide an intuitive analysis of the implications of asymmetric changes in trade barriers and show that a decline in import costs always benefits the liberalizing country. This stands in contrast to variants of the Melitz model with a freely traded (outside) sector, such as Demidova (2008) and Melitz and Ottaviano (2008), where the country that reduces importing trade costs experiences a decline in welfare.



**JEL classification**: F12, F13

**Key words:** firm heterogeneity, small economy, trade liberalization


## 1 Introduction

In this paper we present a version of the Melitz (2003) model for the case of a small economy. We show that unlike the case of the Melitz (2003) setup with large economies, the equilibrium analysis can be carried out with the help of a simple figure that summarizes the key relationships in the model. In particular, we show that the equilibrium can be fully characterized by two conditions that relate the wage with the productivity cut-off for exporters in the small country. First, there is a "competitiveness" condition, according to which a higher wage reduces the country's competitiveness, and this leads to an increase in the productivity cut-off for exporting. Second, there is a "trade balance" condition, according to which an increase in the productivity cut-off for exporting leads to a decline in exports and, hence, a trade deficit. The deficit must be counteracted by a decline in the wage, which increases exports and decreases imports. These two conditions give


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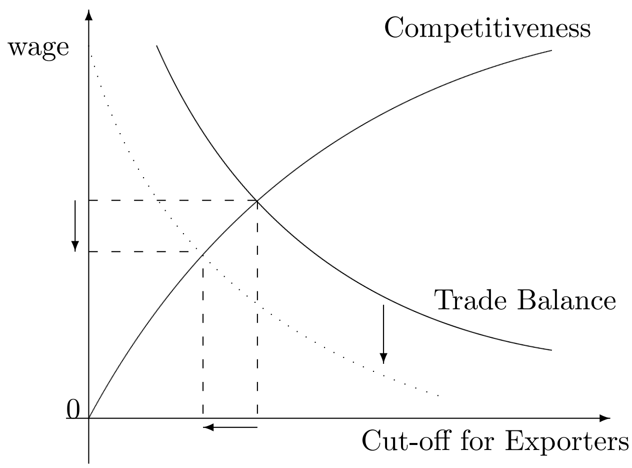
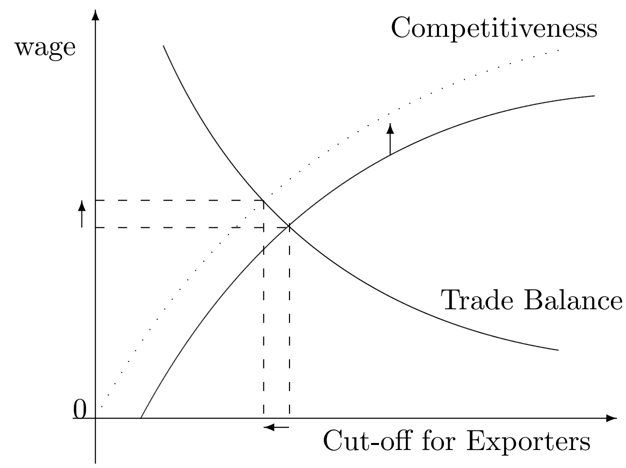
Figure 1: The Equilibrium Conditions




(a) Decline in Costs of Importing




(b) Decline in Costs of Exporting

us two curves, the *competitiveness curve* and the *trade balance curve*, one sloping upwards and one downwards as shown in Figure 1, and their intersection gives the equilibrium.

We illustrate the usefulness of this approach by exploring the implications of asymmetric changes in trade barriers. With the aid of our simple figure, we show that unilateral trade liberalization (i.e., a decline in the variable or fixed cost of importing) by the small economy does not affect the competitiveness curve but it shifts the trade balance curve downwards, since a lower wage is needed to restore trade balance after imports become cheaper. As we see in Figure 1(a), this leads to a decline in the wage and a decline in the productivity cut-off for exporters. The effect on the real wage is unambiguous: we show that welfare always moves in the opposite direction as the productivity cut-off for exporting, hence unilateral trade liberalization increases welfare (i.e., the price index falls by more than the wage).[1] Similarly, a decline in the variable cost of exporting leads to a shift up in the competitiveness curve with no movement in the trade balance curve, implying from Figure 1(b) an increase in the wage and also a decline in the productivity cut-off for exporting. Hence, welfare also increases.

In Section 2 we consider the standard case of two large economies and show that a unilateral trade liberalization by one of these economies shifts both the competitiveness and the trade balance curves down. Changes in the wage and the productivity cut-offs of the liberalizing economy affect the intensity of competition in the other economy, and this is what leads to the shift in the competitiveness curve. Since both curves shift downward, the graphical analysis tells us that the wage must fall but it does not tell us what happens to the cut-off for exports. But this does not mean that the effect is ambiguous: as we establish in Proposition 1, the cut-off for exports falls,

[1] In the text below we show that the free entry condition implies that the productivity cut-offs for domestic production and for exporting move in the opposite directions, and also that, as in Melitz (2003), the productivity cut-off for domestic production is a sufficient statistic for welfare. A direct implication is that a decline in the productivity cut-off for exporting leads to an increase in welfare.

and welfare increases in the liberalizing economy.

The proof of Proposition 1 is somewhat involved, so it is useful to consider simpler scenarios in which one can more easily analyze the effects of unilateral trade liberalization. A common approach in the literature has been to assume the existence of an “outside” sector that pins down the wage (e.g., Grossman, Helpman and Szeidl (2006), Chor (2009), Baldwin and Okubo (2009), and Baldwin and Forslid (2010)). In our graphical analysis this implies that the trade balance curve is a horizontal line determined by productivity, so it does not move with trade liberalization. Since unilateral trade liberalization shifts the competitiveness curve down, the result is an increase in the cut-off for exporters and hence a decline in welfare for the liberalizing country, the result shown in the literature (see Melitz and Ottaviano (2008) and Demidova (2008)). In Section 3 we propose an alternative simplifying assumption, namely that the liberalizing economy is small.[2] In this case, the competitiveness curve does not shift with trade liberalization in the small economy, but the trade balance curve shifts down. Thus, as in Figure 1, the cut-off for exporters decreases, implying gains from unilateral trade liberalization.

Our model is similar to Demidova and Rodríguez-Clare (2009), but here our focus is different: instead of characterizing the optimal policies to deal with the various distortions in the model, we show that the model admits a simple and intuitive analysis of the equilibrium determination and comparative statics.[3] Accordingly, we focus here on the consequences of a reduction in iceberg trade costs rather than tariffs. We emphasize that these two types of frictions have important differences. This is most easily appreciated for the case of a small economy, for which welfare is maximized when iceberg trade costs are eliminated completely whereas a strictly positive tariff is optimal (see Gros, 1987, and Demidova and Rodríguez-Clare, 2009).

## 2 Case of a Large Economy

To demonstrate the advantage of our approach, we will first look at the general Melitz (2003) model of two large but possibly asymmetric economies. We establish that unilateral trade liberalization is welfare improving, but also note that the existence of general equilibrium interactions among the two large economies makes the graphical analysis insufficient in this case.[4]

[2] In the Appendix we show that the case of a small economy can be obtained as the limit of the case with two large economies as one of the economies becomes infinitesimally small.

[3] Our paper is not the first to apply the small economy assumption in the CES/monopolistic competition models. See, for example, Flam and Helpman (1987), who do it in the setting with homogenous firms. However, in Flam and Helpman (1987) wages are pined down exogenously by the presence of a homogenous good sector with constant returns and perfect competition, while we allow for endogenous wages. Another difference with between our paper and Flam and Helpman (1987) is that in their model the price index in the large economy is affected by what happens in the small economy, so their case cannot be seen as the limit of a case with two large countries as one becomes infinitesimally small.

[4] The complexity of the analysis in this setting is caused mainly by firm heterogeneity. In the canonical CES/monopolistic competition case with homogenous firms, the analysis of trade liberalization becomes straightforward, since the number of firms in each economy does not depend on trade costs at all. Hence, to show that both countries gain from unilateral trade liberalization, one only has to look at the trade balance condition to figure out

## 2.1 Model

Consider two countries indexed by $i = 1, 2$ and populated by $L_i$ identical households, each of which has a unit of labor supplied inelastically and earns wage $w_i$. There is a continuum of goods indexed by $\omega \in \Omega$. The representative consumer has Dixit-Stiglitz preferences in each country with elasticity of substitution $\sigma > 1$.

Each country has an (endogenous) measure $M_i^e$ of monopolistically competitive firms that pay a fixed cost $w_i F_i$ to enter the market and draw their random productivity $z$ from the cumulative distribution function $G_i(z)$. Given $z$, a firm from country $i$ faces a cost $w_i/z$ of producing one unit and decides whether to sell in the domestic market and/or export. Firms from $i$ have to use $f_{ij}$ units of labor in country $i$ to export any quantity to country $j$ – this entails a fixed cost $w_i f_{ij}$. Iceberg trade costs are $\tau_{ij} > 1$ so that for a firm in $i$ with productivity $z$ the cost of producing and selling one unit in $j$ is $w_i \tau_{ij}/z$. We assume that $\tau_{ii} = 1$ for $i = 1, 2$.

## 2.2 Characterization of the Equilibrium

Since profits are monotonically increasing in productivity, $z$, there is a productivity cut-off $z_{ij}^*$ such that, among country $i$ firms, only those with a productivity of at least $z_{ij}^*$ decide to sell in market $j$. Letting $\rho \equiv 1 - 1/\sigma$, these cut-offs are defined implicitly by

$$w_j L_j P_j^{\sigma-1} \left(w_i \tau_{ij}/\rho z_{ij}^*\right)^{1-\sigma} = \sigma w_i f_{ij}, \tag{1}$$

where $P_j$ is the price index in country $j$ given by

$$P_j^{1-\sigma} = \sum_{i=1}^{2} M_i^e \int_{z_{ij}^*}^{\infty} \left(\frac{w_i \tau_{ij}}{\rho z}\right)^{1-\sigma} dG_i(z).^5 \tag{2}$$

The free entry condition for firms in country $i$ equalizes the expected profits of entering the market to the entry costs. Following Melitz (2003), we let $J_i(a) \equiv \int_a^\infty \left[\left(\frac{z}{a}\right)^{\sigma-1} - 1\right] dG_i(z)$. Note that (from the definition of cut-offs $z_{ij}^*$) the expected profits for country $i$ firms in country $j$ are $w_i f_{ij} J_i(z_{ij}^*)$. Then the free entry condition in country $i$ is

$$\sum_{j=1}^{2} f_{ij} J_i\left(z_{ij}^*\right) = F_i. \tag{3}$$

Next, let us look at the labor market clearing condition that equalizes total labor demand given by $M_i^e F_i + \sum_{j=1}^{2} L_{ij}$ to labor supply in country $i$, $L_i$, where $L_{ij}$ is (variable and fixed) labor

what happens with the wage, which is the only unknown variable in this case. (The proof can be found in the online Appendix available at the authors' web-sites.)

[5] In establishing these conditions for the cut-offs, we have used four standard results. First, firms set prices equal to unit cost multiplied by the mark-up $1/\rho$. Second, firms' variable profits are revenues divided by $\sigma$. Third, revenues in market $j$ given a price $p$ are $R_j P_j^{\sigma-1} p^{1-\sigma}$, where $R_j$ are total expenditures in $j$. And fourth, $R_j = w_j L_j$, since due to free entry the only source of national income is labor payments.

employed by firms in $i$ to sell to market $j$. Using (1), (3), and the definition of $J_i(z^*_{ij})$, the labor market clearing condition can be written as

$$M_i^e \sigma \sum_{j=1}^{2} f_{ij} \left[ J_i \left( z^*_{ij} \right) + 1 - G_i \left( z^*_{ij} \right) \right] = L_i. \tag{4}$$

Total sales by firms from $i$ in $j$ are $X_{ij} \equiv M_i^e \sigma w_i f_{ij} \int_{z^*_{ij}}^{\infty} \left( z/z^*_{ij} \right)^{\sigma-1} dG_i(z)$, hence

$$X_{ij} = M_i^e \sigma w_i f_{ij} \left[ J_i(z^*_{ij}) + 1 - G_i(z^*_{ij}) \right].$$

Trade balance ($X_{ij} = X_{ji}$) can then be written as

$$M_i^e w_i f_{ij} \left[ J_i(z^*_{ij}) + 1 - G_i(z^*_{ij}) \right] = M_j^e w_j f_{ji} \left[ J_j(z^*_{ji}) + 1 - G_j(z^*_{ji}) \right]. \tag{5}$$

To summarize, there are 10 unknown equilibrium variables: $M_i^e$, $z^*_{ii}$, $z^*_{ij}$, $P_i$, and $w_i$ for $i, j = 1, 2$. We have 9 equilibrium conditions: two free entry conditions, four cut-off conditions, two price index equations, and trade balance. Setting labor in one of the countries as numeraire, we can then use the equilibrium conditions to solve for all the unknown variables.[6]

For future reference, we note here that, as in Melitz (2003), the effect of trade on welfare is completely determined by the behavior of the productivity cut-off for domestic sellers. Free entry implies that there are no profits, so the real wage, $w_i/P_i$, measures welfare per capita in our simple economy. But (1) directly implies that

$$\frac{w_i}{P_i} = \left( \frac{L_i}{\sigma f_{ii}} \right)^{\frac{1}{\sigma-1}} \rho z^*_{ii}.$$

Hence, to know what happens to welfare as a result of trade liberalization, we just need to see what happens to the domestic productivity cut-off, $z^*_{ii}$.

## 2.3 Graphical Analysis

First, let us normalize wage in country 2 to unity, $w_2 \equiv 1$. We now show how to use the equilibrium conditions to re-write 2 equations, the zero profit condition for exporters from country 1 and the trade balance condition as the functions of only 2 unknowns, $w_1$ and $z^*_{12}$. But from (1) we get

$$z^*_{12} = h_{12}(w_1, z^*_{22}) \equiv \tau_{12} \left( \frac{f_{12}}{f_{22}} \right)^{\frac{1}{\sigma-1}} (w_1)^{\frac{1}{\rho}} z^*_{22}, \tag{6}$$

and

$$z^*_{21} = h_{21}(w_1, z^*_{11}) \equiv \tau_{21} \left( \frac{f_{21}}{f_{11}} \right)^{\frac{1}{\sigma-1}} (w_1)^{-\frac{1}{\rho}} z^*_{11}. \tag{7}$$

Furthermore, (3) implies that $z^*_{22}$ can be expressed as a function of $z^*_{21}$, and $z^*_{11}$ can be expressed as a function of $z^*_{12}$. With a slight abuse of notation, we write these two functions as $z^*_{22}(z^*_{21})$ and

[6] As is standard in the literature, we assume that iceberg trade and fixed marketing costs are such that $z^*_{ii} < z^*_{ij}$ for all $i, j = 1, 2$.

$z^*_{11}(z^*_{12})$. Using these functions together with (6) leads to an expression that relates the productivity cut-off for exporting from 1 to 2, $z^*_{12}$, to the wage in country 1, $w_1$,

$$z^*_{12} = \tau_{12} \left( \frac{f_{12}}{f_{22}} \right)^{\frac{1}{\sigma-1}} (w_1)^{\frac{1}{\rho}} z^*_{22} \left( h_{21} \left( w_1, z^*_{11} \left( z^*_{12} \right) \right) \right). \tag{CC}$$

Using again $z^*_{ii}(z^*_{ij})$, from (4) we can express $M^e_i$ as a function of $z^*_{ij}$ only, which we capture by writing $M^e_i(z^*_{ij})$. Then, using (7) to get $z^*_{21}(w_1, z^*_{12}) \equiv h_{21}\left(w_1, z^*_{11}\left(z^*_{12}\right)\right)$, we can re-write the trade-balance condition (5) as an equation in $w_1$ and $z^*_{12}$,

$$\begin{aligned} & M^e_1(z^*_{12}) w_1 f_{12} \left[ J_1\left(z^*_{12}\right) + 1 - G_1(z^*_{12}) \right] \\ = \quad & M^e_2(z^*_{21}(w_1, z^*_{12})) f_{21} \left[ J_2(z^*_{21}(w_1, z^*_{12})) + 1 - G_2(z^*_{21}(w_1, z^*_{12})) \right]. \end{aligned} \tag{TB}$$

This is also an equation in $w_1$ and $z^*_{12}$, which together with Condition CC gives us a system of 2 equations in 2 unknowns. We can prove the following result:

**Lemma 1** ***Condition CC implies a positive relationship between*** $w_1$ ***and*** $z^*_{12}$***, while Condition TB implies a negative relationship between*** $w_1$ ***and*** $z^*_{12}$***.***

**Proof. See the Appendix.**

As shown in Figure 1, Conditions CC and TB give us two curves, the "competitiveness curve" that is sloping upwards (reflecting the negative effect of the wage has on the country's competitiveness, which, in turn, raises the productivity cut-off for exporters) and the "trade balance curve" that is sloping downwards (reflecting the negative effect of the wage on net exports, which must be compensated by a decline in the export cut-off to restore trade balance). Their intersection gives the unique equilibrium values of $w_1$ and $z^*_{12}$.

## 2.4 Unilateral Trade Liberalization

We now explore the effect of unilateral trade liberalization in country 1, which we now call "Home." We refer to country 2 as "Foreign." In particular, we consider a reduction of inward variable and/or fixed trade barriers in Home, $\tau_{21}$ and/or $f_{21}$. This leads to a shift downwards in the CC and TB curves (see the Appendix for the proof). In the case of the competitiveness curve, a fall in import trade barriers in Home encourages additional entry in Foreign and intensifies competition in the Foreign market. Thus, to keep firms with a given productivity indifferent about selling in Foreign, the wage in Home must fall. In the case of the trade balance curve, lower import trade barriers in Home increase Foreign imports, so to restore trade balance for a given exporting productivity cut-off at Home, the wage at Home must fall.

The fact that both curves move down implies that, for the case of unilateral liberalization in a large economy, our graphical analysis does not provide us with the complete description of the new equilibrium. Thus, one needs to go through the complicated mathematical derivations to get the answer. Nevertheless, knowing from our graphical analysis that $w_1$ unambiguously falls with falling import trade barriers significantly helps with the derivations, so we can prove that:

**Proposition 1** *Welfare increases for a country that unilaterally reduces importing trade barriers.*

**Proof. See the Appendix.**

This result stands in sharp contrast to that in Demidova (2008) for the setting with CES preferences and Melitz and Ottaviano (2008) for the setting with linear demand, where lowering import trade barriers reduces welfare. The difference arises from the presence of an "outside" good in these papers and the absence of such a good in the current paper. The presence of an outside good allows for a Home Market Effect (HME) on specialization patterns. Specifically, liberalization in Home makes Foreign a better export base, which results in the additional entry of firms there and a decline in entry of firms in Home. As a result, Foreign specializes in the differentiated good sector while Home specializes in the homogenous good sector. As shown in Venables (1987), this results in a welfare loss in Home. Proposition 1 shows that this result no longer holds when there is no outside good pinning down wages in both countries. Without this good, HME is no longer operative on specialization patterns. Instead, trade liberalization in Home leads to a decline in its relative wage, but this is smaller than the decline in the price index, hence, welfare rises.

Nevertheless, our approach proves to be a useful tool even in models in which an "outside" good pins down the wage as long as there is no complete specialization.[7] In this case the trade balance curve becomes a horizontal line, with the wage determined by productivity levels in the outside sector in the two countries. A reduction in importing barriers by Home (lower $\tau_{21}$ and/or $f_{21}$) shifts down the competitiveness curve while not affecting the horizontal TB curve, resulting in a higher $z_{12}^{*}$, and hence in lower welfare at Home. Similarly, lower exporting barriers (lower $\tau_{12}$ and/or $f_{12}$) shift the competitiveness curve up and increase welfare.

In the next Section we will show how the assumption that the Home country is a small economy used in Demidova and Rodríguez-Clare (2009) helps to significantly simplify the analysis.

# 3 Case of a Small Economy

Here we assume Home (i.e., country 1) can be treated as a small economy. Compared to Section 2, the small economy assumption requires two changes. First, we assume that foreign demand for a domestic variety is given by $Ap^{-\sigma}$. The term $A$ includes both the national income and the price index in Foreign (i.e., country 2). In line with the small economy assumption, $A$ is not affected by changes at Home, i.e., $A$ is exogenous in our small-country setting. Second, the measure $M_2^e$ of monopolistically competitive firms in Foreign is exogenous. However, since $f_{21} > 0$, not all foreign firms sell at Home, so the measure of foreign varieties available at Home is endogenous.

In the Appendix we show that our small economy model can be obtained from the model of two large countries as a limit case, where the share of labor in Home, $n \equiv L_1/(L_1 + L_2)$, goes to zero. Formally, we show that if two large countries are symmetric in everything except for size, and if the

[7] It can be easily shown that if Home becomes small enough relative to the Foreign ecomomy, it completely specializes in the production of the outside good. In that case, our approach, which is based on the existence of the differentiated good sector at Home, is no longer valid.

productivity distribution in both countries is Pareto, then in the limit (as $n \to 0$) we obtain the three key assumptions of the small economy model, namely: (1) the domestic productivity cut-off for firms in Foreign is not affected by changes in Home; (2) the mass of firms in Foreign is not affected by changes in Home; and (3) the demand in Foreign for Home goods exported at the price $p$ can be expressed as $Ap^{-\sigma}$, where $A$ is a constant not affected by changes in Home.

### 3.1 Characterization of the Equilibrium

As before, productivity cut-offs $z_{11}^*$ and $z_{21}^*$ are determined by (1), but $z_{22}^*$ is now taken as exogenous, while $z_{12}^*$ is determined by

$$A\left(w_1\tau_{12}/\rho z_{12}^*\right)^{1-\sigma} = \sigma w_1 f_{12}. \tag{8}$$

In turn, the free entry, labor market clearing, and trade balance conditions at Home remain the same. To summarize, in the case of a small economy, there are 5 unknown variables in the equilibrium, $M_1^e$, $z_{11}^*$, $z_{12}^*$, $z_{21}^*$, and $w_1$, defined implicitly by 5 equilibrium equations: equations (3) and (4) for $i = 1$, and equations (5), (7), and (8).

### 3.2 Graphical Analysis

Next, we show how to reduce the system of 5 equilibrium conditions with 5 unknowns to 2 equations with 2 unknowns, $w_1$ and $z_{12}^*$. The first equation is obtained from (8),

$$z_{12}^* = \tau_{12} f_{12}^{1/(\sigma-1)} w_1^{1/\rho} \left(\sigma/A\right)^{1/(\sigma-1)}/\rho. \tag{CC}$$

Note that this no longer depends on $\tau_{21}$ or $f_{21}$. The reason is that these conditions no longer affect country 2 (Foreign) if country 1 (Home) is small. This will simplify the comparative statics below.

The second equation is the trade balance condition. It is the same as in the case of two large economies except that $M_2^e$ is now exogenous,

$$M_1^e(z_{12}^*)w_1 f_{12}\left[J_1(z_{12}^*) + 1 - G_1(z_{12}^*)\right] = M_2^e f_{21}\left[J_2(z_{21}^*(w_1, z_{12}^*)) + 1 - G_2(z_{21}^*(w_1, z_{12}^*))\right]. \tag{TB}$$

Conditions CC and TB form a system of 2 equations in $w_1$ and $z_{12}^*$. Again, it can be shown that Condition CC implies a positive relationship between $w_1$ and $z_{12}^*$, while Condition TB implies a negative relationship between $w_1$ and $z_{12}^*$. With the same intuition as before, Conditions CC and TB give us two curves, the "competitiveness curve" and the "trade balance curve," as in Figure 1.

### 3.3 Unilateral Trade Liberalization

Consider again a reduction of variable and/or fixed trade barriers for foreign exporters. Unlike the case with a large Home economy, the competitiveness curve is not affected, and as shown in Figure 1(a), only the trade balance curve moves down, implying an unambiguous decline in the equilibrium levels of $w_1$ and $z_{12}^*$. As before, the decline in $z_{12}^*$ implies an increase in $z_{11}^*$ and, hence, an increase in the real wage in Home. The reason that the graphical analysis is now sufficient to establish the result is that the CC curve does not depend on $\tau_{21}$ or $f_{21}$. In turn, this is because if Home is small so there is no feedback from changes in Home to the Foreign demand curve for Home goods.

We can also use this analysis to explore the impact of a reduction in the variable trade costs that Home faces to export goods to Foreign, i.e., a decline in $\tau_{12}$. This causes an upward shift in the competitiveness curve, as shown in Figure 1(b), as a higher wage in Home is required to leave the export cut-off $z_{12}^*$ unchanged when $\tau_{12}$ falls. But there is no shift in the trade balance curve, and hence, we immediately see that the decline in $\tau_{12}$ leads to an increase in Home's wage and a decline in the export cut-off $z_{12}^*$. The latter implies an increase in $z_{11}^*$ and, hence, an increase in Home's real wage. Moreover, it can be shown that the log derivative of the export productivity cut-off $z_{12}^*$ with respect to inward and outward trade barriers $\tau_{12}$ and $\tau_{21}$ is the same[8], i.e., $d\ln z_{12}^*/d\ln\tau_{12} = d\ln z_{12}^*/d\ln\tau_{12}$). Hence, we have the following result resembling the Lerner Symmetry Theorem: in the case of a small economy, a proportional change in import trade costs has the same welfare effect as an (equal-sized) proportional change in outward trade costs.

A decline in the fixed cost of exporting by Home firms to Foreign, $f_{12,}$ is, unfortunately, not simple. Now both the competitiveness and trade balance curves shift with changes in $f_{12}$. Not only does $f_{12}$ directly affect both curves, but it also affects the relationship between $z_{11}^*$ and $z_{12}^*$ implied by (3), i.e., the function $z_{11}^*\left(z_{12}^*\right)$ in Condition TB also depends on $f_{12}$. This makes it very difficult to sign the derivative $dz_{12}^*/df_{12}$.

## 4 Conclusion

The complexity of the Melitz model has led many researchers to make compromises in the analysis of trade liberalization in the presence of monopolistic competition, heterogenous firms, and fixed trade costs. Some have assumed that trade liberalization was symmetric in spite of the fact that liberalization was really asymmetric, often even unilateral. Some have instead added an outside good sector with zero trade costs as a way to fix relative wages, thereby ignoring general equilibrium forces that are important for the welfare analysis. In this paper we proposed an alternative approach that has a long history in the international trade literature, namely, that the country of interest is a small economy. This may miss important feedback effects when liberalization takes place in large economies, but for many cases of interest it provides a useful benchmark. And the analytical benefits are significant – for example, the analysis of unilateral trade liberalization can be done with the help of a simple figure that helps to understand the key forces at play.

[8] To see this, consider Condition TB, which does not depend on $\tau_{12}$ and $\tau_{21}$ directly, but depends on $z_{12}^*$, $z_{11}^*\left(z_{12}^*\right)$, and $z_{21}^*$. The last variable is a function of $z_{12}^*$ and the product of $\tau_{12}\tau_{21}$ (from (6) and (7) the equation for $z_{21}^*$ is $z_{21}^* = \tau_{12}\tau_{21}\left(\frac{f_{21}f_{12}}{f_{11}f_{21}}\right)^{\frac{1}{\sigma-1}}\frac{z_{11}^* z_{22}^*}{z_{12}^*}$, where from (3), $z_{11}^*\left(z_{12}^*\right)$ and $z_{22}^*\left(z_{21}^*\right)$ do not depend on $\tau_{12}$ and $\tau_{21}$.) This means that any change in either $\tau_{12}$, or $\tau_{21}$ that results in the same change in $\tau_{12}\tau_{21}$ has the same effect on $z_{12}^*$.

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# 5 Appendix

## 5.1 Proof of Lemma 1

First, let us look at CC: $z_{12}^* - \tau_{12}\left(\frac{f_{12}}{f_{22}}\right)^{\frac{1}{\sigma-1}}(w_1)^{\frac{1}{\rho}} z_{22}^* = 0$. We need to show that

$$\frac{dw_1}{dz_{12}^*} = -\frac{\partial LHS/\partial z_{12}^*}{\partial LHS/\partial w_1} > 0,$$

where $\partial LHS/\partial z_{12}^* = 1 - \tau_{12}\left(\frac{f_{12}}{f_{22}}\right)^{\frac{1}{\sigma-1}}(w_1)^{\frac{1}{\rho}} \frac{dz_{22}^*}{dz_{21}^*}\frac{dz_{21}^*}{dz_{11}^*}\frac{dz_{11}^*}{dz_{12}^*}$. By using (3) to derive $dz_{ii}^*/dz_{ij}^*$, and (7) to derive $dz_{21}^*/dz_{11}^*$, we get

$$\partial LHS/\partial z_{12}^* = 1 - \left(\frac{f_{12}f_{21}}{f_{11}f_{22}}\tau_{12}\tau_{21}\left(\frac{f_{12}f_{21}}{f_{11}f_{22}}\right)^{\frac{1}{\sigma-1}}\right)^2 \frac{J_1'\left(z_{12}^*\right)J_2'\left(z_{21}^*\right)}{J_1'\left(z_{11}^*\right)J_2'\left(z_{22}^*\right)},$$

where $J_i'(a) = \frac{1-\sigma}{a}\int_a^\infty \left(\frac{\varphi}{a}\right)^{\sigma-1} dG_i(\varphi)$. Using CC and (7), we get

$$\partial LHS/\partial z_{12}^* = 1 - (\tau_{12}\tau_{21})^{2(1-\sigma)} \frac{\int_{z_{12}^*}^\infty \varphi^{\sigma-1}dG_1(\varphi)\int_{z_{21}^*}^\infty \varphi^{\sigma-1}dG_2(\varphi)}{\int_{z_{11}^*}^\infty \varphi^{\sigma-1}dG_1(\varphi)\int_{z_{22}^*}^\infty \varphi^{\sigma-1}dG_2(\varphi)} > 0,$$

since $\tau_{12}\tau_{21} > 1$, $1-\sigma < 0$, $z_{11}^* < z_{12}^*$, and $z_{22}^* < z_{21}^*$. Next, note that

$$\partial LHS/\partial w_1 = -\frac{z_{12}^*}{\rho w_1} - \tau_{12}\left(\frac{f_{12}}{f_{22}}\right)^{\frac{1}{\sigma-1}}(w_1)^{\frac{1}{\rho}}\frac{dz_{22}^*}{dz_{21}^*}\frac{dz_{21}^*}{dw_1} < 0,$$

since from (3) and (7), $dz_{22}^*/dz_{21}^* < 0$ and $dz_{21}^*/dw_1 = -z_{21}^*/\rho w_1 < 0$. Hence, from CC, $dw_1/dz_{12}^* > 0$.

Now let us turn to the TB condition. Using (4) to solve for $M_i^e$, we can rewrite TB as

$$L_2\left(\frac{f_{22}\left[J_2\left(z_{22}^*\right)+1-G_2\left(z_{22}^*\right)\right]}{f_{21}\left[J_2\left(z_{21}^*\right)+1-G_2\left(z_{21}^*\right)\right]}+1\right)^{-1} = w_1 L_1\left(\frac{f_{11}\left[J_1\left(z_{11}^*\right)+1-G_1\left(z_{11}^*\right)\right]}{f_{12}\left[J_1\left(z_{12}^*\right)+1-G_1\left(z_{12}^*\right)\right]}+1\right)^{-1},$$

Letting $\psi_i \equiv \left(f_{ii}\int_{z_{ii}^*}^\infty \left(\varphi/z_{ii}^*\right)^{\sigma-1}dG_i(\varphi)\right)/\left(f_{ij}\int_{z_{ij}^*}^\infty \left(\varphi/z_{ij}^*\right)^{\sigma-1}dG_i(\varphi)\right)$ and solving for $w_1$ from (7) yields

$$\left(\tau_{21}\left(f_{21}/f_{11}\right)^{\frac{1}{\sigma-1}}\right)^\rho \left(z_{21}^*\right)^{-\rho}\left(\psi_2+1\right) = \left(z_{11}^*\right)^{-\rho}\left(\psi_1+1\right). \tag{9}$$

Equation (3) implies that an increase in $z_{ii}^*$ leads to a decline in $z_{ij}^*$ (for $j \neq i$), hence, given the definition of $\psi_i$, we see that $\psi_i$ is decreasing in $z_{ii}^*$. Again, by using (3) for country 1, the RHS of (9) can be written as an increasing function of $z_{12}^*$, while by using (3) for country 2, the LHS of (9) can be written as an increasing function of $z_{21}^*$. It then follows that if $z_{12}^*$ rises, then $z_{21}^*$ must rise as well. From (3) for country 1, $z_{11}^*$ must fall with rising $z_{12}^*$. Hence, we conclude from (7) that $w_1$ should fall when $z_{12}^*$ increases.

## 5.2 Proof of Proposition 1

**Shift in the curves.** First, let us show that for any given $z_{12}^*$, a decrease in $\tau_{21}$ and/or $f_{21}$ shifts down the competitiveness curve. To see this, note that if $z_{12}^*$ is fixed, then from (3), $z_{11}^*$ is fixed as well. But since from CC and (7), $z_{12}^* z_{21}^* = \tau_{12}\tau_{21}\left(f_{12}f_{21}/f_{11}f_{22}\right)^{\frac{1}{\sigma-1}} z_{11}^* z_{22}^*$, $z_{22}^*$ must rise and $z_{21}^*$ must fall (from (3) they move in the opposite directions). Hence, from (6) $w_1$ falls for any fixed $z_{12}^*$.

Now we need to show that for any fixed $w_1$, a decrease in $\tau_{21}$ and/or $f_{21}$ shifts the trade balance curve to the left, i.e., $z_{12}^*$ falls for any given $w_1$. We will make use of the following two equations, which are just a reformulation of (7) and (9):

$$\tau_{21}\left(\frac{f_{21}}{f_{11}}\right)^{\frac{1}{\sigma-1}}\frac{z_{11}^*}{z_{21}^*} = (w_1)^{\frac{1}{\rho}}, \tag{10}$$

$$(\psi_1+1) = w_1(\psi_2+1). \tag{11}$$

We need slightly different argument for the case of a decline in $\tau_{21}$ than for the case of a decline in $f_{21}$. Consider first a decline in $\tau_{21}$. We proceed in two steps: first, we show that $z_{21}^*$ decreases and then we use this to establish that $z_{12}^*$ also decreases.

For the first step, we proceed by contradiction, assuming that $z_{21}^*$ increases given a constant $w_1$. From (10), $z_{11}^*$ must rise as well. However, if $z_{21}^*$ rises, then from (3), $z_{22}^*$ falls, resulting in increasing $\psi_2$, and (from (11)) increasing $\psi_1$, which from (3) implies that $z_{11}^*$ falls, a contradiction. Hence, $z_{21}^*$ falls with a fall in $\tau_{21}$.

Having proved that a decline in $\tau_{21}$ results in a fall in $z_{21}^*$, we now proceed to the second step and prove that $z_{12}^*$ decreases. We proceed as follows. First, by (3) the fall in $z_{21}^*$ leads to an increase in $z_{22}^*$. This leads to a decline in $\psi_2$, which by (11) must be accompanied by a decline in $\psi_1$, implying an increase in $z_{11}^*$ and (again by (3)) a decline in $z_{12}^*$.

Now consider a decline in $f_{21}$. The previous logic does not work here because $f_{21}$ enters the free entry condition (3) for country 2. Consider then a decline in $f_{21}$ and assume by contradiction that $z_{12}^*$ rises. Then from (3) for $i=1$, $z_{11}^*$ falls and, in turn, $\psi_1$ rises so that from (11), $\psi_2$ must rise as well. Moreover, from (10) we see that $(f_{21})^{\frac{1}{\sigma-1}}/z_{21}^*$ must rise with falling $z_{11}^*$, hence, since $f_{21}$ falls, we need to have $z_{21}^*$ decrease. But from the definition of $\psi_2$ we have

$$\psi_2\left((f_{21})^{\frac{1}{\sigma-1}}/z_{21}^*\right)^{\sigma-1}\int_{z_{21}^*}^{\infty}\varphi^{\sigma-1}dG_2(\varphi) = f_{22}\int_{z_{22}^*}^{\infty}(\varphi/z_{22}^*)^{\sigma-1}dG_2(\varphi). \tag{12}$$

Since both $\psi_2$ and $(f_{21})^{\frac{1}{\sigma-1}}/z_{21}^*$ increase, while $z_{21}^*$ falls, then the LHS of (12) must increase, implying that the RHS must also increase, hence, $z_{22}^*$ must fall. Then from (3), $f_{21}J_2(z_{21}^*)$ should fall as well. But note that

$$f_{21}J_2(z_{21}^*) = \left((f_{21})^{\frac{1}{\sigma-1}}/z_{21}^*\right)^{\sigma-1}\left[\int_{z_{21}^*}^{\infty}\varphi^{\sigma-1}dG_2(\varphi) - (z_{21}^*)^{\sigma-1}(1-G(z_{21}^*))\right].$$

We established above that $(f_{21})^{\frac{1}{\sigma-1}}/z_{21}^*$ increases and $z_{21}^*$ falls, hence, the RHS of this expression would increase, implying that $f_{21}J_2(z_{21}^*)$ would increase, a contradiction. Thus, we proved that for any given $w_1$, $z_{12}^*$ falls with a fall in $f_{21}$.

**Welfare change.** We know from Figure 1 that if both curves shift down, $w_1$ falls with a fall in $\tau_{21}$ and/or $f_{21}$. We now prove that $z_{12}^*$ must decrease. We proceed by contradiction. If $z_{12}^*$ increases then then from (3), $z_{11}^*$ falls. This implies that $\psi_1$ increases, and from (11) combined with the decline in $w_1$ we see that $\psi_2$ must increase. At this point, we need slightly different arguments for the case of a decline in $\tau_{21}$ and the case of a decline in $f_{21}$.

Consider first the case of a decline in $\tau_{21}$. The increase in $\psi_2$ implies a decrease in $z_{22}^*$, which combined with the decline in $\tau_{12}$ and $w_1$ implies that the RHS of (6) falls, whereas by the assumption that $z_{12}^*$ increases the LHS of 6) increases, a contradiction.

Now consider the case of a decline in $f_{21}$. Using (6) and (7), we can rewrite $\psi_2$ as

$$\psi_2 = \frac{(\tau_{12}\tau_{21})^{\sigma-1} f_{12} (z_{12}^*)^{1-\sigma} \int_{z_{22}^*}^{\infty} \varphi^{\sigma-1} dG_2(\varphi)}{f_{11} (z_{11}^*)^{1-\sigma} \int_{z_{21}^*}^{\infty} \varphi^{\sigma-1} dG_2(\varphi)}. \tag{13}$$

Assume by contradiction that $z_{12}^*$ increases. From (3) for country 1 then $z_{11}^*$ falls. Since $w_1$ falls, then (6) implies that $z_{22}^*$ must increase. We know that $\psi_2$ increases, so the LHS of (13) must also increase. But since $z_{12}^*$ and $z_{22}^*$ both increase and $z_{11}^*$ falls, the only way that the LHS of (13) can increase is for $z_{21}^*$ to increase. However, $z_{22}^*$ and $z_{21}^*$ cannot rise at the same time, since in this case given a fall in $f_{21}$, the LHS of (3) for country 2, $f_{22}J_2(z_{22}^*) + f_{21}J_2(z_{21}^*)$, falls (recall $J_i(.)$ is decreasing), while its RHS remains constant, leading to a contradiction. Hence, as in the case of a decline in $\tau_{21}$, $z_{12}^*$ falls with a decline in $f_{21}$. Therefore, from (3), $z_{11}^*$ rises, raising welfare at Home.

To simplify the exposition, in the proofs above we have disregarded the possibility that the competitiveness curve does not shift or that $z_{12}^*$ stays constant given a change in $\tau_{21}$ or $f_{21}$. It is straightforward to extend the previous proofs by contradiction to show that this is indeed the case – that is, that neither the competitiveness curve, nor $z_{12}^*$ can stay unchanged. This establishes the strict monotonicity for the result in Proposition 1.

### 5.3 Justification of Small Economy Assumptions

Here we will show that the assumptions we use to treat Home as a small economy can be obtained from the model of two large countries, Home and Foreign, with Home becoming small relative to the Foreign one (the "limit" case). In particular, if two countries are endowed with $n$ and $(1-n)$ shares of the world's labor, $L$,

$$L_1 = nL, \quad L_2 = (1-n)L, \quad n \in [0,1],$$

then the "limit" case we want to explore is the one when $n \to 0$.

To simplify our analysis, we assume that 2 countries are symmetric in everything except for their sizes, i.e., $f_{11} = f_{22} = f$, $f_{12} = f_{21} = f_x$, $F_1 = F_2 = F_e$, $\tau_{12} = \tau_{21} = \tau$. Also, we assume that the productivity distribution in both countries is now specified as Pareto: $G(z) = 1 - \left(\frac{b}{z}\right)^{\beta}$ for $z \geq b$. Then, the free entry condition in country $i$ can be written as

$$(\theta - 1) b^{\beta} \left[ f (z_{ii}^*)^{-\beta} + f_x (z_{ij}^*)^{-\beta} \right] = F_e, \tag{FE}$$

where $\theta = \beta / (\beta - (\sigma - 1))$ . Moreover, from (6) and (7),

$$z_{ij}^* = \tau \left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}} \left(\frac{w_i}{w_j}\right)^{\frac{\sigma}{\sigma-1}} z_{jj}^* \equiv B \left(\frac{w_i}{w_j}\right)^{\frac{\sigma}{\sigma-1}} z_{jj}^*, \quad \text{where } B \equiv \tau \left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}} > 1.$$

Note that by using FE in the definition of $M_i^e$, we get $M_i^e = (\theta - 1) b^\beta L_i / \sigma F_e$. Hence, if we denote $\frac{w_1}{w_2}$ by $w$, then we get the new TB condition:

$$\frac{n}{1-n} = w^{2\beta \frac{\sigma}{\sigma-1} - 1} \left[\frac{z_{11}^*}{z_{22}^*}\right]^{-\beta}. \tag{TB}$$

To summarize, for given $n$, the equilibrium in the model with 2 countries can be described by 2 free entry and 1 trade balance conditions with 3 unknown variables, $z_{11}^*$, $z_{22}^*$, and $w$.

What happens in the model described above when $n \to 0$? Solving FE for $z_{11}^*$ and $z_{22}^*$ gives

$$\left[\frac{z_{11}^*}{z_{22}^*}\right]^{-\beta} = \frac{1 - \frac{f_x}{f} B^{-\beta} w^{-\beta \frac{\sigma}{\sigma-1}}}{1 - \frac{f_x}{f} B^{-\beta} w^{\beta \frac{\sigma}{\sigma-1}}},$$

so that the TB condition can be rewritten as

$$\frac{n}{1-n} = w^{2\beta \frac{\sigma}{\sigma-1} - 1} \frac{1 - \frac{f_x}{f} B^{-\beta} w^{-\beta \frac{\sigma}{\sigma-1}}}{1 - \frac{f_x}{f} B^{-\beta} w^{\beta \frac{\sigma}{\sigma-1}}}. \tag{14}$$

As $n \to 0$, the LHS of (14) goes to 0. Moreover, the RHS of (14) rises with $w$ (here we use the fact that $\frac{f_x}{f} B^{-\beta} < 1$). Hence, as $n$ falls, $w$ falls as well, and when $n \to 0$, the RHS of (14) goes to 0. Note that if $n < 1/2$, then $w < 1$. (If $w > 1$, then from FE, $z_{11}^* < z_{22}^*$. But then in (14), the LHS<1, while the RHS>1, resulting in contradiction.). Thus, the denominator in the RHS of (14) is always positive and bigger than $1 - \frac{f_x}{f} B^{-\beta}$. Hence, as $n \to 0$, we must have $w^{2\beta \frac{\sigma}{\sigma-1} - 1} \left(1 - \frac{f_x}{f} B^{-\beta} w^{-\beta \frac{\sigma}{\sigma-1}}\right) \to 0$. Can $w$ be below $\left[\frac{f_x}{f} B^{-\beta}\right]^{\frac{\sigma-1}{\beta\sigma}}$ for some $n \in (0, 1/2)$? The answer is no, since in this case the RHS of (14) would become negative, while $n/(1-n) > 0$. Thus, as $n \to 0$, then $w$ falls to $\left[\frac{f_x}{f} B^{-\beta}\right]^{\frac{\sigma-1}{\beta\sigma}}$. Moreover, from FE, if $n$ falls, then $z_{22}^*$ falls and $z_{11}^*$ rises.

Note that due to the Pareto distribution assumption, $z_{22}^*$ cannot fall below $b$, the minimum value for $\phi$, but from the solution of FE, it seems that $z_{22}^* \to 0$ as $n \to 0$. How to explain this? The reason is that as $n$ continues to fall, $z_{22}^*$ reaches its minimum so that all foreign firms survive. As $n$ continues to fall, $z_{22}^*$ remains at level $b$, and the zero profit condition for country 2 is violated, so that FE is no longer true for country 2.[9] This also means that we proved assumption (1): productivity cutoff $z_{22}^*$ is not affected by changes at Home, when $n$ is small enough.

Now let us derive the new FE conditions for $n$ small enough so that $z_{22}^* = b$ and $\pi_{22}(z_{22}^*) > 0$. While for Home we have the same FE condition as before, for the Foreign country,

$$\frac{1}{\sigma} L_2 P_2^{\sigma-1} \rho^{\sigma-1} \theta b^{\sigma-1} - f + f_x (\theta - 1) b^\beta (z_{21}^*)^{-\beta} = F_e,$$

[9] Note that this logic also applies to the other types of the productivity distributions.

which from the zero profit condition for exporters from Home can be rewritten as

$$wf_x\left(\frac{w\tau}{z_{12}^*}\right)^{\sigma-1}\theta b^{\sigma-1}-f+f_x\left(\theta-1\right)b^{\beta}\left(z_{21}^*\right)^{-\beta}=F_e. \qquad \text{(New FE)}$$

By using the new FE conditions for small enough $n$, we get

$$M_1^e=\frac{\left(\theta-1\right)b^{\beta}nL}{\sigma F_e},\quad M_2^e=\frac{\left(1-n\right)L}{\sigma\left(F_e+f+b^{\beta}f_x\left(z_{21}^*\right)^{-\beta}\right)},$$

which allows us to rewrite the TB condition as

$$\frac{n}{1-n}=\frac{F_e\left(z_{12}^*/z_{21}^*\right)^{\beta}}{\left(\theta-1\right)b^{\beta}w\left(F_e+f+b^{\beta}f_x\left(z_{21}^*\right)^{-\beta}\right)}.$$

As $n\to 0$, the LHS falls to 0 as well. Since the minimum value for $F_e+f+b^{\beta}f_x\left(z_{21}^*\right)^{-\beta}$ cannot be smaller than $F_e+f$, then for the RHS$\to$ 0, we need $\left(z_{12}^*/z_{21}^*\right)^{\beta}/w\to 0$ as $n\to 0$. Using this property in the new FE condition for country 2, which we can rewrite as

$$f_x\left(\frac{w\tau}{z_{12}^*}\right)^{\sigma-1}\theta b^{\sigma-1}\left(z_{12}^*\right)^{\beta}+f_x\left(\theta-1\right)b^{\beta}\left[\left(z_{12}^*/z_{21}^*\right)^{\beta}/w\right]=\left(F_e+f\right)\frac{\left(z_{12}^*\right)^{\beta}}{w},$$

implies that we can ignore the second term in the LHS above, i.e., for small enough $n$,

$$f_x\left(\frac{w\tau}{z_{12}^*}\right)^{\sigma-1}\theta b^{\sigma-1}\left(z_{12}^*\right)^{\beta}\sim\left(F_e+f\right)\frac{\left(z_{12}^*\right)^{\beta}}{w},\quad\text{or}\quad w^{\sigma}\left(z_{12}^*\right)^{1-\sigma}\sim const.$$

However, from the zero profit condition for exporters from Home, $R_2P_2^{\sigma-1}\propto w^{\sigma}\left(z_{12}^*\right)^{1-\sigma}$. Hence, we proved assumption (3): at some low level of $n$, we can treat $R_2P_2^{\sigma-1}$ as a constant, i.e., the foreign demand for Home goods exported at the price $p$ can be expressed as $Ap^{-\sigma}$. This also means that since for small $n$, $P_2^{1-\sigma}=M_2^e\theta\rho^{\sigma-1}b^{\beta}+M_1^e\theta b^{\beta}\left(\rho/\tau w\right)^{\sigma-1}\left(z_{12}^*\right)^{-\beta+(\sigma-1)}\sim M_2^e\theta\rho^{\sigma-1}b^{\beta}$ (as $L_1$ is very small) and $R_2\sim L$, then treating $R_2P_2^{\sigma-1}$ as a constant implies treating $M_2^e$ as a constant, i.e., we proved assumption (2).