

# Robust New Product Pricing

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## Abstract

We study the pricing decision for a monopolist launching a new innovation. At the time of launch, we assume that the monopolist has incomplete information about the true demand curve. Despite the lack of objective information the firm must set a retail price to maximize total profits. To model this environment, we develop a novel two-period non-Bayesian framework, where the monopolist sets the price in each period based only on a *non-parametric set of all feasible demand curves*. Optimal prices are dynamic as prices in any period allow the firm to learn about demand and improve future pricing decisions. Our main results show that the direction of dynamic introductory prices (versus static) depends on the type of heterogeneity in the market. We find (1) when consumers have homogeneous preferences, introductory dynamic price is higher than the static price (2) when consumers have heterogeneous preferences and the monopolist has no ex-ante information, the introductory dynamic price is the same as the static price and (3) when consumers have heterogeneous preferences and the monopolist has ex-ante information, the introductory dynamic price is lower than the static price. Further, the degree of this initial reduction increases with the amount of ex-ante heterogeneity.

**Keywords:** Non-Bayesian learning, ambiguity, pricing, new products

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*“A Bayesian analysis may be “rational” in the weak axiomatic sense, yet be terrible in a practical sense if an inappropriate prior distribution is used.”*

-Berger [1985] (Statistical Decision Theory and Bayesian Analysis, page 121)

# 1 Introduction

## 1.1 Overview

Consider the pricing decision of a monopolist launching a new non-storable product or technology with unit per-period demand. In order to understand the demand curve, the manager conducts market research, this could be in the form of concept testing (Schwartz [1987]), experiments (e.g. Green and Srinivasan [1978]) or surveys (Dolan [1993]) to assess consumers valuations. These provide the manager with some information about the underlying demand curve for their new product. However, it is unrealistic for the manager to have complete information about the demand curve at the time of launch (e.g., Lodish [1980], Besbes and Zeevi [2009], Kahn [2010], Harrison et al. [2012]). In this paper we investigate how a monopolist should set new product prices with limited pre-launch information.

This question relates to a large literature that studies new product introduction. Most papers in this literature assumes firms have complete information about the underlying demand curve (e.g., Robinson and Lakhani [1975], Wernerfelt [1986], Liu and Zhang [2013] please see appendix for a more complete list of papers), and derive the optimal firm policy.<sup>1</sup> Researchers have recognized that firms are uncertain about the underlying demand curve for new products (e.g., Rothschild [1974], Lodish [1980], Braden and Oren [1994], Desai et al. [2007], Bonatti [2011], Hitsch [2006], Biyalogorsky and Koenigsberg [2014] please see appendix for a more complete list of papers). However, in solving for optimal firm policy, these papers assume that the firm has a prior (represented as full knowledge or a prior distribution) over the uncertainty in the demand curve. For example, Desai et al. [2007] and Biyalogorsky and Koenigsberg [2014] assume that demand can be in one of two states and firms know precisely the probability of each state; alternatively Braden and Oren [1994] and Hitsch [2006] assume that firms have a prior over the possible uncertainty. With this information, the manager

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<sup>1</sup>In this literature the dynamics in new product pricing can be optimal either due to dynamics in demand (e.g., evolving consumer preference and preference heterogeneity) or supply (e.g., inventory concerns and competition).

can use Bayesian decision theory to pick the policy that maximizes expected profits.

The practical reliability of a Bayesian decision theory analysis depends critically on prior assumption used (Manski [2005], Berger [1985]). A critical input to new product pricing decisions is an accurate demand forecast. Survey studies that have tracked the accuracy of forecasts, suggest very large forecasting errors for new products and technologies. Gartner and Thomas [1993] report new product forecast errors vary from -2900% to +1500% with a mean of -46.9%; and Kahn [2002] report a forecast accuracy for new to market products as 40%. This suggests that firms may not always have appropriate priors. The literature on judgment and decision making suggests that one reason for this is that managers may have behavioral biases when selecting priors for new products (Tyebjee [1987], Forlani et al. [2002], Bolton [2003], Bolton and Reed [2004], Schwartz and Cohen [2004], Lawrence et al. [2006]). Kahn [2010] summarizes “it is very unlikely that new-product forecasting will be free of all biases”.

Motivated by the quote by Berger [1985] to start this paper, we develop and investigate a novel dynamic non-Bayesian pricing methodology. Despite the lack of objective information the firm must set a retail price to maximize total profits. To achieve this the retail price must (i) consider current profits and (ii) allow the firm to learn about demand in order to extract higher profits in the future (Rothschild [1974], Grossman et al. [1977], Mirman et al. [1993], more generally labeled as learning by doing Arrow [1962]). Our framework is robust in the sense that the monopolist’s price does not depend on subjective information. Instead, at each point in time the monopolist bases her pricing decision only on the *set* of all feasible demand curves.<sup>2</sup> This kind of uncertainty is also known as *Knightian uncertainty* (Knight [2012]) or *ambiguity*. The prices suggested by our proposed methodology can be used by firms who may not be willing to make a prior assumption, or alternatively provide managers information regarding the implications of the subjective assumptions on pricing decisions.

We present a two-period model where the monopolist prices to a unit mass of consumers. Within this framework, we assume each consumer has unit demand with a constant product valuation over time. We consider two environments where consumers either have homogeneous or heterogeneous preferences. We assume that at the time of purchase the monopolist does not have the information

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<sup>2</sup>For example, if the information available to the firm is that 50% of consumer purchase at \$5 and 10% purchase at \$10, then any downward sloping demand curve that is consistent with these data is a feasible demand curve.

to price discriminate and a set a single price in each period. For example, consider pricing at a retail store. These assumptions imply that consumers are *non-strategic* in the sense that they do not have an incentive to misrepresent their valuation to obtain lower future prices. Importantly if the monopolist has full information about consumer valuations, these demand assumptions would not predict dynamic prices. For the heterogeneous model, we make one additional distinction regarding ex-ante heterogeneity information available to the firm. We assume that firms have access to some known characteristics about consumers (e.g. segmentation, location, age, or income) based on which they can group consumers. Further, based on the pre-launch market research, firms have partial preference information for each group.<sup>3</sup> Aggregating this information across groups of consumers allows the firm to partially identify the set of feasible demand curves (see Handel et al. [2013] for econometric identification and estimation methodology). Importantly, these are restrictions on only on the ex-ante information available to the firm as we allow for full non-parametric heterogeneity in the *true* valuations within each group.

## 1.2 Robust Pricing

To study the firm’s pricing decisions under ambiguity we develop a novel dynamic non-Bayesian framework that simultaneously considers current profitability and the value of learning. The monopolist’s objective function is to maximize aggregate profits. However, she only observes the *set* of all feasible demand curves. Without a subjective prior, the manager cannot integrate over this set to calculate the maximum expected profits. Instead, we assume that the monopolist selects a price in each period using *dynamic minimax regret*, a decision criterion that compares prices based strictly on the set of feasible demand curves. This dynamic decision rule is based on the minimax regret criterion which was introduced by Wald [1950] and has been axiomatized by Milnor [1954] and Stoye [2011].<sup>4</sup>

Minimax regret has been used to characterize robust decision making in a variety of social choice settings (see e.g. Manski [2005]) as well as in firm decisions (see Bergemann and Schlag [2008], Bergemann and Schlag [2011] and Handel et al. [2013] in Economics and Perakis and Roels

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<sup>3</sup>Our notion of observable heterogeneity permits a range of pricing environments. Specifically, if all consumers are individually identifiable our setting is equivalent to one in which the firm has panel data.

<sup>4</sup>As summarized by Schlag [2006] “Minimax Regret is the unique criterion that satisfies Ordering, Symmetry, Strong Domination, Continuity, Column Duplication, Convexity, INABA [IIA] and S [Strategic] Independence ”

[2010], Ball and Queyranem [2009], Besbes and Zeevi [2009] and Besbes and Zeevi [2011] in Operations). In the operations literature, Perakis and Roels [2010], find that the minimax regret approach outperforms traditional heuristics used in the literature. Moreover, the notion of Minimax regret is the foundational building block of popular computer science models such as multi-arm bandit problems and machine learning (see Lai [1987], Bubeck and Cesa-Bianchi [2012]). Minimax regret centers around the notion of *regret*, defined in our environment as the profits foregone by the monopolist from not charging the optimal price for the true demand curve. This notion of regret from the statistical decision literature (e.g. Savage [1951]) is completely distinct from the notion of regret discussed in the psychology literature (Janis and Mann [1977]). Specifically, in the absence of ambiguity, the objective function under minimax regret is exactly the same as the standard expected profit maximization.

Our research objective of dynamic robust pricing is similar to that considered in the operations literature (Besbes and Zeevi [2009]) and the computer science literature (Kleinberg and Leighton [2014]). Both areas consider a continuous time model where consumers enter with a random (Poisson) rate where firms can change prices continuously. The proposed solution in operations (Besbes and Zeevi [2009]) is an algorithm with an experimental stage where the firm can learn about demand and then use minimax regret to select a price. Here the experimental stage is divorced from the profit stage, where as in our setup, we consider the potential profit from the learning stages. In the computer science literature, Kleinberg and Leighton [2014] propose a multi-arm bandit solution (parametric approximations to the minimax regret problem) and show that such an algorithm will converge to the first best solution in finite time.

### 1.3 Contributions

To the best of our knowledge, this is the first work on dynamic pricing from micro-economic foundations that accounts for both (i) setting prices without subjective Bayesian information and (ii) learning about demand through pricing. Our main results show that the monopolist can offer a lower, unchanged or higher introductory price in a dynamic environment (as compared to a static environment) depending on the type of heterogeneity in the market. We find (1) when consumers have homogeneous preferences, introductory dynamic price is higher than the static price (2) when consumers have heterogeneous preferences and the monopolist has no ex-ante information, the in-

introductory dynamic price is the same as the static price and (3) when consumers have heterogeneous preferences and the monopolist has ex-ante information, the introductory dynamic price is lower than the static price. Further the degree of this initial reduction increases with the amount of ex-ante heterogeneity. The difference in results between the homogeneous and heterogeneous consumers models is driven by the fact that the homogeneous consumers model restricts the set of feasible demand functions to be mass points. Here increasing initial price is attractive as consumer valuations must be either above and below the higher price. If we increase price and consumers do not purchase, then reducing future prices significantly decreases future regret. In the heterogeneous consumers model, this is not the case as worst case demand would have consumers with valuations both above and below the first period price. Moreover, under the worst case demand consumers who do not purchase will have lower ex-ante valuations than consumers who do purchase. Here decreasing price allows the firm to bound the maximum regret from these ex-ante lower value consumers and potentially target the ex-ante higher valuation consumers with increasing price in period 2. This result depends critically on ex-ante information. When the firm does not have no ex-ante information, lowering price is no longer valuable.

The remainder of the paper proceeds as follows. Section 2 describes the static model of robust firm pricing. Section 3 presents our main results for the dynamic model of robust firm pricing. Section 4 concludes.

## 2 Static Monopoly Pricing Under Ambiguity

Before studying the monopolist’s dynamic decision problem, we review the benchmark static model for monopoly pricing under ambiguity. This section closely follows previous work by Bergemann and Schlag [2008] and Bergemann and Schlag [2011] on robust monopoly pricing without a subjective prior.<sup>5</sup>

In this section we start with an overview of minimax regret in a static setting. We then solve for the optimal static minimax regret prices in an environment where consumers are homogeneous in

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<sup>5</sup>Bergemann and Schlag study a case where one consumer is drawn from any feasible probability distribution of consumers over a known support of potential valuations. The monopolist has no subjective information concerning the relative likelihood of these feasible distributions, and selects a random pricing rule to minimize her maximum expected regret over this space of uncertainty. We present a simplified version of this model, where the simplification results in a deterministic pricing rule.

their valuations. Finally we consider a model where consumer preferences are heterogeneous. Here we will also allow for heterogeneity in ex-ante information for the firm, whereby firms can have different ex-ante partially identified preferences for consumers.

## 2.1 Overview of static minimax regret

In a static setting where the monopolist knows that true distribution of consumer valuations  $F$  lies within a set of feasible demand curves  $\Psi$ . Regret for any chosen price  $p$  is defined with respect to each  $F \in \Psi$  as defined as:

$$R(p, F) = \text{First Best Profit}(F) - \text{Actual Profit}(p, F)$$

Regret here can be interpreted as the measure of the consumer surplus that the firm is unable to capture. For any  $(p, F)$  this regret will result from overpricing (underpricing) corresponding to the consumers with valuations below (above)  $p$ . In the Bayesian setup, this overpricing and underpricing for each  $(p, F)$  pair is weighted by a subjective Bayesian prior over  $\Psi$  and regret minimization with respect to this weighting is equivalent to expected profit maximization. With no Bayesian prior to weight the space of feasible demand curves, minimax regret evaluates each possible price by its *maximum* regret over the set  $\Psi$ . For any  $p$ , maximum regret occurs at a given  $F_{wc}(p) \in \Psi$  where actual profits under  $p$  are furthest away from the first best profits under  $F_{wc}(p)$ . Intuitively, maximum regret is the worst possible case of foregone profits over the set of feasible demand curves given  $p$ . Once maximum regret is determined for each price, the monopolist solves her problem by selecting the price that minimizes this maximum regret. Thus, minimax regret trades off losses from overpricing with losses from underpricing in a manner that is robust to subjective uncertainty over the set of feasible demand curves. Under the interpretation that regret represents the consumer surplus the firm is unable to capture (or lost consumer surplus), here we can interpret the minimax regret price as price that minimizes the lost surplus for the any distributions of valuations.

## 2.2 Model setup and key informational assumptions

In our model we assume the monopolist sets prices to maximize profits for a zero marginal cost product. In this section we will discuss the information sets and decisions variables we assume for

consumers and the monopolist.

The monopolist faces a unit mass of consumers with unit per period demand. We assume each consumer has a stable valuation  $v_i$  which is known privately only to the consumer. In any purchase occasion, the consumer observes the price set by the firm  $p$  and derives a utility  $u_i = v_i - p$ . She decides to purchase the good if only if  $u_i \geq 0$  or equivalently  $v_i \geq p$ .

We assume that the firm conducts pre-launch market research that provides *partial information* about  $v_i$ . In particular, the only information available to the firm is that  $v_i$  lies between  $v_{iL}$  and  $v_{iH}$ . Where the firm knows  $v_{iL}$  and  $v_{iH}$  and has no prior information about the likelihoods of valuations in this range. In a related paper, Handel et al. [2013] provide an econometric framework that the monopolist can use to estimate  $v_{iL}$  and  $v_{iH}$  from pre-launch market research data (e.g., conjoint studies). For notational convenience, we assume that  $v_{iL} = v_L < \frac{v_{iH}}{4}$  for all consumers.

<sup>6</sup> Therefore, the information available to the firm about consumer valuations can be summarized as  $v_i \in [v_L, v_{iH}] \equiv \delta_i$  for every consumer  $i$ . The firm can aggregate these  $\delta_i$  across consumers to define the set of possible distribution of consumer valuations. We allow for fully non-parametric heterogeneity across consumers. For example, consider a case where the firm is facing two consumers (A and B) and knows  $v_A \in [v_L, v_{AH}]$  and  $v_B \in [v_L, v_{BH}]$ . The set of feasible aggregate valuations is defined as  $\{(v_1, v_2) | v_1 \in [v_L, v_{AH}], v_2 \in [v_L, v_{BH}]\}$ .

The type of uncertainty where the decision maker does not have information to place a subjective distribution over the support of possible outcomes is defined as *ambiguity*. Under ambiguity, it is not possible to calculate expected profits. Instead, the monopolist must make a decision based strictly on knowledge of the support of potential valuations. In this paper, we will derive the optimal prices assuming the firm will use the minimax regret criterion. Here, regret is a statistical characterization of the tradeoff between the potential losses from over pricing and the potential losses from under pricing.

### 2.3 Homogeneous Consumers Model

In this section we will assume that all consumer are homogeneous in their valuations. Under this assumption, we can equivalently consider the monopolist selling to one representative consumer.

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<sup>6</sup>Note our results are robust to this assumption, this assumption allows us to simplify notation (see Lemma 1). In our results, we will find that the optimal minimax regret does not depend on  $v_L$ .



The firm knows that the consumer has valuation  $v \in [v_L, v_H]$  but has no prior information about the likelihoods of these different feasible valuations. The consumer knows her valuation, and will purchase the product if  $v \geq p$ .

In our framework, the monopolist does trade off losses from not making a sale with losses from underpricing by choosing a price that minimizes her maximum regret. Regret is defined as the profit that the monopolist forgoes by pricing sub-optimally given a true valuation. The monopolist's regret is her *first-best* profit, denoted  $\pi^*$  given the resolution of ambiguity minus the profit actually earned in that scenario under the chosen minimax regret price. In our setting, the monopolist's first best profit given true valuation  $v$ , is  $v$ . The actual profit earned, denoted  $\pi(p, v)$  is  $p$  if  $v \geq p$  and 0 otherwise. Thus, regret conditional on  $p$  and true valuation  $v$  is:

$$R(p, v) = \begin{cases} v - p & \text{if } v \geq p \\ v & \text{if } v < p \end{cases}$$

If the monopolist sells the product at  $p$ , regret is how much more she could have earned if charging the consumer's true value, this is the regret of under-pricing. If the monopolist does not sell the product, her regret equals the consumer's value  $v$ , the difference between the zero profit she earned and the maximum amount that she could have earned, this is the regret of over-pricing. The monopolist trades off these two different types of losses by choosing  $p$  to minimize her maximum regret over the entire space of ambiguity. She solves the general problem:

$$MMR = \min_p \max_{v \in [v_L, v_H]} R(p, v) = \min_p \max_{v \in [v_L, v_H]} v - p \mathbf{1}(v \geq p)$$

The following lemma restates the static optimal pricing rule found in Bergemann and Schlag (2006) under deterministic pricing:

**Lemma 1.** *The monopolist's static minimax regret price  $p^* = \frac{v_H}{2}$*

*Proof.* The regret from under-pricing ( $v - p; v \geq p$ ) is maximized when  $v = v_H$ . While the regret from over-pricing ( $v; v < p$ ) is maximized at  $v = p - \epsilon$  for  $\epsilon \rightarrow 0$ . Thus, for a given  $p$ , the maximum regret <sup>7</sup>,  $MR(p) = \max[v_H - p, p - \epsilon] \epsilon \rightarrow 0$ . Since  $v_H - p$  is decreasing in  $p$ , this implies that minimax regret is attained when the regret from over-pricing with that from under-pricing are equal,

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<sup>7</sup>Please note that while this is labeled maximum regret, this is more formally the supremum of regret.

when  $v_H - p = p$  or  $p^* = \frac{v_H}{2}$ .

Note: If we remove the assumption that  $v_L \leq \frac{v_H}{4}$ , then the monopolist's solution,  $p^* = \max[\frac{v_H}{2}, v_L]$ .  $\square$

## 2.4 Heterogeneous Consumers Model

The firm in this setting prices to a continuum of consumers, each of whom is known to have a valuation  $v_i \in [v_L, v_{iH}]$ . Define  $v_{H+} = \text{Max}(v_{iH})$ , or the highest upper bound across consumers. Define  $v_{H-} = \text{Min}(v_{iH})$ , or be the lowest upper bound across consumers. To simplify exposition, we assume  $v_{H-} > \frac{v_{H+}}{2}$ . We assume, that the distribution of  $v_{iH}$  can be described by the continuously differentiable distribution  $G(v_H)$  with bounded density  $g(v_H)$ . Since we assume the firm knows  $v_{iH}$  for every consumer  $i$ , therefore the firm knows  $G(v_H)$  perfectly.

We interpret the ex-ante heterogeneity in preference bounds as arising from a situation where the monopolist has some demographic information to characterize the population of consumers while having only partial information about preferences conditional on known demographic information. We assume that at the time of purchase the firm cannot distinguish between consumers and therefore does not have the required information to price discriminate at the time of purchase.

In order to solve its pricing problem, the firm must contend with ambiguity over the set of feasible demand curves, which can be derived from knowledge about possible sets of valuations for each consumer. Each potential demand curve,  $F(p)$  describes the proportion of buyers who will buy the product at a given price  $p$ . Each  $F(p)$  is a weakly decreasing function mapping the space of feasible valuations to  $[0, 1]$ . We define the set of feasible demand curves,  $\Psi$ , as the set of weakly decreasing functions that satisfy the following restrictions:

$$\Psi \equiv F(p) : \begin{cases} F(p) = 1 & \text{if } p < v_L \\ F(p) \leq 1 - G(p) & \text{if } v_L \leq p \leq v_{H+} \\ F(p) = 0 & \text{if } p > v_{H+} \end{cases}$$

$\Psi$  is the set of all possible true demand curves that the monopolist could be facing given his knowledge of the distribution of consumer valuation supports. For each price  $p$ , as many as  $1 - G(p)$  consumers could have actual values more than  $p$  while it is possible for up to all consumers to have value less than  $p$  so long as  $p > v_L$ .  $\Psi$  is depicted in Figure 1.

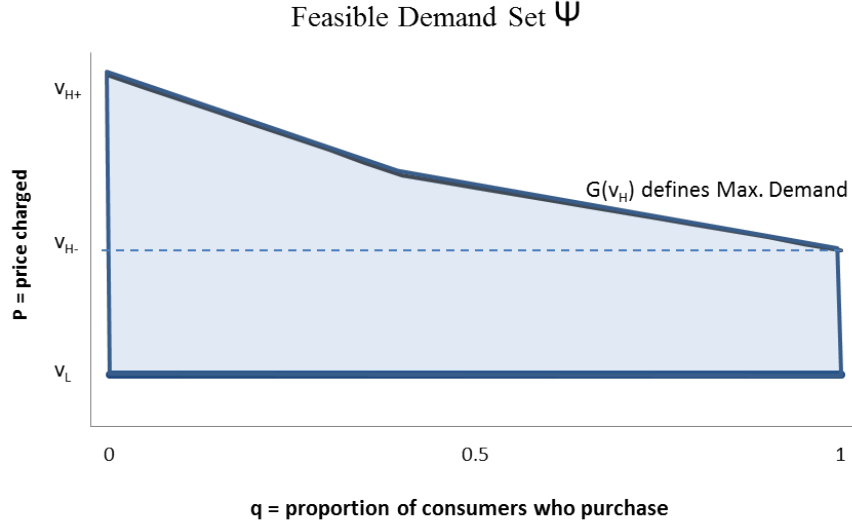


Figure 1: The set of feasible demand curves is every possible downward sloping demand curve that lies in the blue region.

Now, the monopolist chooses  $p$  to minimize her maximum regret relative to the set of feasible demand distributions. We normalize the monopolist's constant marginal cost to zero and assume that she is not capacity constrained.  $\pi^*(F)$  is the first best profit given complete information about  $F(\cdot)$ .<sup>8</sup> This equates to charging all consumers their valuations

$$\pi^*(F) = \int_{v_L}^{v_{H+}} v dF(v)$$

The monopolist's actual profit given price  $p$  and demand distribution  $F(\cdot)$  is:

$$\pi(p, F) = pF(p)$$

Thus, the monopolist's regret for charging price  $p$  with true demand  $F(\cdot)$  is:

$$\begin{aligned} R(p, F) &= \pi^*(F) - \pi(p, F) \\ &= \int_{v_L}^{v_{H+}} v dF(v) - pF(p) \end{aligned}$$

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<sup>8</sup>We note that in the first-best solution we assume that the firm does have the complete information about  $F(\cdot)$  and also observes the consumers' type at the point of purchase. Therefore under the first-best solution the firm can price discrimination.

For a given price  $\hat{p}$ , we define the monopolist's *worst case demand* as the potential demand curve  $F(\cdot) \in \Psi$  that will yield maximum regret. For each price  $\hat{p}$  charged, a different  $F(\cdot)$  can yield maximum possible regret, making worst case regret. More formally:

$$F_{wc}(\hat{p}) = \arg \max_{F(\cdot) \in \Psi} R(\hat{p}, F)$$

$$MR(\hat{p}) = R(\hat{p}, F_{wc}(\hat{p}))$$

The price that the monopolist chooses under ambiguity given the set of feasible demand curves  $\Psi$  solves the minimax regret problem:

$$\min_p \max_{\Psi} R(p, F) \equiv \min_{\hat{p}} R(\hat{p}, F_{wc}(\hat{p}))$$

The monopolist's minimax regret problem is equivalent to the problem where the monopolist minimizes regret under  $F_{wc}(\hat{p})$  with respect to  $\hat{p}$ . Intuitively, for each possible price the monopolist could charge, he considers the worst-case outcome conditional on that price within the set of feasible demand functions. The minimax regret price is the price that minimizes regret under its worst-case demand. Given the constant lower bound  $v_L$  for all  $\delta_i$ ,  $F_{wc}(\hat{p})$  can be found using a threshold value assignment rule that depends on  $v_{iH}$ .  $v_{wc}(v_{iH}, \hat{p})$  is the worst case valuation for specific consumer  $i$  given price  $\hat{p}$ . The following lemma shows (A) how we can determine  $v_{wc}$  as a function of  $p$  and (B) the solution to the monopolist's minimax max regret problem  $\min_p MR(p, v_{wc})$

**Lemma 2.** *A) For  $\hat{p} \leq v_{H-}$ ;  $F_{wc}(\hat{p})$  is composed from potential individual valuations using the cutoff rule:*

$$v_{wc}(v_{iH}, \hat{p}) = \begin{cases} \hat{p} - \epsilon & \text{if } v_{iH} < 2\hat{p} \\ v_{iH} & \text{if } v_{iH} \geq 2\hat{p} \end{cases}$$

*For  $\hat{p} > v_{H-}$ ; we have  $v_{wc}(v_{iH}, \hat{p}) = \max(v_{iH}, \hat{p} - \epsilon)$*

*B) The monopolist's static regret minimizing price is  $\frac{Med(v_H)}{2}$*

*Proof in the Appendix.*

### 3 Dynamic Monopoly Pricing Under Ambiguity

To study the monopolist’s multi-period pricing problem we use a dynamic version of minimax regret that extends the static criterion to account for the value of learning (see Hayashi [2011] for axiomatic foundations). We structure the learning dynamics assuming the monopolist is forward looking and non-Bayesian. Specifically, the monopolist understands how her set of feasible demand curves could be narrowed in each period conditional on **(i)** the price that she charges and **(ii)** the possible purchase quantities she could observe given that price.

In each period, the monopolist evaluates multi-period regret by computing foregone profits over *all* remaining periods for a given price, feasible demand curve, *and* future price.<sup>9</sup> The future price the monopolist considers depends on her decision-making dynamics. We structure decision-making dynamics with the assumption that the monopolist is *sequentially rational*. In our context, sequential rationality means that in each period the monopolist **(i)** dynamically minimizes maximum regret given her current information set and **(ii)** knows that in all subsequent periods she will do the same.<sup>10</sup> This assumption corresponds to the scenario where the monopolist’s management team meets each period to determine current prices and cannot credibly commit to future prices.<sup>11</sup> Thus, when the monopolist computes multi-period maximum regret for a given current price, she endogenizes her future pricing behavior at each possible contingent information set. After computing multi-period maximum regret in this way for each possible price, the monopolist selects the price that minimizes this object.

The dynamic minimax regret solution for an  $N$  period problem (or  $N$  period continuation problem) can be found recursively using backwards induction. In the final period, this criterion reduces to static minimax regret. In the two-period framework, the monopolist solves its static minimax regret pricing problem for all potential second period information sets, and incorporates this information into its dynamic problem in the first period. We assume that for that there is no

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<sup>9</sup>Though the monopolist may learn over time that an initially feasible demand curve is not true demand, from the current perspective the regret calculation with respect to any feasible demand curve presumes that that curve *is* true demand and, as a result, does not need to consider removal of that demand function over time.

<sup>10</sup>Sequential rationality in this single agent problem is similar to the concept of sub-game perfection in dynamic multi-player games.

<sup>11</sup>If the firm commits to a full sequence of prices it will not be able to take advantage of its ability to learn as it will evaluate the entire sequence of prices with respect to only the initially information available. Specifically, it will minimize multi-period maximum regret in period 1, but, in period 2, will be forcing itself to choose a potentially sub-optimal price. As a result, we view the assumption that the firm does not commit, and acts sequentially rational, as an appropriate way to set up dynamic decision-making and learning with the minimax regret criterion.

discount factor, i.e. both period profits are weighted equally in period one. The analytical challenge in analyzing this situation is that the worst case distribution (demand curve that causes the most regret) must be consistent with decision making in each time period.

We will start with the analysis of a homogeneous consumers model and then consider the heterogeneous consumers model.

### 3.1 Dynamic Minimax Regret with Homogeneous Consumers

We introduce dynamic minimax regret in a two period model with homogeneous consumers, here we consider the monopolist selling to one representative consumer. Here the consumer's first period purchase decision provides information to the monopolist that she will incorporate into her second period pricing rule. As in the static problem, we assume at the beginning of the first period that the monopolist knows that the consumer has valuation  $v \in [v_L, v_H]$ . We will denote the firm's first period information set as  $\delta_1$ . After the monopolist sets a first period price  $p_1$ , she observes whether or not the consumer purchases (we , and updates her range of possible valuations. If the consumer purchases, the monopolist knows that the consumer's valuation must be at least  $p_1$ . If the consumer does not purchase, the monopolist knows that the consumer's valuation is lower than  $p_1$ . The monopolist's second period information sets,  $\delta_2$ , under these contingencies are:

$$\delta_2 = [v_L, p_1] \text{ if } v < p_1$$

$$\delta_2 = [p_1, v_H] \text{ if } v \geq p_1$$

Once the monopolist has an updated information  $\delta_2$  at the beginning of the second period, she chooses the second period price  $p_2^*(\delta_2)$  that minimizes maximum regret as described in the static model in section 2.2. In the first period the monopolist minimizes maximum regret over a state space composed of all possible valuations  $\delta_1$  and incorporates the way it will price in the second period conditional on the information set it must have at that point in time, contingent on  $v \in \delta_1$  and  $p_1$ . For any given  $v \in \delta_1$  and price  $p_1$ , there is *only one purchase history* that could be consistent with  $v$ . In other words, if  $v$  is the *true* valuation the monopolist knows in the first period, the information set she will have in period two. The monopolist's multiperiod regret will be the difference between her ideal profit (twice the consumer's actual valuation) and her actual profit summed over both

periods. However, the space of uncertainty which she is concerned with in the first period will include only feasible second period behavior/information if  $v$  is in fact the true valuation. More formally, the firm's multiperiod regret as a function of  $p_1$  and true valuation  $v$  is:

$$R(p_1, v) = 2v - p_1 \mathbf{1}[v \geq p_1] - p_2(\delta_2(p_1, v)) \mathbf{1}[v \geq p_2^*(\delta_2(p_1, v))]$$

The ability to learn enters this formulation through the ability of the monopolist to impact the second period information set with its choice of  $p_1$ . When choosing  $p_1$  the monopolist considers the impact that this choice will have on  $\delta_2$ ,  $p_2(\delta_2)$ , and first period regret. The sequential optimality assumption implies that the monopolist cannot commit to  $p_2$  in period one but does know what she will choose in that period conditional on her information set.

Maximum regret as a function of  $p_1$  is:

$$MR(p_1) = \max_{v \in \delta_1} R(p_1, v) = \max_{v \in \delta_1} 2v - p_1 \mathbf{1}[v \geq p_1] - p_2^*(\delta_2(p_1, v)) \mathbf{1}[v \geq p_2^*(\delta_2(p_1, v))]$$

Given maximum regret conditional on  $p_1$ , the monopolist selects the first period price  $p_1^*$  that minimizes this maximum regret:

$$p_1^* = \arg \min_{p_1} \max_{\delta_1} R(p_1, v) = \arg \min_{p_1} \max_{\delta_1} 2v - p_1 \mathbf{1}[v \geq p_1] - p_2^*(\delta_2(p_1, v)) \mathbf{1}[v \geq p_2^*(\delta_2(p_1, v))]$$

Given this setting we can solve for the monopolists optimal solution. Our main result stated in Theorem 1, where we find that learning will lead the monopolist to increase her introductory price.

**Theorem 1. Higher introductory prices when having homogeneous consumers** *The monopolist facing homogeneous consumers will set a higher introductory price in a dynamic setting relative to the static setting.  $p_1^* = \frac{4v_H}{7} > p^* = \frac{v_H}{2}$ . With  $MR(p_1^*) = \frac{6v_H}{7} = \frac{6}{7}MR(p^*)$ .*

*Proof in the Appendix.*

When we make the model dynamic to incorporate the value of learning under ambiguity, in the one consumer model the monopolist always chooses a first period price that is higher than the optimal price in the static model. If the consumer purchases the price remains at the higher level in the second period, if the consumer does not purchase the price is lowered. In the dynamic framework, the monopolist's minimax regret is  $\frac{1}{7}$  less than it would be applying static minimax

regret in a multiperiod setting, incorporating learning in both settings.

A firm lowering initial price would want to price in that direction only if information learned from the consumer purchasing could later reduce regret coming from high valuation consumers (otherwise aggregate regret could not be lower than repeating the static solution twice). However in this case, lowering initial price provides *no information to change* the second period regret. Consider the case that the consumer has valuation a  $v = v_H$ , if  $p_1^* < \frac{v_H}{2}$  then the consumer will purchase and  $\delta_2 \equiv [p_1^*, v_H]$ . By lemma 1  $p_2^*(\delta_2) = p^*(\delta_1) = \frac{v_H}{2}$ , the static minimax regret pricing rule. The regret in the second period will be  $v_H - p_2^*(\delta_2) = \frac{v_H}{2}$ . This is exactly the same as the static maximum regret.

Conversely, when the firm increases initial price, it learns valuable information that it can use in the contingency of the consumer having a valuation just below the price charged.<sup>12</sup> The ability to learn this information gives the firm flexibility to extract more of the profit if the consumer has a high valuation. Because it knows if the consumer has a low valuation, it can re-optimize in the second period and significantly lower regret. In effect, because there is less total value to be lost from the consumer having a low valuation. Here the firm can first try to extract profits assuming the consumer has a high valuation and if that fails, the firm can lower the second period price.

### 3.2 Dynamic Minimax Regret with Heterogeneous Consumers

In this section, we extend the analysis to consider dynamic minimax regret to heterogeneous consumers. Here each consumer's first period purchase decision provides information to the monopolist that she will incorporate into her second period pricing rule. As in the static problem, we assume at the beginning of the first period that the monopolist knows that each consumer  $i$  has valuation  $v_i \in [v_L, v_{iH}]$ . We will define  $\delta_{i1} \equiv [v_L, v_{iH}]$  as the identified set for consumer  $i$  in time period 1. We describe the first period purchase decisions of each consumer with the binary variable  $b_i = \mathbf{1}[v_i \geq p_1]$ . The monopolist's second period information on each consumer,  $\delta_{i2}$  is obtained by narrowing  $\delta_{i1}$  for

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<sup>12</sup>In the static case, the regret from overpricing is highest when the consumers valuation is just below the price charged



each consumer after the first period conditional on  $b_i$

$$\delta_{i2}(p_1, b_i) \equiv [v_L, p_1] \text{ if } b_i = 0$$

$$\delta_{i2}(p_1, b_i) \equiv [p_1, v_{iH}] \text{ if } b_i = 1$$

Once the monopolist has updated information  $\delta_{i2}$  for each  $i$ , she determines the space of feasible demand curves in period two and sets  $p_2$  as described in the static model of pricing under ambiguity to heterogeneous consumers. In order to derive the set of feasible demand curves contingent on consumer first period purchase decisions, we need to introduce some additional notation. Denote by  $q$  the proportion of consumers that purchased the product in the first period. Define  $G(v_H|b = 1)$  as the distribution of valuation upper bounds in second period conditional on consumers having purchased in the first period. For consumers that purchase, their (a) lower bound of preferences will be  $p_1$  and (b) upper bound will remain the same in period two (when  $p_1 < v_{H-}$ , this will be true from the assumption that  $v_{H-} > \frac{v_{H+}}{2}$ ). Since consumers with different  $v_{iH}$  are identifiable from one another, the monopolist derives the distribution  $G(\cdot|b = 1)$  as follows:

$$G(v_H|b = 1) = \frac{\int_{v_{H-}}^{v_H} g(s|b = 1)ds}{q}$$

A decision not to purchase the product in the first period reduces the upper bound of feasible valuations for consumer  $i$  from  $v_{iH}$  to  $p_1$ , implying that  $G(v_H|b = 0)$  is degenerate with all mass at value  $p_1$ . The firm will maintain the lower bound of  $v_L$  for these consumers. Using these properties, we derive the set of feasible second period demand curves,  $\Psi_2$ , as the set of weakly decreasing functions satisfying the following restrictions (graphically shown in figure 2):

$$\Psi_2 \equiv F_2(p) : \begin{cases} F_2(p) = 1 & \text{if } p < v_L \\ F_2(p) \geq q & \text{if } v_L \leq p < p_1 \\ F_2(p) = q & \text{if } p = p_1 \\ F_2(p) \leq q(1 - G(p|b = 1)) & \text{if } p_1 < p \leq v_{H+} \\ F_2(p) = 0 & \text{if } p > v_{H+} \end{cases}$$

In addition to being a description of the set of feasible demand curves in the second period,

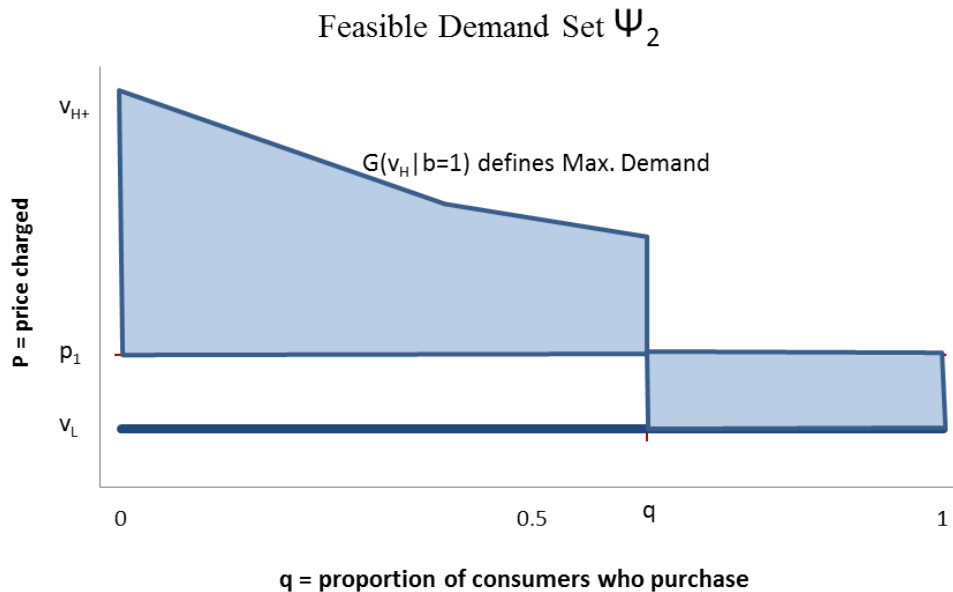


Figure 2: A representation of  $\Psi_2$ . The shaded region represents the space in which all downward sloping demand curves are feasible in period 2. Notice that all demand curves must go through point  $(p_1, q)$  which is the exact price and quantity sold in period one.

we note that  $\Psi_2$  implicitly describes the first period purchase history of consumers. That is, that knowledge of  $\Psi_2$  implies knowledge of  $q$  and  $G(v_H|b = 1)$ .

In the heterogeneous consumers setting the structure of the second period information set is different from that in the static model therefore with dynamic consistency we solve this in two steps. In section 3.2.1 we study second period pricing problem conditional on the information structure  $\Psi_2$ , and use the results from that to study the first period pricing problem in section 3.2.2 where we will derive our main results.

Before presenting our main results we add two regularity assumptions on the distribution  $G(v_H)$ . These assumptions are conservative and ensure that there are no sections of  $G(v_H)$  where there is low density  $g(v_H)$  relative to  $1 - G(v_H)$ , similar to a monotone likelihood ratio assumption. These relationships for the median of  $v_H$  of truncated distributions of  $G(v_H)$  hold for all standard distributions that occur on a bounded interval  $G(\cdot)$  (truncated normal, uniform, etc.). Condition (I) states that the median of successive truncations of  $G$  does not change too quickly as  $q$  changes. Condition (II) states that the median of successive truncations of  $G$  does not change too quickly as the truncation threshold  $x$  changes.<sup>13</sup>

<sup>13</sup>These two assumptions can be mapped directly into one another given a specified relationship between values and

**Assumption 1** (Median Regularity).

$$(I) \quad \frac{\partial MED(v_H|v_H > Q_{1-q})}{\partial q} > -\frac{MED(v_H|v_H > Q_{1-q})}{q} \text{ for } q > 0$$

$$(II) \quad \frac{\partial MED(v_H|v_H > x)}{\partial x} < 1$$

Where  $Q_x$  denotes the  $x$  quantile of  $G(v_H)$ .

### 3.2.1 Second Period Pricing

The second period monopoly pricing problem is conceptually identical to the static pricing problem under ambiguity with consumer heterogeneity described in section 2.3. However, learning from prior period gives the space of feasible demand curves  $\Psi_2$  a different structure in the second period problem. We solve the monopolist's second period problem, considering the two cases of selecting  $p_2 > p_1$  or  $p_2 \leq p_1$ . This is a useful framework because, conditional on choosing  $p_2 > p_1$ , the monopolist essentially ignores consumers who did not purchase in the first period, as the maximum regret for consumers who did not purchase in the first period will be  $p_1$  which does not depend on  $p_2$ . Therefore the prices in the second period are set to minimize maximum second period regret among only consumers who purchased in the first period. Conversely, if  $p_2 < p_1$ , the monopolist sells for sure to all first period purchasers in the second period. However the regret from first period purchasers increases relative to first period as a result of the reduced price. On the other hand, lowering  $p_2$  can only reduce regret for consumers who did not purchase in the first period.

Lemma 3 describes the monopolist's second period solution, conditional on  $p_2$  being either larger or smaller than  $p_1$ . For each feasible  $\Psi_2$  and corresponding  $q$  that could arise conditional on  $p_1$ , the monopolist will choose  $p_2$  to minimize maximum regret across the restricted solutions described in Lemma 3. Specifically, If the minimax regret solution conditional on  $p_2 > p_1$  yields lower (higher) maximum regret than the solution conditional on  $p_2 \leq p_1$ , the monopolist will choose the  $p_2^*$  and face maximum regret found in the solution restricting  $p_2 > p_1$  ( $p_2 \leq p_1$ ). In the solution to the full model, we use these properties of the second period solution as inputs into finding the dynamic

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quantiles of  $G$ . Instead of unifying these assumptions with such a mapping, for exposition and clarity we state both. Lastly, while condition (II) says that the change in the median relative to changes in the truncation value cannot be too large in absolute value, condition (I) makes sure that this derivative with respect to quantiles does not jump too quickly relative to the actual median normalized by the fraction of individuals purchasing.

minimax regret solution.

**Lemma 3.** *I) Conditional on selecting second period price  $p_2 > p_1$ , the monopolist's second period minimax regret price  $p_2^*$  and maximum regret given that price are:*

$$p_2^* = \frac{\text{Med}(v_H|b=1)}{2}$$

$$MR_2 = (1-q)p_1 + q(p_2 G(2p_2|b=1) + \int_{2p_2^*}^{v_{H+}} v_H - p_2 dG(v_H|b=1))$$

*II) Conditional on selecting second period price  $p_2 \leq p_1$ , the monopolist's second period minimax regret price  $p_2^*$  and maximum regret given that price are:*

$$p_2^* = \begin{cases} \frac{p_1}{2} & \text{if } q < 0.5 \\ \in [\frac{p_1}{2}, p_1] & \text{if } q = 0.5 \\ p_1 & \text{if } q > 0.5 \end{cases}$$

$$MR_2 = (1-q)p_2^* + [\int_{p_1}^{v_{H+}} (v_H - p_2) dG(v_H|b=1)]q$$

### 3.2.2 First Period Pricing

When setting a dynamic minimax regret first period price, the monopolist knows what second period price she will charge contingent on any information set  $\Psi_2$ . For each  $p_1$  the monopolist could charge, she knows which  $\Psi_2$  are *possible* in the second period. Furthermore, the monopolist knows that the demand function  $F(\cdot)$  is the same for both periods. This has two main implications for our solution. First, since the worst-case demand function must be the same for both periods, the two periods need to be solved jointly. Second, when evaluating dynamic regret from a given  $F(\cdot) \in \Psi_1$  for a specific  $p_1$ , the monopolist knows exactly what consumer purchase decisions in the first period she will observe under this demand function.

The monopolist's first period problem dynamic minimax regret problem is:

$$\min_{p_1} \max_{F(\cdot) \in \Psi_1} 2\pi^* - \pi_1(p_1, F) - \pi_2(p_2^*(\Psi_2(p_1, F)), F)$$

The monopolist's ideal profit  $2\pi^*$  is still the single-period first best profit given known demand  $F(\cdot)$  earned in each period. The first period profit  $\pi_1$  depends on the first period price and true  $F(\cdot)$ . The

second period profit  $\pi_2$  depends on the second period price, set contingent on the information learned from first period purchase decisions, and true demand. The monopolist minimizes maximum regret dynamically by selecting the price that yields the lowest possible maximum aggregate regret over all feasible demand curves. The knowledge of *how* she will set the second period price impacts the multi-period maximum regret calculation for each first period price. For instance, if the monopolist sets a high initial price and nobody purchases, she knows that she will respond by setting a much lower price in the second period. This implies that from a *first period perspective* maximum multi-period regret from setting a high price may not come from a feasible demand curve that leads to no consumers initially purchasing because second period maximum regret will be low in this contingency.

In order to solve the monopolist's first period dynamic minimax regret pricing problem, we need to find what feasible demand curve will yield the highest maximum regret for a given  $p_1$  conditional on sequentially optimal behavior in the second period. We define worst-case demand in period  $t$  conditional on  $p_t$ :

$$F_{wc}(p_t) = \arg \max_{F(\cdot) \in \Psi_t} R(p_t, F)$$

In the previous subsection we studied the monopolist's second period solution, which depends on  $F_{wc}(p_2)$ .  $F_{wc}(p_2)$  is determined within the context of the second period static minimax regret problem. In this section, we will characterize properties of  $F_{wc}(p_1)$  in order to find the dynamic minimax regret pricing solution  $p_1^*$ . Determining  $F_{wc}(p_1)$  is much more challenging than  $F_{wc}(p_2)$  because it must account for the dynamic price and outcome path engendered by  $p_1$ , not just static outcomes. To analyze  $F_{wc}(p_1)$  we must know (i)  $F_{wc}(p_2)$  for each feasible  $\Psi_2$  and  $p_2$  (ii)  $p_2^*$  conditional on each value of  $(p_1, \Psi_1, F)$  and (iii) what multi-period maximum regret will be for  $p_1$  for each  $F(\cdot)$  given the learning and second period pricing that will occur. Once we know  $F_{wc}(p_1)$  for each  $p_1$ , the monopolist selects the first period price  $p_1$  that minimizes multi-period maximum regret with respect to  $F_{wc}(p_1)$ . In the previous subsection we discussed how to characterize (i) and (ii), while this section uses those results to address (iii) and characterize the dynamic minimax regret first period solution.

We prove our main results in the following steps:

1. **Characterize the the worst case demand function.** We identify a subset of demand curves within  $\Psi_1$  that could be worst case demand, conditional on  $p_1$ . We consider the two cases  $p_1 < p_2^*$  and  $p_1 \geq p_2^*$  separately and for each consider what functions could be the worst case demand functions. This step simplifies our problem by determining the effect of the two possible sequential minimax regret pricing rules on  $F_{wc}(p_1)$ . Restricting  $p_2$  relative to  $p_1$  allows us to narrow the space of feasible  $F_{wc}(p_1)$  to two demand curves conditional on  $p_1$ , which we use to determine which  $p_2$  yields maximum regret. Specifically here we establish the worst case demand curve does not have any weight on valuations who purchase in only one period. Our result characterized formally by the following claim.

**Claim 1.** *I) When  $p_2^*(\Psi_2(F_{wc}(p_1), p_1)) > p_1^*$ ,  $F_{wc}(p_1)$  does not contain consumers who purchase only in the first period. II) When  $p_2^*(\Psi_2(F_{wc}(p_1), p_1)) \leq p_1^*$ , there exists  $F_{wc}(p_1)$  that does not contain consumers who purchase only in the second period. Proof in the Appendix.*

2. **Showing dynamic consistency.** Using the results from section 3.2.1, we show dynamic consistency along the dynamic minimax regret price path. This shows that along the optimal price path, the monopolist prefers the same  $p_2^*$  (i.e. same pricing direction relative to  $p_1$ ) before and after learning the worst case demand in period one. Our result characterized formally by the following lemma.

**Lemma 4.** *Conditional on  $p_1$  and on  $\Psi_2$  consistent with  $F_{wc}(p_1)$  occurring, along the dynamic minimax regret price path, the monopolist would choose  $p_2 = p_2(\Psi_2(F_{wc}(p_1), p_1))$  ex ante. Proof in the Appendix.*

3. **Optimal pricing** We use these results as inputs into the main theorem where we solve for optimal first period prices.

**Theorem 2.** *When  $G$  is non-degenerate  $p_1^* < p^*$ , i.e. the monopolist will lower introductory price in a dynamic setting relative to the static setting. Further, for any  $\Psi_1$ ,  $p_2^*(\Psi_2(F_{wc}(p_1), p_1)) > p_1^*$  along the dynamic minimax regret price, i.e. if a purchase history consistent with worst-case demand is observed, the monopolist will price upwards over time. When  $G$  is degenerate,  $p_1^* = p^* = p_2(\Psi_2(F_{wc}(p_1, p), p_1))$ .*

*Proof in the Appendix.*

This result characterizes two important features of the monopolist's dynamic minimax regret

pricing rule in a setting with arbitrary consumer heterogeneity. First, the ability to learn about demand causes the monopolist to lower introductory price relative to a static setting with consumer heterogeneity. Second, after the introductory price, if the worst possible partial realization of uncertainty occurs through learning from first period purchases, the monopolist will increase her price in the second period.

Intuitively, both these results are true because of the way information learned through first period purchases impacts dynamic maximum regret in an environment with heterogeneity. For any  $p_1$ , the monopolist's worst-case demand will have some consumers who do not purchase with values just below  $p_1$ , and some consumers that do purchase and have values equal to their maximum valuations. Crucially, under worst-case demand, consumers who do not purchase will have low  $v_H$  relative to those who do purchase. Setting  $p_2 > p_1$  allows the monopolist to establish low maximum regret levels from consumers who do not purchase over both periods (low  $v_H$ ), which lets the monopolist then focus exclusively on minimizing maximum regret for high valuation consumers in the second period. Setting  $p_2 < p_1$  does not similarly bound maximum regret from consumers that purchase. When the monopolist lowers price over time she can restrict the valuation of someone who purchases to above  $p_1$ , but when lowering price deterministically increases maximum regret from this set of consumers by  $p_1 - p_2$  for each consumer. This result depends critically on ex-ante consumer heterogeneity. We have shown that lower initial price are valuable when ex-ante observable heterogeneity exists in a dynamic setting with ambiguity and that the monopolist will decrease initial prices. However, where there is no ex-ante heterogeneity ( $G$  is degenerate), lowering initial prices is not valuable because there are no consumers with relatively low  $v_H$  to target with a lower first period price. We derive the the dynamic MMR result with uniform heterogeneity in the following section.

### 3.2.3 Dynamic Pricing with Uniform Heterogeneity

We illustrate the solution to the heterogeneous model by explicitly solving the monopolist's dynamic minimax regret problem when pricing to a mass of heterogeneous consumers with  $v_{iL} = v_L \leq \frac{v_{H-}}{2}$  and  $G(v_H) \rightarrow U[v_{H-}, v_{H+}]$ . This is the case where the highest possible demand function is a straight downward sloping line. This form of demand ambiguity could arise in a setting where the monopolist observes a uniform distribution of an important demographic variable in its target

population (this could be from the pre-launch experiments), and believes that the most someone could value their product is linked directly to that variable.

Given the result of Theorem 2, we know that the monopolist will price charge a lower introductory price when heterogeneity is uniform and that, under a first period purchase history consistent with worst case demand, the monopolist will choose a higher second period price. The following theorem solves explicitly for the monopolist's solution.

**Theorem 3.** *The dynamic minimax regret first period pricing rule for the monopolist under uniform heterogeneity is  $p_1^* = \frac{23v_{H+}+49v_{H-}}{144} < \frac{v_{H-}+v_{H+}}{4} = p_{MMR}$ .  $p_2(\Psi_2(F_{wc}(p_1^*, p), p_1^*)) = \frac{714v_{H+}+294v_{H-}}{2016} > p_{MMR}$ .*

*Proof in the Appendix.*

**Corollary 1.** *For  $G(\cdot) \rightarrow U[v_{H-}, v_{H+}]$  and  $G'(\cdot) \rightarrow U[v_{H-} - \alpha, v_{H+} + \alpha]$ ,  $p_1^*(\Psi_1(G)) > p_1^*(\Psi_1(G'))$  for  $\alpha > 0$ .*

Note that  $p_1^* = p^*$  when  $G$  is degenerate as shown in Theorem 2. Corollary 1 describes how the extent of introductory price reduction relates to the degree of consumer heterogeneity in the population. This result follows directly from knowing that  $p_1^* = \frac{23v_{H+}+49v_{H-}}{144}$ . Therefore as the distribution of ex-ante heterogeneity becomes more dispersed, the monopolist's introductory price decreases.

### 3.2.4 Alternatives to minimax regret

An alternative to minimax regret in the decision theory literature, is to use the maxmin criterion, or set a price that maximizes the firm's minimum possible payoff over the range of possible valuations. Formally  $p^{MM} = \arg \max_p \min_{F(\cdot) \in \Psi} \pi(p, F)$ . In our setting the maxmin price is set to  $v_L$  for the static, dynamic first and second period cases. To see this, consider a distribution where all consumers are of type  $v_L$  (a feasible distribution), any price larger than  $v_L$  will yield zero profit. Therefore the minimum profits for any price higher than  $v_L$  is 0, while the minimum profits for a price  $v_L$  is  $v_L$ . We believe the maxmin criterion is less appealing than minimax regret in a dynamic pricing context for two reasons. First, maxmin focuses only on the potential losses from not selling the product (overpricing) and does not consider the potential losses from foregone profits by selling at a price far lower than the consumer's valuation (underpricing). In contrast, minimax regret explicitly considers the tradeoff between underpricing and from overpricing. Second, unlike dynamic minimax



regret, there is no price experimentation or learning with dynamic maxmin.

In appendix section 8, we derive the optimal Bayesian price assuming an uninformative prior. Explicitly here the decision maker assumes, based on a *subjective prior*, that each consumers valuations  $v_i \sim U[v_{Li}, v_{Hi}]$ . In a dynamic setting the Bayesian decision maker can update her information set based on the price charged ( $p$ ) and purchase decision of the consumer. If consumer  $i$  purchases at a price  $p$ , the monopolist will update her information to  $v_i \sim U[p, v_{Hi}]$ . If consumer  $i$  does not purchase at a price  $p$ , the monopolist will update her information to  $v_i \sim U[v_{Li}, p]$ . We derive the uninformative prior Bayesian price as:

$$p^B = \frac{1}{2} \left( v_L + \frac{v_{H+} - v_{H-}}{\log(v_{H+} - v_L) - \log(v_{H-} - v_L)} \right)$$

We find that the optimal Bayesian price depends on  $v_L$ ,  $v_{H+}$  and  $v_{H-}$ , whereas the dynamic minimax regret price depends only on  $v_{H+}$  and  $v_{H-}$ .<sup>14</sup> Moreover, we show that  $\frac{\partial p^B}{\partial v_L} < 0$  (Appendix section 8). Therefore if the firm gets new data that increases  $v_L$  for all consumers, the Bayesian price derived  $p^B$  will decrease (till  $p^B = v_L$ ). Consider the two time period pricing decisions for a decision maker using Bayesian updating. The implication here is that if the firm sets a price in the first time period ( $p_1^B$ ) and all consumers purchase, the second period price ( $p_2^B$ ) will be equal to  $p_1^B$ . This is unlike the dynamic minimax regret solution where  $p_2^{MMR}$  will be higher than  $p_1^{MMR}$  if all consumer purchase in the first time period.

We will evaluate the implications of these decision criterion on firm profits in the next section.

### 3.2.5 Numerical Example with Uniform Heterogeneity

We present a numerical example that illustrates the solution with uniform heterogeneity. Consider a new product launch for a non-storable good with considerable ambiguity. Assume, the monopolist knows that all consumers could have the same minimal value  $v_L = 2$  and the highest possible valuations  $v_{iH}$  are distributed  $U[10, 18]$  ( $v_{H-} = 10$ ,  $v_{H+} = 18$ ). The static minimax regret price ( $p_{MMR}^*$ ), the first period dynamic minimax regret price ( $p_1^*$ ), the maximin price ( $p^{MM}$ ) and Bayesian

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<sup>14</sup>One of the implications here is that adding heterogeneity in  $v_L$  across consumers will impact the Bayesian price but not the minimax regret price

price with an uninformative prior ( $p^B$ ) are given below

$$\begin{aligned}
p^{MMR} &= \frac{\text{median}(v_H)}{2} = \frac{v_{H+} + v_{H-}}{4} = 7.0 \\
p_1^{MMR} &= \frac{23v_{H+} + 49v_{H-}}{144} = 6.3 \\
p^{MM} &= v_L = 2.0 \\
p^B &= \frac{1}{2} \left( v_L + \frac{v_{H+} - v_{H-}}{\log(v_{H+} - v_L) - \log(v_{H-} - v_L)} \right) = 6.8
\end{aligned}$$

To understand the profit implications of the Minimax regret price, we simulate true preferences from a parametric distribution and compute the resultant firm profits. Note importantly, consistent with our model we assume that the firm only knows that  $v_i \in [v_L, v_{Hi}]$ , and does not know the distribution within this set. We generate consumers' true valuations  $v_i$  from a Beta distribution with parameters  $\alpha$  and  $\beta$  between  $v_L$  and  $v_{Hi}$ .<sup>15</sup> We compare the profit results for the following six prices: (1) static minimax regret price ( $p^{MMR} = 7.0$ ); (2) first period dynamic minimax regret price ( $p_1^{MMR} = 6.3$ ); (3) second period dynamic minimax regret price ( $p_2^{MMR}$ , derived numerically as a function of  $p_1^{MMR}$  and first period purchase decisions); (4) maxmin price, there is a single solution for the static and both periods in the dynamic model ( $p^{MM} = 2.0$ ); (5) static Bayesian price assuming an uninformative prior ( $p^B = 6.8$ ); and (6) second period Bayesian updating with an uninformative prior ( $p_2^B$ , derived numerically as a function of  $p^B$  and first period purchase decisions). We simulate outcomes for three sets of parameters ( $\alpha$  and  $\beta$ ), to see how the shape of the true preferences impacts the resultant profits. For each set of parameters, we simulate preferences for 500,000 consumers.

The results of the simulation are shown in figure 3. The rows of the figure represent the three simulations (labeled A though C) with different parameter ( $\alpha, \beta$ ) values to generate the true preferences ( $v_i \sim \text{Beta}(\alpha, \beta)$  between  $v_L$  and  $v_{Hi}$ ). The first column of the figure plots the distribution of valuations across the population. The second column plots the prices charged based on the different pricing methods described above. The third column plots the realized profits. We will discuss the results for each simulation in detail.

- In simulation A ( $\alpha = 2, \beta = 9$ ), true preference are skewed to the left. In a static setting, the maxmin price ( $p^{MM} = 2.0$ ) ensures all consumers purchase and results in higher profits

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<sup>15</sup>We thank the associate editor for suggesting this simulation.

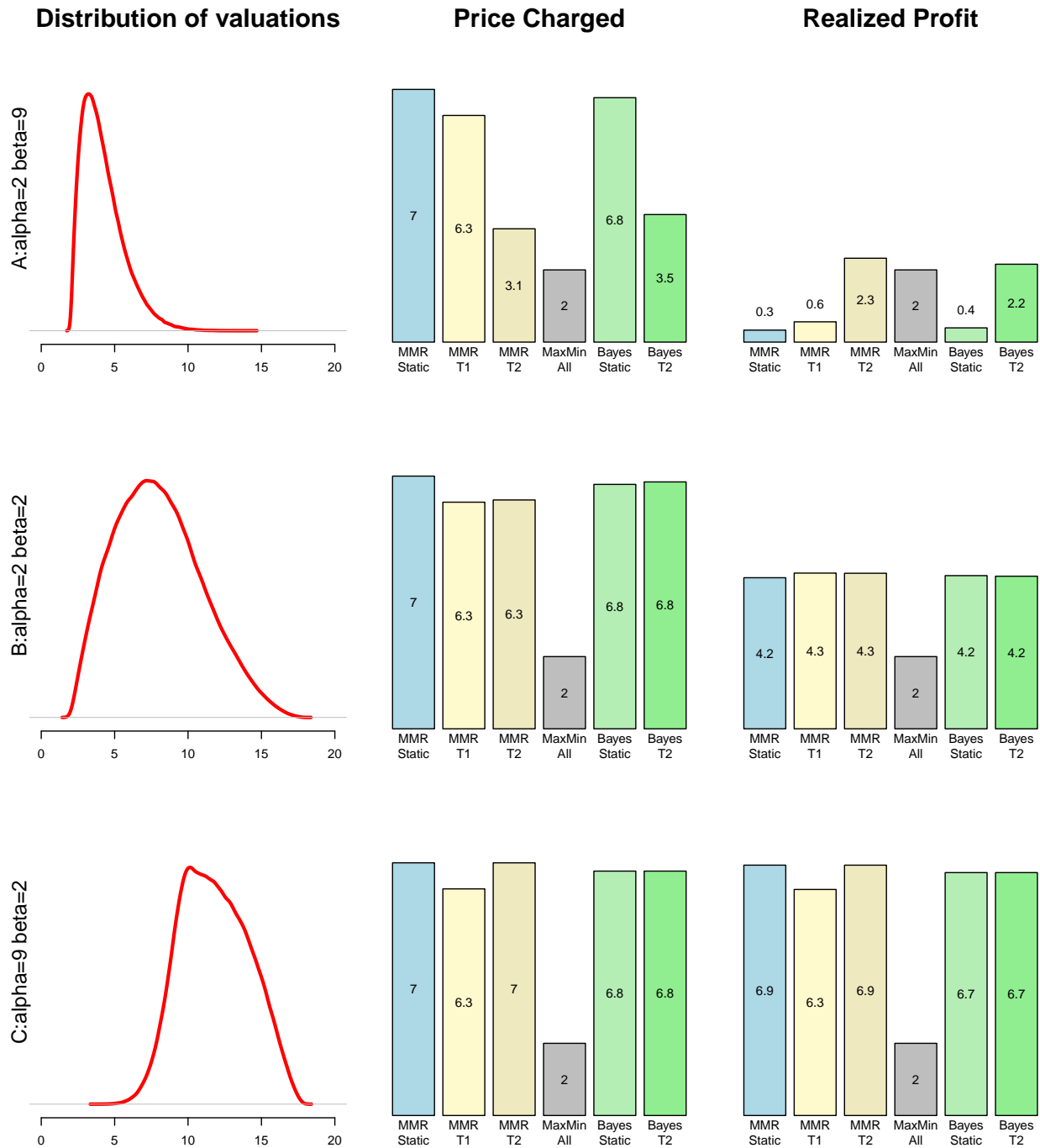


Figure 3: Simulation experiment with the following setting  $v_L = 2$ ;  $v_{Hi} \sim U[10, 18]$ ;  $v_i \sim \text{Beta}(\alpha, \beta)$  between  $v_L$  and  $v_{Hi}$ . We consider three sets of parameters ( $\alpha$  and  $\beta$ ) labeled A through C (rows of the figure above). In each simulation we simulate the true preferences for 500,000 consumers from the assumed distribution. The charts in the first column represents the true distribution of preferences for the simulation. The charts in the second column and third column represent the prices charged and the corresponding realized profits.

than the both the minimax regret price ( $p^{MMR}$ ) and the uninformed Bayesian price ( $p^B$ ). In dynamic setting, under minimax regret the firm observes only 9% of consumers purchase in the first period and lowers price in the second period ( $p_2^{MMR} = \frac{p_1^{MMR}}{2} = 3.1$ ). We find this results in higher profits under the second period dynamic minimax regret price than the maxmin price. With Bayesian updating, the firm will also lower price in the second period ( $p_2^B$ ). We find that the dynamic minimax regret prices result higher profits than the Bayesian updating prices in both time period.

- In simulation B ( $\alpha = 2, \beta = 2$ ), true preference are symmetric. In a static setting, both the minimax regret price ( $p^{MMR}$ ) and the uninformed Bayesian price ( $p^B$ ) result in higher profits than the conservative maxmin price ( $p^{MM}$ ). In the dynamic setting, under minimax regret the firm observes 69% of consumers purchase in the first period and will not change the price in the second period ( $p_2^{MMR} = p_1^{MMR} = 6.3$ ). With Bayesian updating, the firm will also maintain the same price in the second period ( $p_2^B$ ). Again we find that the dynamic minimax regret prices result higher profits than the Bayesian updating prices in both time period.
- In simulation C ( $\alpha = 9, \beta = 2$ ), true preference are skewed to the right. In a static setting, the minimax regret price ( $p^{MMR}$ ) results in higher profits than the uninformed Bayesian price ( $p^B$ ) and the conservative maxmin price ( $p^{MM}$ ). In the dynamic setting, under minimax regret the firm observes nearly all consumers purchase in the first period and will increase the price in the second period ( $p_2^{MMR} = p^{MMR} = 7.0$ ). This is unlike the case with Bayesian updating, where despite high demand, the firm will maintain the same price in the second period ( $p_2^B$ ) (as discussed above). We find that the dynamic minimax regret second period price result higher profits than the Bayesian updating price.

In summary, these simulations highlight the way dynamic minimax regret can learn about demand from the first period and adjust prices in the second period. Across all three simulations, we find that this ability to learn results in higher second period profits under dynamic minimax regret as compared to maxmin and Bayesian updating with an uninformed prior.

Assumption about heterogeneity		Main Result		
Valuations	Ex-ante Information	Static	Dynamic	
		$p^*$	$p_1^*$	$p_2^*$ (under $F_{wc}$ )
No		$\frac{v_H}{2}$	$> p^*$	$p_1^*$ or $\frac{p_1^*}{2}$
Yes	No	$\frac{Median(v_H)}{2}$	$= p^*$	$= p_1^*$
Yes	Yes	$\frac{Median(v_H)}{2}$	$< p^*$	$> p_1^*$

Table 1: Main Results for Robust New Products Prices

### 3.3 Summary of Results

Overall our results suggest that the difference between the first period dynamic price and the static price depends critically on consumer heterogeneity. The main results are characterized in the Table 1. The results show that the role of leaning information varies with heterogeneity in valuations and ex-ante heterogeneity in information. In a setting where all consumers have homogeneous preferences, the monopolist will set high (compared to the static model) period 1 prices, with lowering the price in period 2 if consumers do not purchase. While in a setting with heterogeneity in preferences and ex-ante information, the monopolist will set low (compared to the static model) period 1 prices, and if the worst case demand is realized then will increase prices in period 2. The main difference lies in the fact that the homogeneous consumers model restricts the set of feasible demand functions to be mass points, which alters the intuition in the heterogeneous consumers model. In the homogeneous consumers model, increasing initial price is attractive as consumer valuations must be either above and below the higher price. If we increase price and consumers do not purchase, then reducing future prices significantly decreases future regret. In the heterogeneous consumers model, this would not be the case because worst case demand would have consumers with valuations both above and below  $p_1$ . Moreover, under the worst case demand consumers who do not purchase will have lower ex-ante valuations than consumers who do purchase. Here decreasing price allows the firm to bound the maximum regret from these ex-ante lower value consumers and potentially target the ex-ante higher valuation consumers with increasing price in period 2. This result depends critically on ex-ante observable heterogeneity. When the firm does not have information about ex-ante heterogeneity, lowering price is no longer valuable.

## 4 Conclusion

This paper focuses on how monopolists will price when they face ambiguity about demand, but can reduce that ambiguity over time as they acquire information from consumer purchase decisions. The manager will make pricing decisions in a manner that optimally trades off information learned about demand with current profits in an environment. We show that incorporating learning causes the monopolist to reduce introductory prices and then adaptively price based on the information that is learned. When the first period purchase outcomes are as bad as possible from the monopolist's perspective, she will price upwards over time. We present a novel non-Bayesian framework for studying dynamic decision making. Marketers introducing new products face significant ambiguity when pricing new products. The current literature assumes that the manager uses a Bayesian prior to price in early periods. In contrast, we assert that the manager knows only that each consumer's valuation lies with a range of possible valuations and, has no subjective information on which of these are more likely. An alternative view of our results is to understand the implications of subjective beliefs on pricing decisions.

Our main result show that the monopolists can offer a lower, unchanged or higher introductory price in a dynamic environment (as compared to a static environment) depending on the type of heterogeneity in the market. We find (1) when consumers have homogeneous preferences, introductory dynamic price is higher than the static price (2) when consumers have heterogeneous preferences and the monopolist has no ex-ante information, the introductory dynamic price is the same as the static price and (3) when consumers have heterogeneous preferences and the monopolist has ex-ante information, the introductory dynamic price is lower than the static price. Further the degree of this initial reduction increases with the amount of ex-ante heterogeneity. The extant literature shows that dynamics in pricing are optimal either due to dynamics in demand (e.g., evolving preference) or supply (e.g., inventory constraints). We add a demand learning objective for the firm that can also lead to dynamic optimal prices. While the focus of our paper is on pricing, the methods described in our paper can be applied to other marketing mix instruments (e.g., advertising) where the firm might have incomplete data. We hope future empirical and analytical research will explore robust marketing mix decisions.

In this paper, we acknowledge that we study a stylized problem with three main limitations.

First, while we study a monopoly pricing problem, situations where pricing under ambiguity is relevant might involve multiple firms pricing strategically. Second, our model incorporates firm side learning, while consumer valuations are static over time. In new product settings, consumers might also exhibit preference formation or learning (e.g., Erdem and Keane [1996]). Third, we are restricted to non-storable goods. Considering storable goods a research might consider goods where consumers have incentives to stockpile (Hendel and Nevo [2006]) and goods (e.g., technology) where firms might consider temporal price discrimination (e.g., Nair [2007]).

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