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HEDGING HOUSING RISK

By

Peter Englund
Min Hwang
John M. Quigley

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UNIVERSITY OF CALIFORNIA, BERKELEY

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Peter Englund Min Hwang John M. Quigley[**]

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Abstract: An unusually rich source of data on housing prices in Stockholm is used to analyze the investment implications of housing choices. This empirical analysis derives market-wide price and return series for housing investment during a 13-year period, and it also provides estimates of the individual-specific, idiosyncratic, variation in housing returns. Because the idiosyncratic component follows an autocorrelated process, the analysis of portfolio choice is dependent upon the holding period. We analyze the composition of household investment portfolios containing housing, common stocks, stocks in real estate holding companies, bonds, and t-bills. For short holding periods, the efficient portfolio contains essentially no housing. For longer periods, low risk portfolios contain 15 to 50 percent housing. These results suggest that there are large potential gains from policies or institutions that would permit households to hedge their lumpy investments in housing. We estimate the potential value of hedges in reducing risk to households, yet yielding the same investment returns. The value is surprisingly large, especially to poorer homeowners.

Keywords: Portfolio Risk, House Price Index, Hedging.

JEL Codes: G2, D6, R0

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[**] Englund, Stockholm School of Economics, Peter.Englund@hhs.se, Hwang, University of California, Berkeley, min@econ.berkeley.edu, Quigley, University of California, Berkeley, quigley@econ.berkely.edu

I. *Introduction*

Housing is a major component of household consumption expenditure. The average household in Western Europe and North America spends 25 to 35 per cent of its income on housing, and young homeowners often spend even larger proportions. For a variety of reasons, most housing is owner-occupied. Differences in tax treatment and credit availability contribute to the substantial differences in the fraction of owner-occupied housing across countries. However, apart from these institutional factors, there are other reasons why most households choose to own their home, at least over a part of their life cycles. Homeownership gives the occupants full freedom to tailor the house to their specific tastes and needs. Homeowners also stand to benefit from the consequences of sound decisions about maintenance and renovation. These offer considerable financial advantages (See Sweeney, 1974, for a discussion of the maintenance cost advantages of homeownership.) The advantages of homeownership have to be weighed against the transaction costs of changing homes; these costs are typically larger for homeowners. Less frequent movers will be more likely to prefer homeownership, whereas households with shorter expected durations may be better off renting.

Choosing to own a home is not only a consumption decision. It also entails a portfolio choice. In fact, most homeowners have strongly unbalanced portfolios. This is illustrated in Table 1 based on micro data, the PSID for the United States and the HINK[1] database for Sweden. Despite different institutional environments, the life cycle pattern is relatively similar in these two countries. In both countries the average household invests well above 100 per cent of net wealth in its home up until 50 years of age. The age profile appears to be somewhat steeper for the U.S., where households below 30 years of age invest more than three times their net wealth in owner-occupied housing.

[1] This data set is based on tax returns for a random sample of 10,000 Swedish households. Asset valuations made for tax purposes are translated into approximate market values.

This wealth composition does not seem to be the outcome of an unrestricted portfolio choice. Rather, it suggests that current institutional arrangements do not allow an optimal sharing of the risks associated with homeownership. There is some econometric evidence to suggest that the high risks of homeownership have real consequences. For example, studies by Rosen et al. (1984) and Turner (2000) both conclude that house price volatility discourages homeownership. Turner also finds that high-income households, presumably those with smaller investment shares in housing, are less sensitive to price volatility.

Recently, different proposals have been put forth to improve the possibilities to pool and share these risks. Case, Shiller and Weiss (1993) have proposed a market in futures contracts tied to regional house price indexes, allowing households to hedge by taking short positions in these derivatives contracts. Caplin et al. (1997) have suggested setting up housing partnerships that would allow households to share the risk of owning the dwellings in which they live with other investors. So far, neither of these proposals for hedging and risk sharing has met with much success in practice. This is undoubtedly partly due to legal and practical problems. Setting up housing partnerships would require new legislation, and the attractiveness of a derivatives market in price index futures depends on the quality and integrity of the indexes. These new markets would take time to develop, and despite their immediate appeal, it is not clear to what extent households are adversely affected by their unbalanced investment portfolios.

This paper provides a detailed assessment of the potential gains for homeowners from improved hedging opportunities. Specifically, we investigate the benefits in mean-variance space which arise from introducing opportunities to take positions in a price index for owner-occupied homes. The next section of the paper briefly reviews available evidence on housing returns in a portfolio context. Section III presents the data that our analysis draws on and the methods we use for estimating house price indexes and the returns to investing in an owner-

occupied home. In order to generate long-term returns we estimate a VAR model. This model and the properties of returns at different horizons are discussed in section IV. Further details are reported in Appendix B. In section V we use the variance-covariance structure of returns to derive mean-variance efficient portfolios under a series of conditions. We deal with three issues: the optimal allocation of investment to housing in efficient portfolios unrestricted by consumption motives; the inherent risk in housing chosen only for consumption motives; and the potential for risk reduction which could be afforded by instruments to hedge housing investments. Households that hold a large fraction of their portfolios in their own homes – typically younger and poorer households – pay a high cost for this choice in terms of extra risk. Introducing a market in the housing index leads to a considerable reduction of this risk.

II. *Evidence on housing returns and optimal portfolio choice*

Empirical studies of the quantitative importance of the portfolio-imbalance problem are scarce. Exceptions are papers by Goetzmann (1993), Flavin and Yamashita (1998), Eichholtz et al. (2000), and Gatzlaff (2000).[2] Despite using different methods to measure the returns to housing, all three studies find low correlations between the returns to housing and other assets – Goetzmann uses repeat-sales price indexes for four U.S. cities estimated by Case and Shiller (1990); Gatzlaff uses indexes for 20 MSAs in Florida estimated by similar techniques; Flavin and Yamashita use panel information on the owners' own assessments of house values. The low correlation between housing and other assets suggests that housing should contribute to diversifying the portfolio and lowering the risk. Although the exact specification varies among these three studies, each indicates a portfolio share for housing

[2] See also Devaney and Rayburn (1988).

around fifty per cent for the minimum variance portfolio.[3] At the riskier end of the efficient frontier, results differ more across studies, not surprisingly since this portion of the frontier is much more sensitive to estimation errors (see, for example, Jorion, 1985).

For a standard portfolio choice problem, the holding period of the investment is not a major concern. Because most asset returns are reasonably well described by random walk processes, their variances and covariances over n periods are approximately equal to n times their one-period counterparts. The solution to a portfolio-choice problem based on quarterly returns is thus almost identical to the solution based on multi-period returns. Despite the recent findings of high-frequency positive autocorrelation and long-term mean reversion in stock prices, the random walk assumption remains a reasonable approximation for most asset returns. Housing is a major exception for two reasons. First, index returns exhibit positive autocorrelation for many markets; see Case and Shiller (1989) for U.S. cities and Englund and Ioannides (1996) for international comparative data. Second, houses are heterogeneous, as are the conditions of sale. Thus, there is a strong idiosyncratic component to the return from investing in an individual house. The importance of the idiosyncratic component can be expected to diminish over time in relation to the price index uncertainty. For this reason, and since transaction costs are important, the assumed holding period (the investment horizon) may be quite important in analyzing a portfolio choice problem in which housing is one of the assets. Goetzmann considers the impact of the holding period. He finds that the two aspects tend to have offsetting effects on the riskiness of housing. The annualized standard deviation of the index-based return tends to increase with the holding period, but the impact of the idiosyncratic component decreases. On balance, according to Goetzmann's study, the holding period does not appear to be a major concern.

[3] Goetzmann derives the efficient frontier based on four assets all with non-negativity constraints. Flavin and Yamashita impose non-negativity constraints on all four assets but also include mortgage loans constrained only

The study by Eichholtz et al. (2000) focuses on the potential of bonds and common stocks as hedges against the risks of homeownership. Interestingly, they find that the demand for stocks and bonds in an optimal portfolio is not significantly affected by homeownership, implying that neither asset provides a good hedge for housing. This suggests that one has to look towards instruments more directly geared at housing returns in order to find good hedges.

III. *Data and Methods*

Research on this issue has been hampered by the lack of reliable time-series data on housing prices and housing returns. In this paper, we draw upon a body of data consisting of observations on *all sales of one-family houses* in Sweden from January 1, 1981 through August 31, 1993. These data that have been used to estimate rather precise quarterly house price indexes for eight major Swedish regions; see Englund et al. (1998) and Quigley and Redfearn (1999). This data series is much shorter than comparable price series that could be used to estimate stock returns. To focus on longer-term returns, this poses special problems. We address those by estimating a vector autoregression system, and we use the estimated VAR model to generate the returns to housing investment over the longer term. The data set on housing includes every arm's length sale of owner occupied housing in Sweden. Contract data reporting the transaction price for each sale have been merged with tax assessment records containing detailed information about the characteristics of each house. Repeat sales are identified, as is the location of each unit down to the smallest geographical unit, the parish (akin to a census tract). The data set is exceptional in its detailed description of each dwelling at the date of sale and its identification of the panel nature of sales of the same property.

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by house value. Gatzlaff includes commercial real estate and REITs among the assets, again with non-negativity constraints.

Assume that the sale price of a housing unit is the product of an index representing the level of services emitted by the unit and an index of price per quality unit. To represent this, suppose

$$V_{it} = P_t + Q_{it} + \omega_{it}. \qquad (1)$$

where V_{it} is the logarithm of the observed selling price of house i at time t, Q_{it} is the log of the quality of house i sold at time t, P_t is the log of the constant quality housing price index at time t, and ω_{it} is a random error reflecting idiosyncratic aspects of a particular transaction, e.g., a "distressed" sale. According to (1), each house emits a quality of service Q_{it} that is priced at P_t at a particular point in time. Q_{it}.

We assume that this composite term, i.e. the idiosyncratic error net of the individual-specific error, is autocorrelated:

$$\varepsilon_{it} = \rho^{t-\tau}\varepsilon_{i,\,t-\tau} + \upsilon_{it}, \qquad (5)$$

where

$$E(\xi_i) = 0 \qquad E(\xi_i^2) = \sigma_\xi^2 \qquad (6)$$

$$E(\upsilon_{it}) = 0 \qquad E(\upsilon_{it}^2) = \sigma_\upsilon^2.$$

The panel nature of the data identifies the key parameters: the price index P_t, the autoregressive term ρ, and the error variances. The methods of estimating these parameters are discussed in Englund, et al. (1998). Appendix Table A1 reports the coefficient estimates of the price series and standard errors for eight regions in Sweden. The price indices are very precisely estimated. The indexes are estimated monthly and aggregated to quarter year intervals in our analysis.

The estimated price indexes for all the eight regions identified in Appendix Table A.1 are depicted in Figure 1. As the diagrams indicate, the course of housing returns across the different regions of Sweden during this period has been fairly well synchronized. Prices were stagnant in nominal until 1985, when they started to increase. House prices reached a peak in 1991 followed by a sharp drop, as a major financial crisis hit Sweden during the early 1990s. Stockholm has the highest and most volatile housing prices.

The return from investing in an owner-occupied home consists of three components: the rate of change of the housing price index, *P*, the rental value of the service flow generated by the housing unit (net of operating costs and depreciation), and the rate of change of the idiosyncratic component of the house price. We impute a rental value using the index of rents for residential apartments in each region, i.e. we assume the apartment index to be valid for

one-family houses as well.[4] We set the rent in the first quarter of 1981 at 1 per cent of house value. This follows the widely used one-in-one-hundred rule.[5] This gives us two return series. The housing index return, r^H, is given by

$$r_t^H = P_t - P_{t-1} + 0.01 \cdot P_t, \quad (7)$$

And the return on an individual housing unit, r^h, by

$$r_t^h = r_t^H + \varepsilon_{it} - \varepsilon_{it-1}. \quad (8)$$

Housing returns are highly correlated across all eight regions, indicating that there are only small diversification benefits from holding a multi-regional housing portfolio within the country. This contrasts with the United States where the benefits from regional diversification are considerable; see Goetzmann (1993). For this reason, we limit ourselves to including the Stockholm housing market in the portfolio analysis. Figure 2A depicts the temporal pattern of the real quarterly return on Stockholm housing defined according to (7) as the weighted change in the monthly price index (see Appendix Table A1) aggregated to quarters plus the quarterly rental service stream minus the change in CPI.

Following Goetzmann (1993) we include general stocks (the AFGX index, produced by a leading business periodical), five-year bonds and three-month treasury bills among the investment alternatives. In order to highlight the opportunities for hedging using currently traded instruments, we also include an index for real estate corporations traded on the Stockholm stock exchange. This index covers a group of companies whose main source of income comes from real estate holdings (office and residential). To varying degrees they also have other lines of business, primarily in construction. In the absence of REITs, this is the

[4] This is the only consistent rent series available, and it is based upon comprehensive data. However, use of this index is problematic since apartment rents are regulated, with the objective of following production costs. Our use of this index probably leads to an underestimate of the short-term variation in the value of rental services. However, the variation in rents should in any case be small relative to the variation in the value of the stock, so this is probably not very serious.

[5] See Kain and Quigley, 1975, for an early statement.

most natural vehicle for investing in real estate for a Swedish investor. Figures 2A and 2B report the real returns on these assets which could be used to form an investment portfolio.

The data cover a dramatic period in Swedish economic history.[6] The 1980s was a decade of major asset revaluations, as seen by the high returns to stocks throughout the decade, and to homes, during the second half of the decade. During the 1980s the Stockholm stock exchange outperformed all major stock markets in industrialized countries. The development of asset prices can be explained by the deregulation of credit markets around 1985 and by an expansionary fiscal policy. Returns to fixed-income instruments were also extraordinary. Since the parity of the krona was maintained at a fixed level after a devaluation in 1982 despite the fact that inflation was higher in Sweden than abroad, the currency had to be defended by high interest rates. The early 1990s saw an end to the asset-price boom with a sharp drop in prices, particularly for real estate, in 1991 and 1992. This was associated with a banking crisis, with total credit losses between 1990 and 1993 on the order of 12 per cent of one year's GDP. A general economic crisis persisted and GDP fell for three consecutive years. In the fall of 1992 the Swedish krona was allowed to float, resulting in a depreciation by twenty per cent by the end of the year. From 1994 the Swedish economy started to recover. The recovery period is not part of our data, however.

IV. *Investment Returns*

Since most households take a long term perspective on their home investment, it is important to analyze the returns to housing over different investment horizons. But with only 13 years of data, we cannot observe long-horizon variances and covariances directly. To circumvent this problem, we assume that returns are generated from a fourth order vector

[6] See Englund (1999) for an account of the events leading up to the banking crisis.

autoregression system on a quarterly basis in the five asset returns, and we estimate the parameters of this system by the method of seemingly unrelated regressions. Once the parameters for the VAR system are estimated, it is fairly straightforward to compute moments over long horizons; see Appendix B for details. The estimated model is presented in Appendix Table B1. The model explains much of the variation in housing returns (R^2adj = 0.72) but, as expected, it works less well for financial assets. Tests for Granger-causality fail to reject the null hypothesis of no causality. Exceptions are the returns to bonds and houses; both are predicted by stock returns, the general index as well as the real estate stock index. Interestingly, high returns to real estate stocks predict high housing returns, while high general stock returns predict low housing returns. Closely related findings, that the returns to securitized real estate help predict the returns to direct investment in individual properties, have been reported for commercial real estate. See Barkham and Geltner (1995).

The first two moments of investment returns at different horizons, based on the estimated VAR parameters, are presented in Tables 2 and 3. As noted in Table 2, expected real returns are generally quite high, ranging from 1.3 per cent per quarter for t-bills and homes to 3.7 per cent for the general stock index, reflecting the strong performance of Swedish asset prices discussed above. The lower panel of Table 2 displays variances at different horizons. To maintain comparability, the table presents long-horizon variances on a per quarter basis, i.e. the variance of n-quarter returns divided by n. In terms of variance, housing returns are considerably riskier than nominal assets, but are less risky than stocks. Real estate stocks have 2-4 times the variance of stocks in general. In comparing the different horizons, we note that the ten quarter horizon displays higher variances for all assets than any other horizons. Generally variances are lower at the 40-quarter than at the one-quarter horizon; the housing index is the only exception.

For housing, we distinguish between index returns and the returns to an individual house, as defined in (7) and (8). We note that the difference is sizeable for short holding periods. At the one-period horizon, the variance of the housing index return equals that of bonds, whereas the variance of the return to an individual home is six times as large. At longer horizons it is less than twice as large. Yet, even at the 40-quarter horizon, the individual home is a relatively risky investment, with a variance half that of common stocks and eight times that of bonds.

Table 3 reports simple correlation coefficients between the investment vehicles for the different time horizons. The returns to housing are positively correlated with real-estate stocks and negatively correlated with t-bills and bonds. All correlations with housing are stronger at longer horizons. The correlation with the general stock index is close to zero. Our results may be compared with those of Goetzmann (1993) and Gatzlaff (2000), both of which apply to one-year horizons. Each of these studies finds a negative correlation between housing and bond returns (-0.54 and -0.23), a small correlation with t-bills (-0.22 and +0.19), and a small negative correlation with the S&P 500 stock index. Only Gatzlaff considers securitized real estate, surprisingly finding a negative correlation (-0.13). Our results confirm the general conclusion from these studies that the correlations between housing and other key assets are sufficiently low as to make housing a potentially attractive addition to a portfolio, at least at lower risk levels. Whether this warrants the observed portfolio shares of a hundred per cent or more is an issue discussed in the next section.

V. *Optimal Portfolios*

We now use the structure of this information to construct mean-variance efficient portfolios. We focus on three sets of issues: the optimal allocation of investment to housing in

efficient portfolios unrestricted by consumption motives; the inherent risk in housing chosen only for consumption motives; and the potential for risk reduction which could be afforded by instruments to hedge housing investments.

V.1. *Unrestricted Portfolios*

We start by considering a benchmark where the amount of housing is chosen freely by mean-variance optimization. We analyze two cases; both include the four financial assets, with housing represented either by a single house or by the housing index (Table 4 and 5, and Figure 3). Portfolio shares, except for a single house, are only restricted to be between plus and minus 500 per cent. The share of a singe house is restricted to be non-negative, reflecting the fact that it is difficult in practice to short-sell individual houses. The benchmark cases are not intended to capture real life investment opportunities, but rather to bring out the implications of the return patterns in the data for portfolio choice. We illustrate the solutions to this problem for two horizons, one quarter and 40 quarters.[7] From Table 4 we see that the optimal portfolio share in an individual house is close to zero in the minimum-variance portfolio at both horizons. Moving out along the frontier, it increases at first, for the one-quarter horizon only modestly to at most 15 per cent, but for the longer horizon more sharply to more than 100 per cent. With further increased risk, the optimal portfolio share in an individual house decreases to become zero at high risk levels. The minimum variance portfolio invests more than a hundred per cent in t-bills and borrows in bonds, but at higher risk levels this is reversed, with borrowing in bills and investment in bonds and shares. Real estate stocks are generally unattractive except far out on the efficient frontier.

[7] Since the correlation structures are not very different between the 10, 20, and 40 quarter horizons, the optimal portfolios are also quite similar.

The results of the same exercise assuming that the investor can invest in the housing index, but not in an individual house,[8] are displayed in Table 5. This case can be thought of as applying to a renter household if an index market were available. Since the housing index offers a lower variance but the same expected return as a single house, it comes as no surprise that the housing index portfolio shares in Table 5 are consistently larger than the corresponding housing shares in Table 4, except when non-negativity restrictions on a single house becomes binding. In fact, the housing index shares are quite large. For the 40 and one quarter horizons, the portfolio shares start at 9 and 15 per cent for the minimum variance portfolios and peak at close to 100 and 200 per cent respectively. This suggests that access to a housing index should prove attractive for a renter household. Other features of the efficient frontiers are much the same as in Table 4. Figure 3 compares the efficient investment frontiers when households can invest in financial instruments and a house with the frontier when they can invest in financial instruments and a housing index. The gains from an index at higher levels of risk are apparent.

We have also computed optimal portfolio shares imposing non-negativity restrictions on stocks but not on bonds and t-bills. This reflects the fact that short-selling is difficult in practice, while short positions in nominal instruments may be interpreted as borrowing. The general pattern of optimal portfolio holdings (see Table 6) along the efficient frontier is roughly the same under these assumptions. With increasing risk, the housing fraction goes from close to zero over positive values (peaking at around 40 percent) to zero at the high-risk end of the frontier.

[8] The case of an unrestricted portfolio with all six assets, i e including both housing index and an individual house, is uninteresting. The optimal investment in an individual house will always be zero unless the position in the index is restricted by a constraint (in our case +/- 500 per cent).

V.2. *Housing Consumption Choice and Risk*

Households do not make their housing choices purely from an investment perspective. Under current institutional arrangements, it is not feasible to disentangle the consumption and investment aspects of housing choices. Taking the consumption choice as exogenous, we now analyze at the optimal portfolio composition conditional on given fractions of wealth invested in housing.[9] We do this for four different cases: "rich" homeowners, for whom we assume the housing share is 100 per cent of net wealth; "average" homeowners (housing share = 200 per cent); "poor" homeowners (housing share = 400 per cent); and renters (housing share = 0). These portfolio shares span the average shares reported in Table 1 for households of different ages. How large is the loss, in mean-variance terms, attributable to these restrictions relative to a fully efficient portfolio? The answer to this question is reported in Figures 4A and 4B which depict mean-variance efficient frontiers for holding periods of 40 quarters and one quarter calculated under the assumption that short selling of stocks is not possible but negative positions in bonds and t-bills are.

Renters experience almost no losses relative to the unrestricted portfolio; efficient frontiers of both classes are almost identical for all different horizons. For the three homeowner categories, return losses get larger with increasing portfolio shares in housing. This is evident from the differences in minimum variance attainable. At the 40-quarter horizon the minimum standard deviation is less than 1.1 percent for the renter portfolio compared to 8.8, 17.8 and 36.7 per cent for the three homeowner portfolios. At the one-quarter horizon, the corresponding minimum variances are even farther apart: 1.4, 11.2, 22.5, and 45.1 per cent respectively. A part of these differences is compensated for by higher expected returns. Comparing at the same standard deviation (47 per cent) at one quarter

[9] Of course the fact that a large housing consumption leads to an unbalanced investment portfolio implies an additional cost to housing consumption, which should be taken into account when choosing housing

horizon, the loss in expected return relative to the unrestricted portfolio is modest, less than 0.3 per cent for a rich owner, but very large (6.3 per cent) for a poor owner.

Table 7 represents the corresponding portfolio compositions for renters and poor homeowners. The renter invests more than hundred per cent in t-bills, financed by borrowing in bonds, at the minimum-variance end of the frontier. However, with increasing risk tolerance, the renter reverses positions between t-bills and bonds; she borrows in t-bills and invests in bonds and stocks. The share of stocks is larger in riskier portfolios while the share of bonds is larger in less-risky portfolios. Only far out on the frontier do we see any investment in real estate stocks. These general patterns hold at all horizons. For the poor homeowner, in contrast, the minimum variance portfolio is a corner solution with maximum short-term borrowing financing the house and an investment in bonds. As in case of the renter, with increasing risk the poor homeowner gradually raises the share of stocks by borrowing in bonds.

V.3. *Hedging housing risk*

Now consider the opportunities for hedging. Among currently available instruments, the most obvious hedging opportunity would be short sales of securitized real estate: shorting real estate stocks. In this section we first explore the gains that the real estate stocks afford. We then consider the extra benefits from allowing positions in the housing index. We illustrate the hedging gains for the four household types in figures 5A through D for a 40-quarter horizon. Each panel graphs three efficient frontiers, one allowing neither hedging opportunity, one allowing short sales in real-estate and other stocks and one allowing both short sales in stocks and positions in the housing index. The corresponding portfolios for the poor homeowner are given in Table 8.

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consumption. Brueckner (1997) analyzes the complete choice problem when portfolio and consumption aspects

Figure 5 shows the gains that real estate stocks and the housing index can respectively bring to homeowners. Starting from portfolios that allow neither short-selling of real-estate stocks nor trading in the housing index, we see that the two frontiers diverge for less-risky portfolios, but converge for riskier portfolios. Comparing the two efficient frontiers with short positions allowed for real estate stocks, but with and without access to the housing index, we note that the frontiers start very close at the minimum variance portfolios, but diverge substantially for riskier portfolios. The housing index brings extra benefits to homeowners even after real estate stocks are used as a hedge. Importantly, while real estate stocks tend to benefit more risk averse homeowners, the housing index can bring substantial gains to less risk averse homeowners in terms of reducing risk for their riskier portfolios.

The portfolio compositions in Table 8 shows that when an index is available, real estate stocks, index and t-bills are used to finance long positions in general stocks and bonds, except for high risk portfolios. At high risks, even bonds are held in short positions, leaving only general stocks in long positions. The role of real estate stocks as a hedge is explained by the high variance and low expected return of real estate stocks (relative to other stocks) and by the relatively strong positive correlation between real estate stocks and houses. The correlation coefficient is around 0.4 at longer horizons. When housing investment is suboptimally large from a portfolio perspective, as it is in these homeowner portfolios, investment shares in real estate stocks become generally smaller (and short positions are larger). The figures illustrate the resulting gains from short positions in real estate stocks in mean-variance terms. For poor homeowners the gains are quite large indeed. At the 40-quarter horizon, the standard deviation of the minimum variance portfolio is reduced from 36.7 to 30.6 per cent, while the expected return is increased from 2.3 to 4.1 per cent. At the one-quarter horizon, gains are much smaller. The variance of the minimum variance portfolio

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are treated simultaneously. See also Heaton and Lucas (2000)

for a poor homeowner is reduced from 45.1 without short selling of stocks to 44.3 per cent with short selling while the increase in expected return is merely 0.2 per cent, from 1.5 to 1.7.

The usefulness of real estate stocks as a hedge is limited by the relatively low correlation with housing returns. The housing price index, in contrast, has a stronger correlation with returns from a single house, ranging from 0.42 at the shortest horizon to 0.77 at longer horizons. Allowing positions in the index has a dramatic impact on the composition of the minimum variance portfolios for the poor homeowners. When a position in the housing index is allowed, the results in Table 8 indicate that there is a large negative position (390 per cent) in that index, a positive position in t-bills and positions close to zero in other instruments. To minimize risk, housing should be financed, almost exclusively, by going short in the housing index. Compared to the case when a housing index is not available, there is some reduction in the minimum variance portfolio, at the one-quarter horizon, from 44.3 per cent to 40.7 per cent (and at the 40-quarter horizon from 30.7 to 24.1 per cent). This safety comes at the expense of a sharp drop in expected returns, however. To account for this, we may compare the expected returns at the minimum variances achievable without the housing index with those with the housing index available. The return increases from 4.1 to 6.9 per cent, at the 40-quarter horizon, and from 1.7 per cent to 5.1 per cent, at the one-quarter horizon.

These results indicate clearly that there is substantial scope for welfare improvement by allowing trade in more direct hedging instruments such as home price index futures. We have also seen that the index appears in positive amounts in the efficient portfolios for renters. Renters as well as institutional investors would seem to be the natural market counterparts of owners. With both a supply side and a demand side, the basic requirements for a market are fulfilled.

VI. Conclusion

We have used an unusually rich source of data on housing prices in Stockholm to analyze the investment implications of housing choices. Our empirical analysis derives market-wide price and return series for housing investment during a 13 year period, and it also provides estimates of the individual specific, idiosyncratic, variation in housing returns. Because index changes and the idiosyncratic component follow autocorrelated processes, the analysis of portfolio choice is dependent upon the holding period specified.

We analyze the composition of household investment portfolios containing housing, common stocks, stocks in real estate holding companies, bonds, and t-bills. For short holding periods, the efficient portfolio contains essentially no housing. For longer periods, low risk portfolios contain 15 to 50 percent housing. These results suggest that there are large potential gains from policies or institutions that would permit households to hedge their lumpy investments in housing. We estimate the potential value of hedges in reducing risk for the same investment returns. The value is surprisingly large, especially for poorer homeowners.

This is the first systematic evidence on the topic. Given the ways in which data on house sales are collected centrally in Sweden, it would seem that one could develop a transparent and reliable price index that should be useful for trading in these derivatives. This market would permit households to hedge their most important investment and to diversify their current risks in owner-occupied housing. Currently, these risks are quite large, especially for young households. Our analysis suggests that financial instruments could reduce these risks quite considerably.

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Table 1

Life-Cycle Housing Investment. Mean house value as a per cent of mean net wealth by age category

Age of household head	United States 1989	Sweden 1991
18-30 (25-34)	351.1	258.0
31-40 (35-44)	236.6	161.7
41-50 (45-54)	158.8	121.1
51-60 (55-64)	96.9	94.6
61-70 (65-74)	75.7	78.7
71 + (75 +)	64.8	80.6

Sources: Flavin and Yamashita (1998) Table 2 and Edin et al. (1995), Table 8b.
Note: Age intervals in parenthesis refer to Sweden.

Table 2

Means and Variances of Real Quarterly Asset Returns

	R.E. Stocks	Gen. Stocks	T-Bills	Bonds	H. Index	Houses
			Expected Returns			
	0.0251	0.0365	0.0129	0.0186	0.0127	0.0127
			Variances			
Horizon						
1 quarter	0.0436	0.0194	0.00021	0.0029	0.0022	0.0126
10 quarters	0.1467	0.0409	0.00031	0.0033	0.0116	0.0193
20 quarters	0.0523	0.0183	0.00018	0.0014	0.0064	0.0122
40 quarters	0.0397	0.0167	0.00014	0.0011	0.0051	0.0087

Note: Returns are generated from the VAR model of Appendix Table B1. For comparability, variances are expressed in quarterly terms by dividing by the number of quarters.

Table 3

Correlation Coefficients Among Asset Returns at Different Time Horizons

Time Horizon		R.E. Stocks	Gen. Stocks	T-Bills	Bond	H. Index	Houses
1 quarter	R.E. Stocks	1	0.8161	-0.1420	0.1612	0.3473	0.1458
10 quarters			0.8961	0.1559	0.0826	0.5171	0.4006
20 quarters			0.7710	0.0813	0.0230	0.5270	0.3822
40 quarters			0.7533	0.0281	0.0104	0.4961	0.3798
1 quarter	Gen. Stocks		1	-0.0696	0.3579	0.1609	0.0676
10 quarters				0.2263	0.2818	0.2231	0.1729
20 quarters				0.0402	0.2520	0.0503	0.0365
40 quarters				-0.0531	0.2334	-0.0292	-0.0223
1 quarter	T-bills			1	0.4100	-0.0694	-0.0292
10 quarters					0.8097	-0.5137	-0.3980
20 quarters					0.6949	-0.3528	-0.2558
40 quarters					0.6720	-0.3009	-0.2303
1 quarter	Bonds				1	-0.1316	-0.0552
10 quarters						-0.6167	-0.4778
20 quarters						-0.4923	-0.3571
40 quarters						-0.4711	-0.3606
	Housing index						
1 quarter						1	0.4199
10 quarters							0.7748
20 quarters							0.7252
40 quarters							0.7654

Table 4

Optimal Unrestricted Portfolios. Four Financial Instruments and a House.

Standard Deviation	Expected Returns	R.E. Stocks	Gen. Stocks	T-Bill	Bond	House
			40-quarter horizon			
0.0089	0.0131	-0.0500	0.0821	1.1226	-0.2127	0.0580
0.0636	0.0304	-0.2696	0.5591	-1.0208	1.3529	0.3784
0.1263	0.0477	-0.4892	1.0361	-3.1642	2.9185	0.6988
0.1895	0.0650	-0.7454	1.6095	-5.0000	4.1588	0.9771
0.2640	0.0823	-1.2203	2.7586	-5.0000	3.4581	1.0036
0.3477	0.0997	-1.6952	3.9078	-5.0000	2.7574	1.0300
0.4352	0.1170	-2.1035	5.0000	-5.0000	2.1456	0.9579
0.6161	0.1343	-0.3167	5.0000	-5.0000	1.3167	0.0000
1.0428	0.1516	2.3417	5.0000	-5.0000	-1.3417	0.0000
1.5396	0.1689	5.0000	5.0000	-5.0000	-4.0000	0.0000
			One-quarter horizon			
0.0136	0.0131	0.0065	0.0177	1.0298	-0.0676	0.0136
0.0802	0.0304	-0.4017	0.9185	0.3201	0.1230	0.0401
0.1586	0.0477	-0.8098	1.8193	-0.3895	0.3135	0.0666
0.2375	0.0650	-1.2179	2.7201	-1.0992	0.5040	0.0930
0.3164	0.0823	-1.6261	3.6209	-1.8089	0.6945	0.1195
0.3953	0.0997	-2.0342	4.5217	-2.5185	0.8850	0.1460
0.4778	0.1170	-2.1526	5.0000	-4.2136	2.2120	0.1542
0.6787	0.1343	-0.3167	5.0000	-5.0000	1.3167	0.0000
1.1250	0.1516	2.3417	5.0000	-5.0000	-1.3417	0.0000
1.6360	0.1689	5.0000	5.0000	-5.0000	-4.0000	0.0000

Table 5

Optimal Unrestricted Portfolios. Four Financial Instruments and the Housing Index.

Standard Deviation	Expected Returns	R.E. Stocks	Gen. Stocks	T-Bill	Bond	H. Index
			40-quarter horizon			
0.0011	0.0138	-0.0812	0.1126	0.9602	-0.1376	0.1460
0.0107	0.0343	-0.4574	0.7320	-1.9567	1.7621	0.9200
0.0213	0.0549	-0.8336	1.3514	-4.8737	3.6618	1.6941
0.0345	0.0754	-1.3996	2.6934	-5.0000	2.9248	1.7814
0.0504	0.0959	-1.9742	4.0681	-5.0000	2.0684	1.8377
0.0676	0.1164	-2.0731	5.0000	-5.0000	1.9932	1.0798
0.0919	0.1370	-1.1705	5.0000	-5.0000	3.5621	-1.3916
0.1204	0.1575	-0.2056	5.0000	-5.0000	5.0000	-3.7944
0.1592	0.1780	1.8488	5.0000	-5.0000	4.1512	-5.0000
0.2335	0.1986	5.0000	5.0000	-5.0000	1.0000	-5.0000
			One-quarter horizon			
0.0132	0.0131	-0.0038	0.0231	0.9483	-0.0532	0.0855
0.0946	0.0337	-0.5094	1.0996	-0.0768	0.2049	0.2817
0.1878	0.0543	-1.0151	2.1761	-1.1020	0.4630	0.4779
0.2813	0.0749	-1.5207	3.2526	-2.1271	0.7211	0.6742
0.3749	0.0955	-2.0263	4.3291	-3.1523	0.9792	0.8704
0.4717	0.1161	-2.2425	5.0000	-5.0000	2.2985	0.9440
0.6093	0.1368	-1.5997	5.0000	-5.0000	4.4252	-1.8255
0.8017	0.1574	-0.2176	5.0000	-5.0000	5.0000	-3.7824
1.1034	0.1780	1.8373	5.0000	-5.0000	4.1627	-5.0000
1.6355	0.1986	5.0000	5.0000	-5.0000	1.0000	-5.0000

Table 6

Optimal Portfolios. Four Financial Instruments and a House.

No Short Selling of Stocks.

Standard Deviation	Expected Returns	R.E. Stocks	Gen. Stocks	T-Bill	Bond	House
			40-quarter horizon			
0.0104	0.0125	0.0000	0.0227	1.1237	-0.1708	0.0244
0.0689	0.0299	0.0000	0.2741	-1.3666	1.8669	0.2255
0.1366	0.0473	0.0000	0.5256	-3.8568	3.9045	0.4267
0.2101	0.0647	0.0000	1.0918	-5.0000	4.6139	0.2943
0.3049	0.0820	0.0000	1.9652	-5.0000	4.0348	0.0000
0.4127	0.0994	0.0000	2.9362	-5.0000	3.0638	0.0000
0.5268	0.1168	0.0000	3.9071	-5.0000	2.0929	0.0000
0.6438	0.1342	0.0000	4.8780	-5.0000	1.1220	0.0000
1.0412	0.1516	2.3324	5.0000	-5.0000	-1.3324	0.0000
1.5396	0.1689	5.0000	5.0000	-5.0000	-4.0000	0.0000
			One-quarter horizon			
0.0136	0.0131	0.0065	0.0177	1.0298	-0.0676	0.0136
0.0992	0.0304	0.0000	0.6017	-0.1783	0.5766	0.0000
0.1976	0.0477	0.0000	1.1796	-1.4124	1.2328	0.0000
0.2963	0.0650	0.0000	1.7576	-2.6465	1.8890	0.0000
0.3951	0.0823	0.0000	2.3355	-3.8807	2.5451	0.0000
0.4940	0.0997	0.0000	2.9497	-5.0000	3.0503	0.0000
0.5989	0.1170	0.0000	3.9172	-5.0000	2.0828	0.0000
0.7107	0.1343	0.0000	4.8848	-5.0000	1.1152	0.0000
1.1250	0.1516	2.3417	5.0000	-5.0000	-1.3417	0.0000
1.6360	0.1689	5.0000	5.0000	-5.0000	-4.0000	0.0000

Table 7

Optimal Portfolios. No Short Selling. No Index.

Panel A. Renters (Housing = 0)

Standard Deviation	Expected Returns	R.E. Stocks	Gen. Stocks	T-Bill	Bond	House
			40-quarter horizon			
0.0106	0.0124	0.0000	0.0251	1.1825	-0.2076	0.0000
0.0714	0.0298	0.0000	0.3166	-0.9726	1.6560	0.0000
0.1417	0.0472	0.0000	0.6081	-3.1277	3.5196	0.0000
0.2126	0.0646	0.0000	0.9916	-4.9916	5.0000	0.0000
0.3044	0.0820	0.0000	1.9607	-5.0000	4.0393	0.0000
0.4123	0.0993	0.0000	2.9326	-5.0000	3.0674	0.0000
0.5265	0.1167	0.0000	3.9044	-5.0000	2.0956	0.0000
0.6436	0.1341	0.0000	4.8762	-5.0000	1.1238	0.0000
1.0407	0.1515	2.3299	5.0000	-5.0000	-1.3299	0.0000
1.5396	0.1689	5.0000	5.0000	-5.0000	-4.0000	0.0000
			One-quarter horizon			
0.0137	0.0130	0.0084	0.0166	1.0453	-0.0703	0.0000
0.0991	0.0304	0.0000	0.6013	-0.1775	0.5762	0.0000
0.1975	0.0477	0.0000	1.1793	-1.4117	1.2324	0.0000
0.2963	0.0650	0.0000	1.7573	-2.6459	1.8886	0.0000
0.3951	0.0823	0.0000	2.3353	-3.8802	2.5449	0.0000
0.4940	0.0996	0.0000	2.9494	-5.0000	3.0506	0.0000
0.5989	0.1170	0.0000	3.9170	-5.0000	2.0830	0.0000
0.7107	0.1343	0.0000	4.8846	-5.0000	1.1154	0.0000
1.1249	0.1516	2.3415	5.0000	-5.0000	-1.3415	0.0000
1.6360	0.1689	5.0000	5.0000	-5.0000	-4.0000	0.0000

Panel B. Poor Homeowners (Housing = 4)

Standard Deviation	Expected Returns	R.E. Stocks	Gen. Stocks	T-Bill	Bond	House
			40-quarter horizon			
0.3673	0.0231	0.0000	0.0000	-5.0000	2.0000	4.0000
0.3842	0.0345	0.0000	0.6364	-5.0000	1.3636	4.0000
0.4161	0.0459	0.0000	1.2729	-5.0000	0.7271	4.0000
0.4598	0.0573	0.0000	1.9093	-5.0000	0.0907	4.0000
0.5124	0.0687	0.0000	2.5457	-5.0000	-0.5457	4.0000
0.5714	0.0801	0.0000	3.1822	-5.0000	-1.1822	4.0000
0.6350	0.0915	0.0000	3.8186	-5.0000	-1.8186	4.0000
0.7020	0.1029	0.0000	4.4550	-5.0000	-2.4550	4.0000
0.8044	0.1143	0.2514	5.0000	-5.0000	-3.2514	4.0000
1.1216	0.1257	2.0000	5.0000	-5.0000	-5.0000	4.0000
			One-quarter horizon			
0.4507	0.0152	0.0000	0.0000	-3.5978	0.5978	4.0000
0.4610	0.0275	0.0000	0.2430	-5.0000	1.7570	4.0000
0.4862	0.0398	0.0000	0.9286	-5.0000	1.0714	4.0000
0.5254	0.0520	0.0000	1.6142	-5.0000	0.3858	4.0000
0.5760	0.0643	0.0000	2.2998	-5.0000	-0.2998	4.0000
0.6352	0.0766	0.0000	2.9855	-5.0000	-0.9855	4.0000
0.7007	0.0889	0.0000	3.6711	-5.0000	-1.6711	4.0000
0.7711	0.1011	0.0000	4.3567	-5.0000	-2.3567	4.0000
0.8581	0.1134	0.1162	5.0000	-5.0000	-3.1162	4.0000
1.1777	0.1257	2.0000	5.0000	-5.0000	-5.0000	4.0000

Table 8

Optimal Portfolios for Poor Homeowners (Housing = 4)

with Short Selling and the Housing Index

Panel A. 40 Quarter Horizon

Standard Deviation	Expected Returns	R.E. Stocks	Gen. Stocks	T-Bill	Bond	Index	House
Short selling no index							
0.3069	0.0412	-1.5267	1.5637	-5.0000	1.9630	0.0000	4.0000
0.3111	0.0506	-1.7774	2.1794	-5.0000	1.5980	0.0000	4.0000
0.3231	0.0600	-2.0281	2.7951	-5.0000	1.2330	0.0000	4.0000
0.3423	0.0693	-2.2787	3.4107	-5.0000	0.8680	0.0000	4.0000
0.3674	0.0787	-2.5294	4.0264	-5.0000	0.5030	0.0000	4.0000
0.3973	0.0881	-2.7801	4.6421	-5.0000	0.1381	0.0000	4.0000
0.4475	0.0975	-2.3227	5.0000	-5.0000	-0.6773	0.0000	4.0000
0.6196	0.1069	-0.8818	5.0000	-5.0000	-2.1182	0.0000	4.0000
0.8582	0.1163	0.5591	5.0000	-5.0000	-3.5591	0.0000	4.0000
1.1216	0.1257	2.0000	5.0000	-5.0000	-5.0000	0.0000	4.0000
Short selling and index							
0.2408	0.0138	-0.0812	0.1126	0.9602	-0.1376	-3.854	4.0000
0.2478	0.0317	-0.4091	0.6525	-1.5824	1.5183	-3.1793	4.0000
0.2677	0.0496	-0.7370	1.1924	-4.1251	3.1742	-2.5046	4.0000
0.3022	0.0675	-1.1784	2.1642	-5.0000	3.2544	-2.2402	4.0000
0.3588	0.0854	-1.6793	3.3624	-5.0000	2.5080	-2.1912	4.0000
0.4298	0.1033	-2.1802	4.5607	-5.0000	1.7616	-2.1421	4.0000
0.5184	0.1212	-1.8654	5.0000	-5.0000	2.3541	-3.4887	4.0000
0.6636	0.1391	-0.4937	5.0000	-5.0000	2.4937	-5.0000	4.0000
1.0447	0.1570	2.2532	5.0000	-5.0000	-0.2532	-5.0000	4.0000
1.5350	0.1749	5.0000	5.0000	-5.0000	-3.0000	-5.0000	4.0000

Panel B. One Quarter Horizon

Standard Deviation	Expected Returns	R.E. Stocks	Gen. Stocks	T-Bill	Bond	Index	House
Short selling no index							
0.4425	0.0167	-0.5631	0.3201	-3.4883	0.7313	0.0000	4.0000
0.4459	0.0288	-0.8463	0.9496	-3.9643	0.8610	0.0000	4.0000
0.4561	0.0409	-1.1295	1.5791	-4.4403	0.9907	0.0000	4.0000
0.4725	0.0530	-1.4127	2.2085	-4.9163	1.1204	0.0000	4.0000
0.4951	0.0651	-1.7700	2.9888	-5.0000	0.7812	0.0000	4.0000
0.5239	0.0772	-2.1431	3.8013	-5.0000	0.3418	0.0000	4.0000
0.5579	0.0893	-2.5162	4.6137	-5.0000	-0.0975	0.0000	4.0000
0.6287	0.1014	-1.7183	5.0000	-5.0000	-1.2817	0.0000	4.0000
0.8619	0.1136	0.1408	5.0000	-5.0000	-3.1408	0.0000	4.0000
1.1777	0.1257	2.0000	5.0000	-5.0000	-5.0000	0.0000	4.0000
Short selling and index							
0.4071	0.0131	-0.0038	0.0231	0.9483	-0.0532	-3.9145	4.0000
0.4152	0.0311	-0.4448	0.9620	0.0543	0.1719	-3.7434	4.0000
0.4387	0.0491	-0.8858	1.9008	-0.8398	0.3970	-3.5722	4.0000
0.4752	0.0670	-1.3267	2.8397	-1.7339	0.6221	-3.4011	4.0000
0.5220	0.0850	-1.7677	3.7785	-2.628	0.8472	-3.2300	4.0000
0.5767	0.1030	-2.2087	4.7174	-3.5221	1.0722	-3.0589	4.0000
0.6440	0.1209	-2.0929	5.0000	-5.0000	2.7937	-3.7008	4.0000
0.7874	0.1389	-0.5166	5.0000	-5.0000	2.5166	-5.0000	4.0000
1.1803	0.1569	2.2417	5.0000	-5.0000	-0.2417	-5.0000	4.0000
1.6788	0.1749	5.0000	5.0000	-5.0000	-3.0000	-5.0000	4.0000

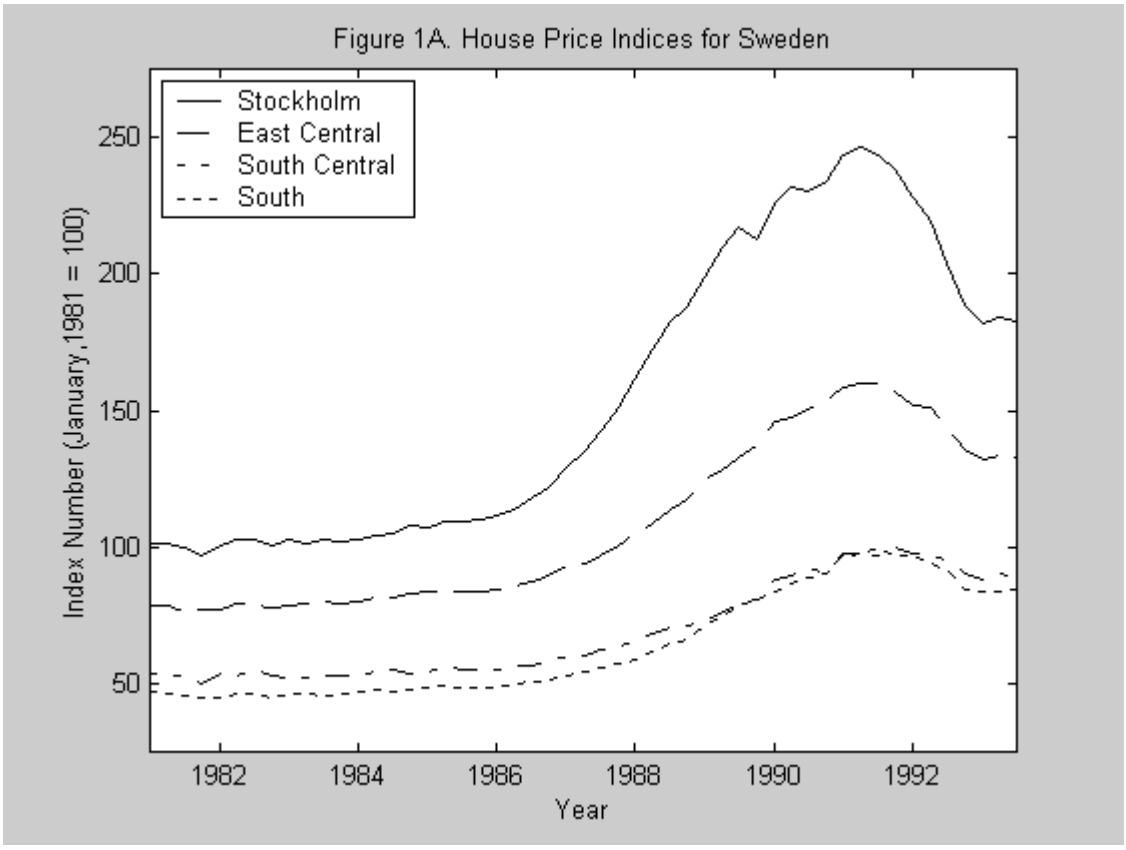
Figure 1A. House Price Indices for Sweden
Stockholm
East Central
South Central
South
Index Number (January,1981 = 100)
250
200
150
100
50
1982
1984
1986
1988
1990
1992
Year

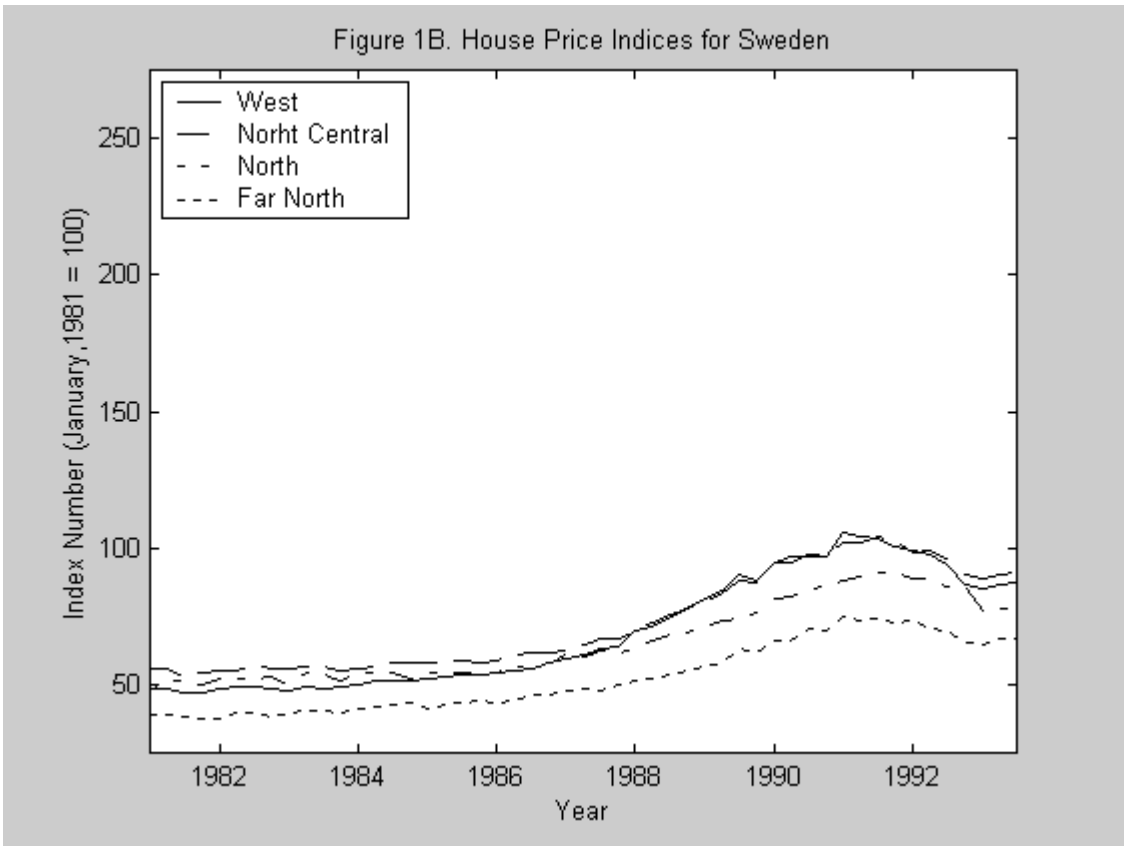
Figure 1B. House Price Indices for Sweden
West
Norht Central
North
Far North
Index Number (January,1981 = 100)
250
200
150
100
50
1982
1984
1986
1988
1990
1992
Year

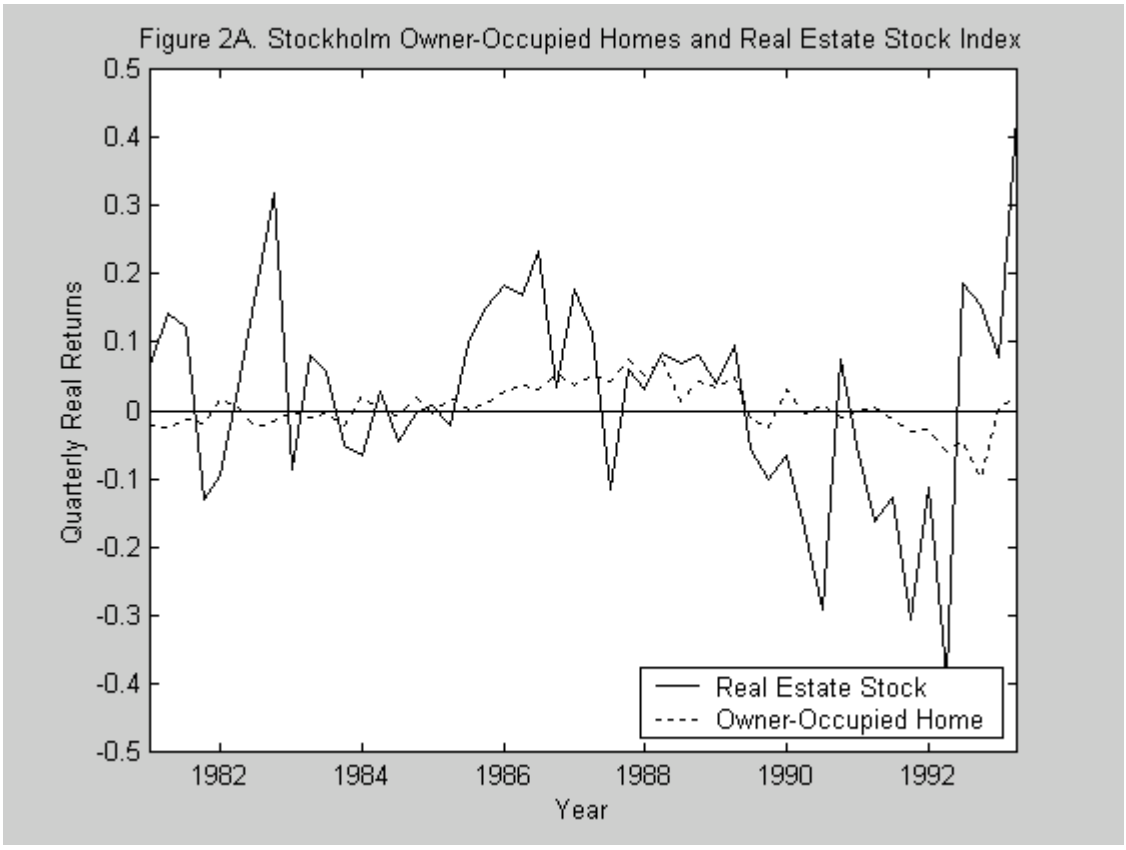
Figure 2A. Stockholm Owner-Occupied Homes and Real Estate Stock Index
Quarterly Real Returns
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
1982
1984
1986
1988
1990
1992
Year
Real Estate Stock
Owner-Occupied Home

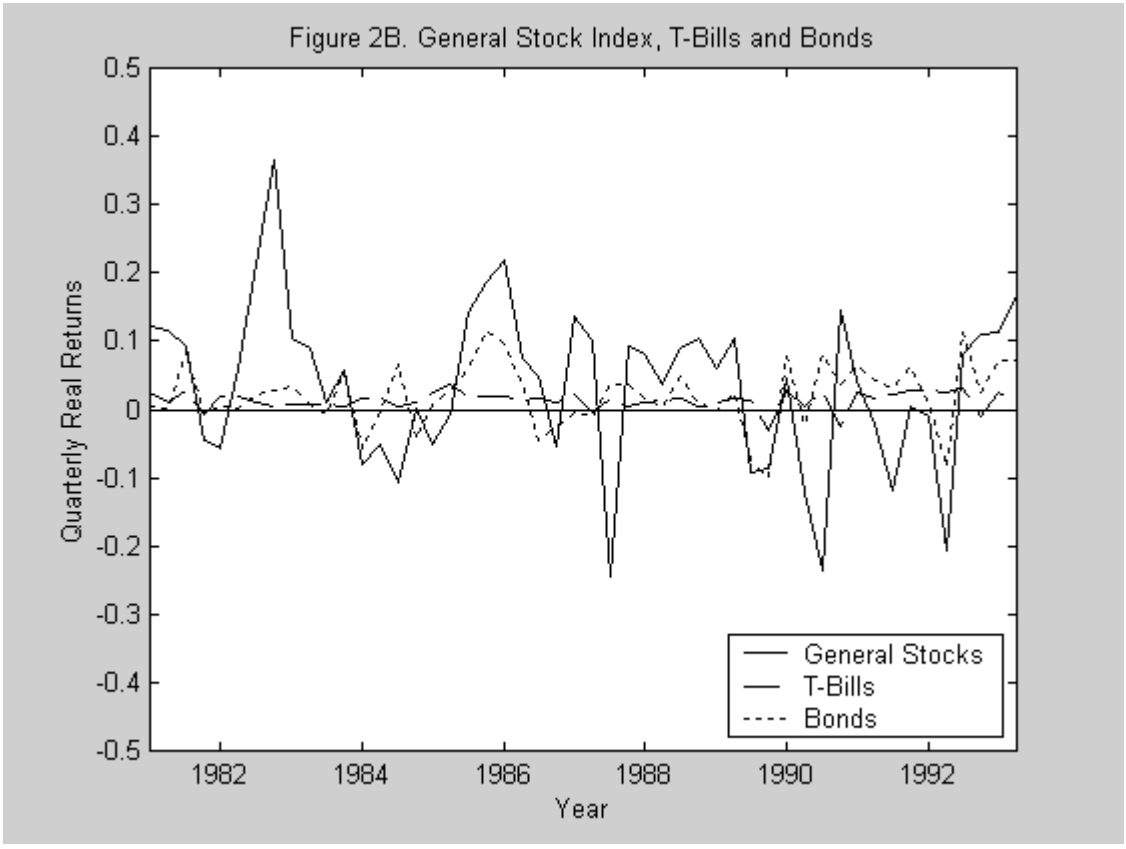
Figure 2B. General Stock Index, T-Bills and Bonds
Quarterly Real Returns
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
1982
1984
1986
1988
1990
1992
Year
General Stocks
T-Bills
Bonds

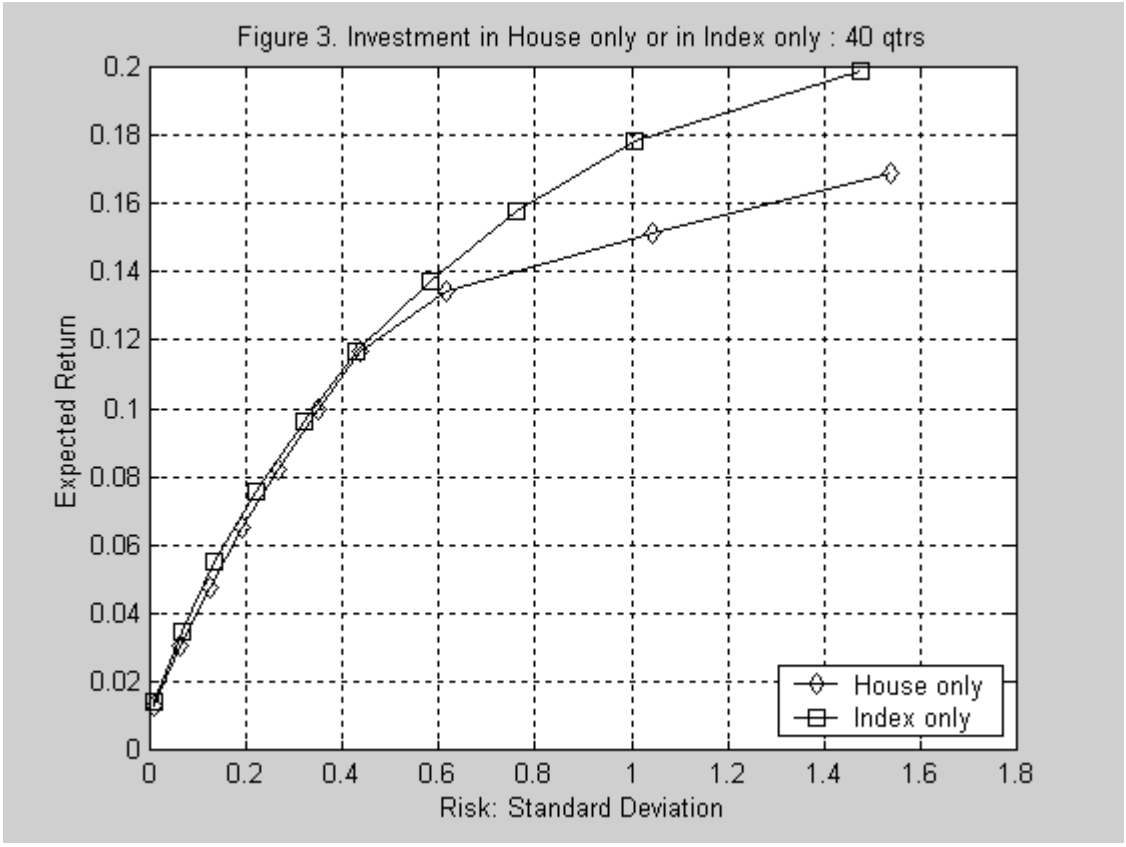
Figure 3. Investment in House only or in Index only : 40 qtrs
Expected Return
Risk: Standard Deviation
House only
Index only
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

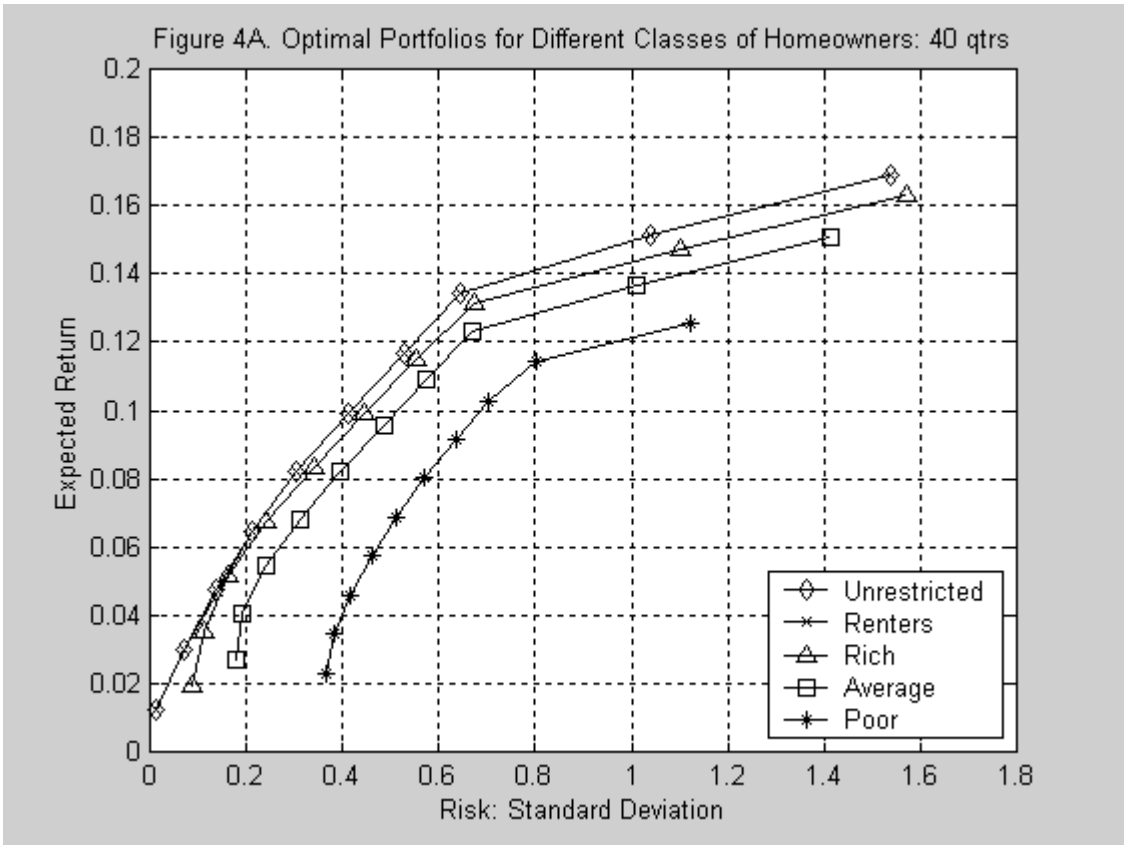
Figure 4A. Optimal Portfolios for Different Classes of Homeowners: 40 qtrs
Expected Return
Risk: Standard Deviation
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
Unrestricted
Renters
Rich
Average
Poor

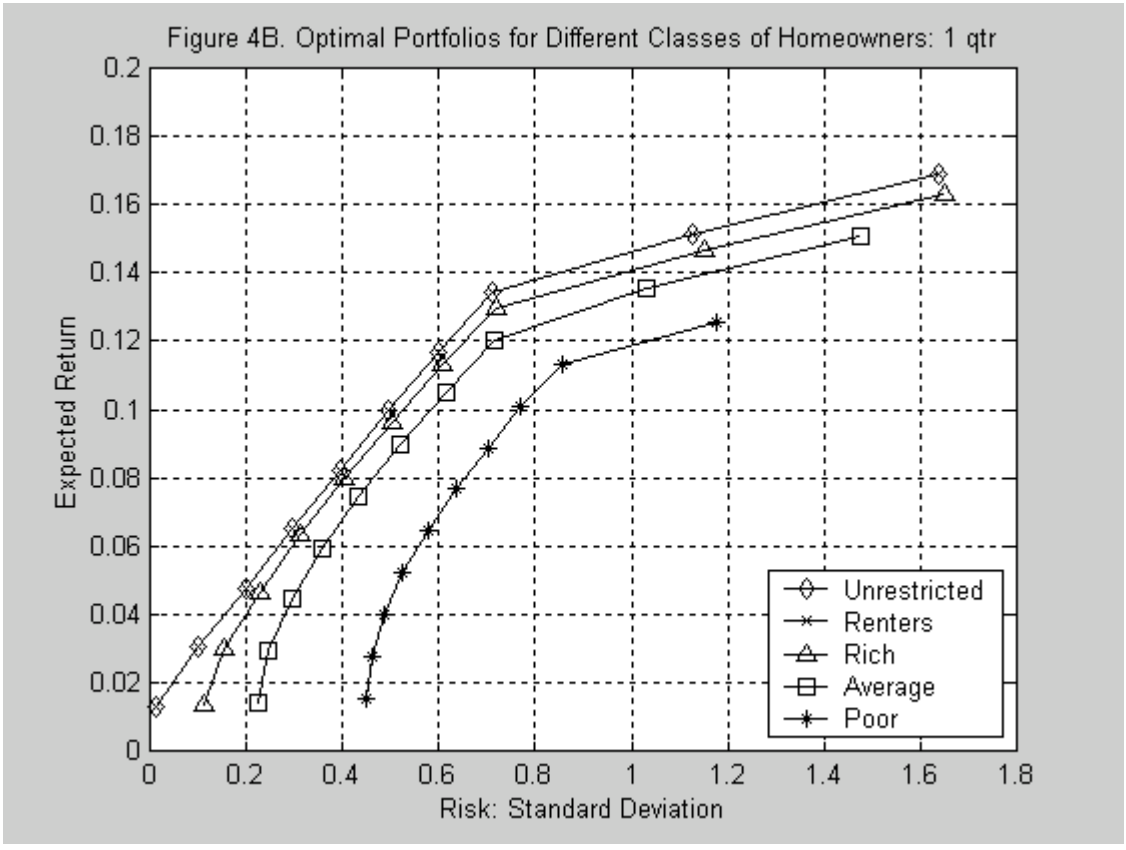
Figure 4B. Optimal Portfolios for Different Classes of Homeowners: 1 qtr
Expected Return
Risk: Standard Deviation
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
Unrestricted
Renters
Rich
Average
Poor

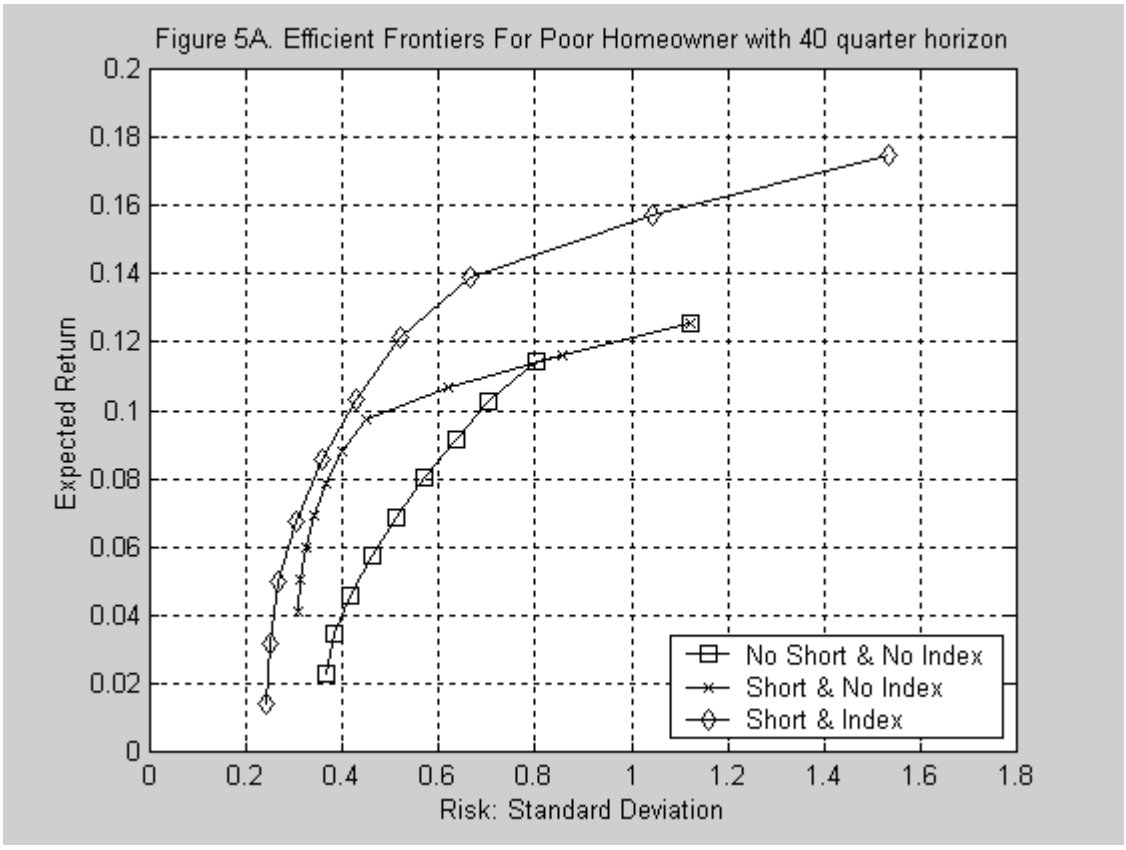
Figure 5A. Efficient Frontiers For Poor Homeowner with 40 quarter horizon
Expected Return
Risk: Standard Deviation
No Short & No Index
Short & No Index
Short & Index
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

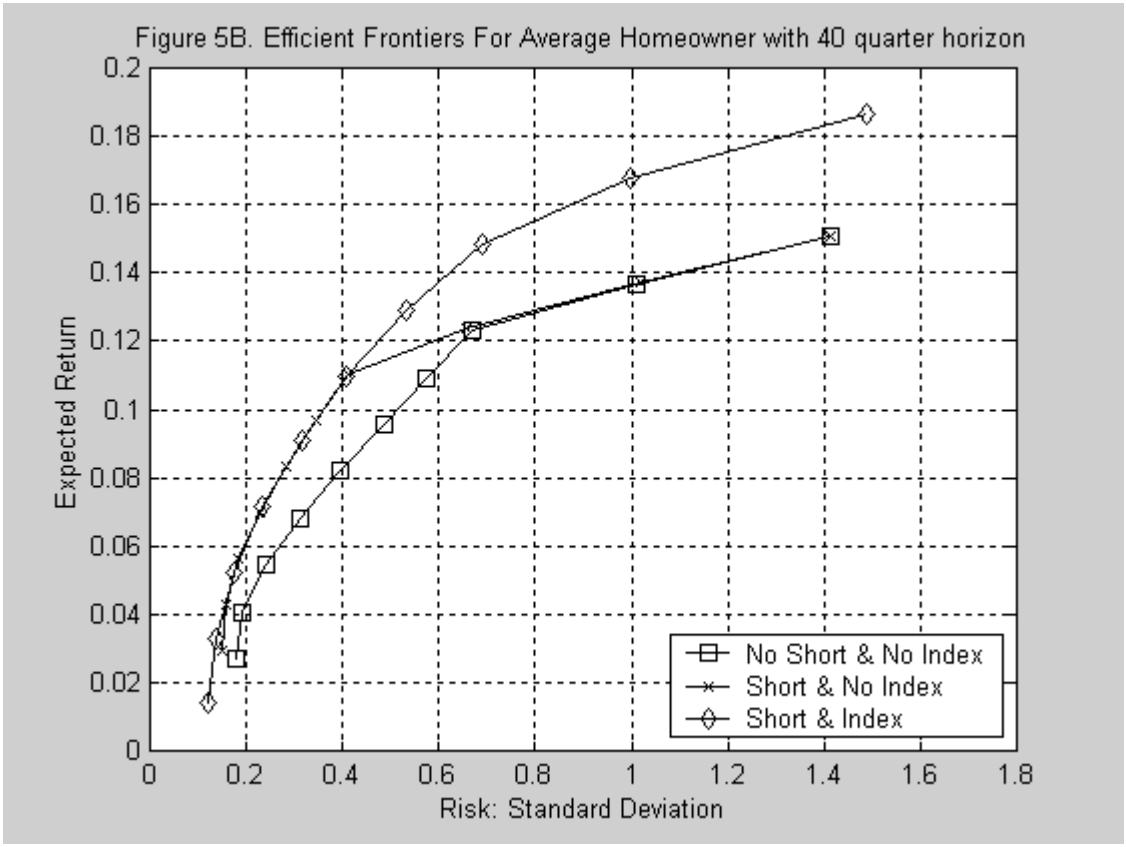
Figure 5B. Efficient Frontiers For Average Homeowner with 40 quarter horizon
Expected Return
Risk: Standard Deviation
No Short & No Index
Short & No Index
Short & Index
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

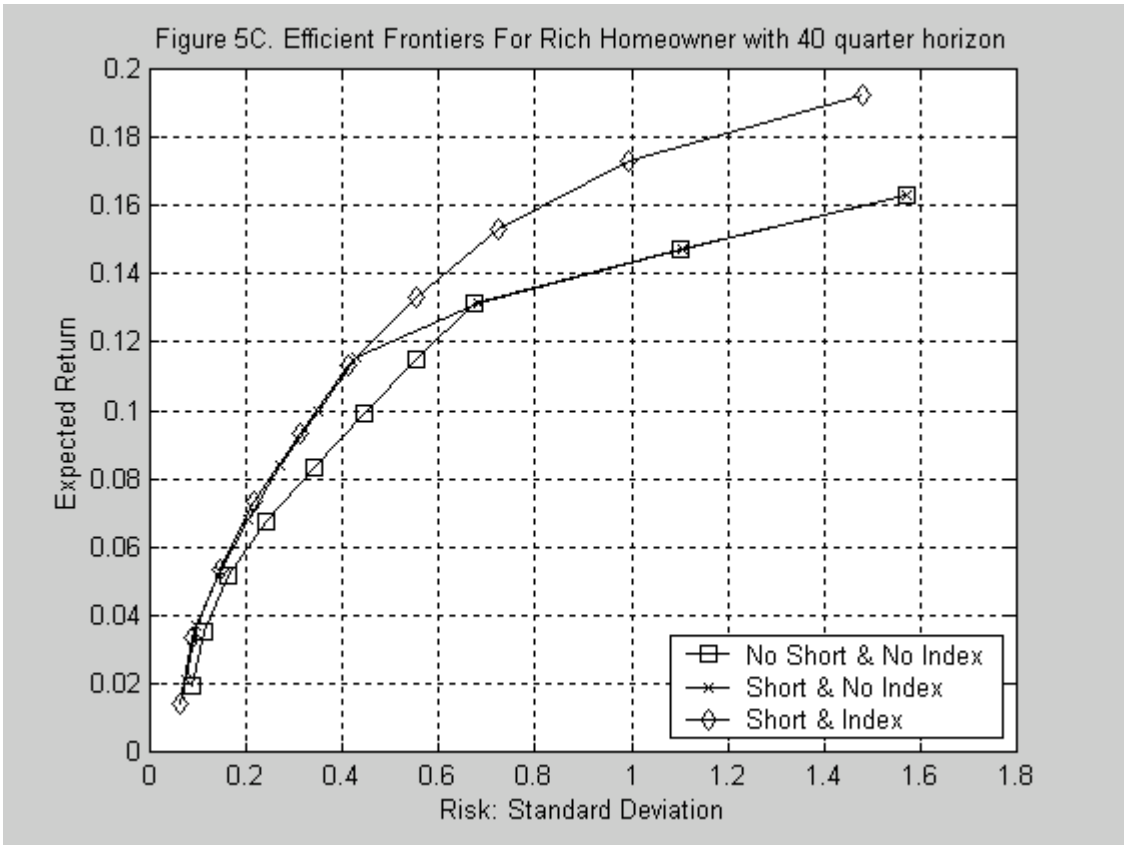
Figure 5C. Efficient Frontiers For Rich Homeowner with 40 quarter horizon
Expected Return
Risk: Standard Deviation
No Short & No Index
Short & No Index
Short & Index
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

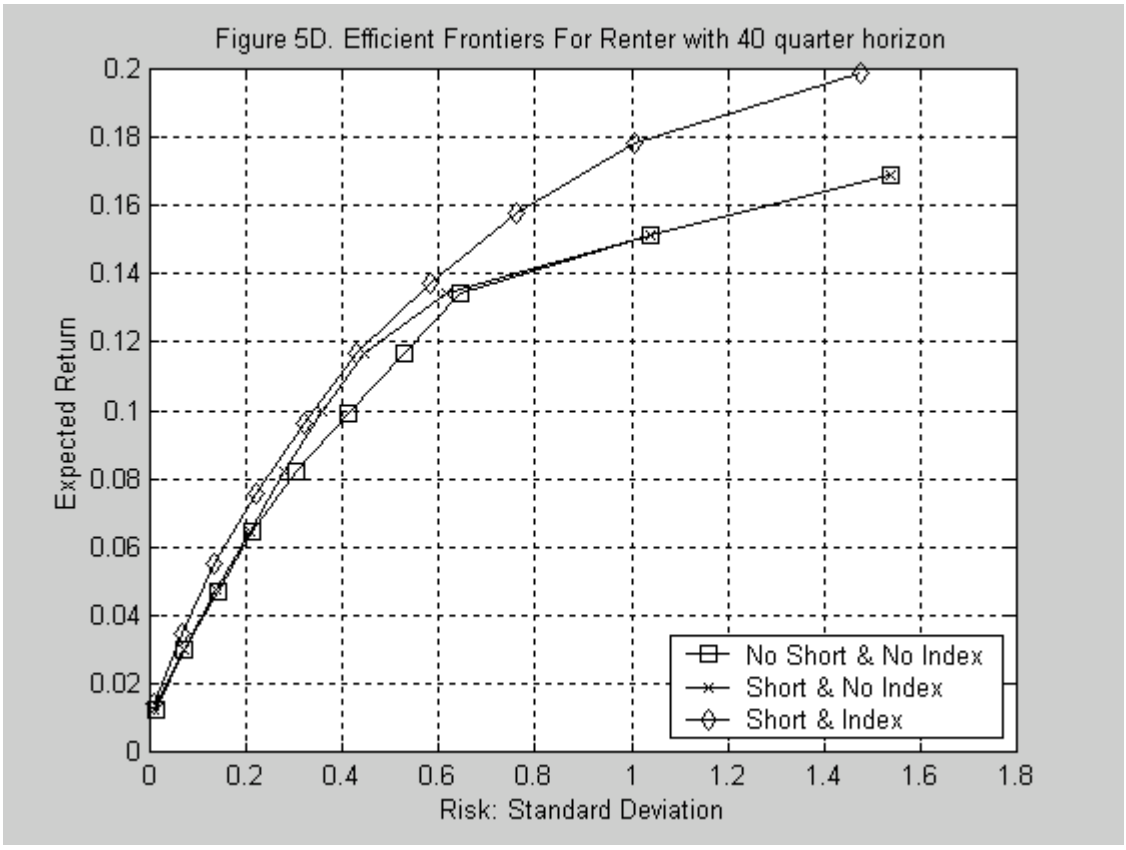
Figure 5D. Efficient Frontiers For Renter with 40 quarter horizon
Expected Return
Risk: Standard Deviation
No Short & No Index
Short & No Index
Short & Index
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

Appendix Table A.1

Monthly Estimates of Regional Price Changes for Sweden, 1981 – 1993
January, 1981 = 1.000
(Entries are the logarithms of estimated price increase during each month)
t-ratios in parentheses

Year/Month	Region							
	Stockholm	East Central	South Central	South	West	North Central	North	Far North
1981: Feb	0.047	-0.023	0.026	-0.043	0.004	-0.009	-0.060	-0.064
	(2.34)	(1.29)	(0.81)	(1.80)	(0.20)	(0.34)	(1.10)	(1.25)
	-0.058	0.032	-0.027	0.033	-0.044	0.024	-0.038	0.047
	(2.54)	(1.61)	(0.77)	(1.28)	(2.02)	(0.82)	(0.61)	(0.77)
	0.038	-0.002	-0.010	-0.020	0.020	-0.031	0.134	-0.034
	(1.66)	(0.10)	(0.31)	(0.80)	(0.91)	(1.13)	(2.31)	(0.52)
	-0.005	-0.005	0.006	-0.020	0.015	0.012	-0.039	0.019
	(0.22)	(0.26)	(0.18)	(0.79)	(0.69)	(0.44)	(0.72)	(0.31)
	-0.029	-0.017	0.009	0.007	-0.010	0.036	0.015	-0.004
	(1.45)	(1.00)	(0.32)	(0.32)	(0.52)	(1.47)	(0.33)	(0.09)
	0.009	-0.021	0.043	-0.021	-0.007	-0.043	0.017	0.007
	(0.42)	(1.23)	(1.59)	(0.99)	(0.36)	(1.82)	(0.40)	(0.15)
	0.005	0.020	-0.048	0.034	-0.012	-0.051	-0.002	-0.031
	(0.21)	(1.00)	(1.63)	(1.45)	(0.57)	(1.99)	(0.04)	(0.61)
	-0.017	-0.014	-0.053	-0.037	-0.008	0.001	-0.047	-0.017
	(0.80)	(0.77)	(1.90)	(1.61)	(0.40)	(0.04)	(1.01)	(0.36)
	-0.013	0.042	0.075	0.007	0.002	0.052	0.006	0.028
	(0.67)	(2.54)	(2.69)	(0.31)	(0.10)	(2.19)	(0.13)	(0.59)
	0.006	-0.030	-0.068	-0.041	0.002	-0.024	-0.053	-0.027
	(0.31)	(1.83)	(2.44)	(1.83)	(0.10)	(0.99)	(1.18)	(0.55)
	-0.031	-0.042	-0.090	0.076	-0.024	-0.006	0.103	-0.004
	(1.30)	(2.14)	(2.80)	(2.81)	(1.08)	(0.20)	(2.04)	(0.07)
1982: Jan	0.057	0.048	0.193	-0.036	0.066	0.045	0.007	0.063
	(2.12)	(2.13)	(5.35)	(1.17)	(2.69)	(1.33)	(0.12)	(0.98)
	-0.026	-0.016	-0.067	-0.009	-0.042	0.002	0.012	-0.025
	(0.99)	(0.73)	(1.95)	(0.31)	(1.77)	(0.06)	(0.21)	(0.38)
	0.029	0.013	0.001	-0.007	0.047	-0.025	-0.052	-0.080
	(1.25)	(0.66)	(0.03)	(0.27)	(2.19)	(0.89)	(1.00)	(1.44)
	-0.001	0.004	-0.022	0.046	-0.011	-0.022	0.038	0.087
	(0.05)	(0.23)	(0.76)	(2.00)	(0.57)	(0.89)	(0.77)	(1.70)
	0.029	0.011	0.072	0.005	-0.001	0.042	-0.006	0.053
	(1.56)	(0.67)	(2.65)	(0.23)	(0.06)	(1.79)	(0.14)	(1.10)
	-0.020	0.023	-0.042	0.001	0.008	0.029	-0.029	-0.045
	(1.15)	(1.54)	(1.79)	(0.05)	(0.47)	(1.36)	(0.76)	(1.05)
	0.005	-0.017	0.061	0.008	-0.001	-0.003	0.023	0.015
	(0.27)	(1.13)	(2.73)	(0.42)	(0.06)	(0.14)	(0.62)	(0.37)
	0.010	-0.011	-0.016	-0.003	0.003	0.017	0.020	0.025
	(0.47)	(0.63)	(0.59)	(0.14)	(0.16)	(0.74)	(0.45)	(0.58)
	-0.018	-0.021	-0.024	-0.023	-0.001	-0.039	-0.025	-0.026
	(0.90)	(1.23)	(0.87)	(1.03)	(0.05)	(1.75)	(0.59)	(0.60)
	0.012	0.035	-0.014	0.005	0.005	0.015	0.024	-0.072
	(0.61)	(2.00)	(0.50)	(0.22)	(0.26)	(0.65)	(0.57)	(1.51)
	-0.039	-0.016	0.020	-0.045	-0.036	-0.025	-0.036	0.077
	(2.00)	(0.94)	(0.74)	(1.87)	(1.90)	(1.08)	(0.83)	(1.61)
	0.005	-0.020	-0.056	0.007	0.019	-0.005	0.056	-0.030
	(0.22)	(1.07)	(1.89)	(0.25)	(0.89)	(0.19)	(1.09)	(0.59)
1983: Jan	0.029	0.042	0.034	0.068	-0.025	-0.008	-0.101	0.054
	(1.16)	(2.06)	(1.00)	(2.27)	(1.07)	(0.26)	(1.64)	(0.90)

	-0.012	-0.024	-0.021	-0.056	0.018	0.028	0.071	-0.013
	(0.51)	(1.21)	(0.65)	(2.03)	(0.81)	(0.91)	(1.18)	(0.20)
	0.019	0.001	0.002	0.054	-0.013	0.004	-0.063	-0.045
	(0.90)	(0.05)	(0.06)	(2.05)	(0.61)	(0.13)	(1.10)	(0.69)
	-0.052	0.015	-0.008	-0.026	0.019	-0.023	0.065	0.022
	(2.64)	(0.85)	(0.26)	(1.04)	(0.96)	(0.85)	(1.19)	(0.37)
	0.054	-0.011	0.018	0.006	0.007	0.012	0.029	0.024
	(3.04)	(0.69)	(0.70)	(0.27)	(0.39)	(0.51)	(0.59)	(0.49)
	-0.006	0.016	0.024	0.017	0.002	0.034	0.026	0.020
	(0.38)	(1.08)	(0.99)	(0.86)	(0.12)	(1.59)	(0.63)	(0.51)
	-0.010	0.020	-0.032	-0.048	-0.011	-0.020	-0.059	-0.017
	(0.58)	(1.31)	(1.29)	(2.43)	(0.64)	(0.90)	(1.55)	(0.43)
	-0.001	-0.015	0.022	0.024	-0.010	-0.004	0.011	-0.049
	(0.05)	(0.87)	(0.81)	(1.13)	(0.51)	(0.16)	(0.25)	(1.09)
	0.020	-0.021	-0.008	-0.011	0.037	0.003	0.049	0.074
	(1.12)	(1.28)	(0.30)	(0.52)	(1.93)	(0.13)	(1.19)	(1.71)
	-0.031	0.012	0.026	0.026	-0.005	-0.039	-0.033	-0.058
	(1.82)	(0.75)	(1.00)	(1.24)	(0.26)	(1.79)	(0.78)	(1.29)
	0.017	-0.002	-0.024	-0.027	-0.007	0.025	-0.059	0.009
	(1.08)	(0.13)	(0.97)	(1.34)	(0.38)	(1.16)	(1.37)	(0.19)
	-0.001	-0.009	-0.010	0.012	-0.016	-0.011	0.001	0.047
	(0.05)	(0.51)	(0.33)	(0.49)	(0.79)	(0.44)	(0.02)	(0.90)
1984: Jan	-0.007	0.013	-0.004	0.011	0.039	0.012	0.090	-0.050
	(0.32)	(0.67)	(0.12)	(0.40)	(1.74)	(0.41)	(1.62)	(0.87)
	0.019	-0.008	0.016	0.020	-0.028	0.008	-0.050	0.012
	(0.90)	(0.41)	(0.49)	(0.79)	(1.31)	(0.26)	(0.89)	(0.21)
	-0.004	0.018	0.008	-0.026	0.022	-0.005	0.051	0.094
	(0.20)	(0.96)	(0.25)	(1.08)	(1.09)	(0.18)	(0.91)	(1.63)
	-0.007	-0.008	0.026	0.001	0.027	0.015	-0.039	-0.088
	(0.36)	(0.45)	(0.90)	(0.05)	(1.42)	(0.60)	(0.77)	(1.67)
	0.021	0.019	-0.031	0.033	-0.009	-0.009	-0.014	0.026
	(1.19)	(1.18)	(1.18)	(1.62)	(0.52)	(0.39)	(0.33)	(0.54)
	-0.005	0.012	0.034	0.007	0.005	0.016	0.069	0.070
	(0.32)	(0.84)	(1.41)	(0.38)	(0.31)	(0.77)	(1.87)	(1.73)
	-0.004	-0.013	0.043	-0.039	0.002	0.024	-0.035	-0.045
	(0.25)	(0.89)	(1.81)	(2.09)	(0.12)	(1.18)	(0.95)	(1.20)
	0.016	0.011	-0.038	0.029	-0.002	0.014	0.023	-0.000
	(0.87)	(0.66)	(1.50)	(1.42)	(0.11)	(0.64)	(0.54)	(-0.00)
	0.001	-0.006	-0.033	-0.010	-0.012	-0.056	-0.054	0.018
	(0.06)	(0.36)	(1.27)	(0.47)	(0.68)	(2.50)	(1.30)	(0.42)
	0.003	0.004	0.016	-0.002	0.003	0.023	-0.008	-0.029
	(0.17)	(0.25)	(0.61)	(0.09)	(0.17)	(1.03)	(0.20)	(0.68)
	-0.002	0.017	-0.015	0.029	0.008	-0.007	0.022	0.036
	(0.12)	(1.14)	(0.58)	(1.42)	(0.47)	(0.32)	(0.54)	(0.82)
	0.053	-0.008	0.006	-0.020	0.037	0.057	-0.042	0.033
	(2.57)	(0.46)	(0.20)	(0.85)	(1.83)	(2.30)	(0.91)	(0.67)
1985: Jan	-0.042	-0.001	0.020	0.021	-0.027	-0.032	0.069	-0.070
	(1.90)	(0.05)	(0.64)	(0.82)	(1.25)	(1.12)	(1.26)	(1.24)
	0.021	-0.007	-0.030	0.008	0.020	-0.020	-0.014	-0.005
	(1.02)	(0.39)	(0.98)	(0.33)	(1.02)	(0.72)	(0.26)	(0.09)
	-0.033	0.037	0.050	-0.015	0.003	0.027	-0.007	-0.039
	(1.73)	(2.08)	(1.66)	(0.62)	(0.16)	(1.03)	(0.14)	(0.71)
	0.015	-0.020	-0.009	0.022	-0.004	-0.027	-0.028	0.061
	(0.85)	(1.20)	(0.34)	(0.97)	(0.23)	(1.14)	(0.57)	(1.07)
	0.015	0.006	0.009	-0.004	0.001	0.030	0.060	-0.009
	(0.92)	(0.41)	(0.37)	(0.20)	(0.06)	(1.40)	(1.39)	(0.17)
	0.018	0.018	0.021	0.006	0.018	0.012	-0.006	0.039
	(1.13)	(1.26)	(0.87)	(0.31)	(1.11)	(0.57)	(0.16)	(0.90)
	-0.022	0.005	-0.013	-0.012	0.018	-0.000	-0.027	0.024
	(1.33)	(0.33)	(0.53)	(0.61)	(1.11)	(-0.00)	(0.68)	(0.57)

	0.021	-0.031	-0.036	-0.006	-0.030	-0.006	-0.033	-0.067
	(1.16)	(1.80)	(1.36)	(0.27)	(1.65)	(0.25)	(0.76)	(1.47)
	-0.023	-0.011	0.020	-0.001	0.003	-0.027	0.074	0.041
	(1.32)	(0.66)	(0.80)	(0.05)	(0.17)	(1.17)	(1.73)	(0.97)
	0.020	0.011	-0.004	0.014	-0.006	-0.001	-0.027	0.015
	(1.25)	(0.69)	(0.16)	(0.67)	(0.36)	(0.05)	(0.67)	(0.34)
	-0.003	0.004	-0.016	-0.005	0.016	-0.006	-0.030	0.036
	(0.19)	(0.26)	(0.62)	(0.24)	(0.97)	(0.28)	(0.74)	(0.81)
	-0.005	-0.006	0.014	-0.015	-0.010	0.009	-0.003	-0.084
	(0.24)	(0.34)	(0.49)	(0.61)	(0.51)	(0.37)	(0.06)	(1.79)
1986: Jan	-0.008	0.029	0.010	-0.006	0.007	0.026	0.024	0.025
	(0.35)	(1.51)	(0.33)	(0.22)	(0.33)	(0.95)	(0.43)	(0.48)
	0.042	-0.033	-0.002	0.031	0.010	-0.004	0.010	0.026
	(2.00)	(1.79)	(0.07)	(1.24)	(0.50)	(0.15)	(0.19)	(0.48)
	-0.001	0.007	-0.029	-0.011	0.014	-0.015	-0.021	-0.060
	(0.05)	(0.37)	(1.03)	(0.46)	(0.71)	(0.58)	(0.41)	(1.18)
	-0.012	0.008	0.034	0.030	-0.014	0.037	0.025	0.042
	(0.62)	(0.47)	(1.38)	(1.40)	(0.81)	(1.61)	(0.55)	(0.92)
	0.017	0.029	0.022	-0.002	0.035	0.004	0.008	0.049
	(1.02)	(2.11)	(0.96)	(0.11)	(2.28)	(0.20)	(0.22)	(1.20)
	0.024	0.002	0.001	-0.003	-0.003	0.009	0.059	-0.043
	(1.65)	(0.16)	(0.05)	(0.18)	(0.20)	(0.48)	(1.92)	(1.24)
	0.002	0.009	0.008	0.022	0.008	0.009	-0.033	0.049
	(0.13)	(0.69)	(0.38)	(1.32)	(0.53)	(0.50)	(1.02)	(1.42)
	0.006	0.001	-0.031	-0.020	-0.010	-0.008	-0.018	0.020
	(0.34)	(0.07)	(1.32)	(1.05)	(0.60)	(0.41)	(0.50)	(0.52)
	0.017	-0.017	0.012	0.000	0.007	0.009	0.000	-0.048
	(0.98)	(1.19)	(0.51)	(0.00)	(0.43)	(0.47)	(0.00)	(1.24)
	0.004	0.028	0.028	0.027	0.033	-0.011	-0.005	0.052
	(0.23)	(1.91)	(1.25)	(1.41)	(2.03)	(0.57)	(0.13)	(1.35)
	0.018	-0.002	0.021	-0.015	0.001	0.020	0.092	-0.007
	(1.03)	(0.14)	(0.93)	(0.78)	(0.06)	(1.02)	(2.40)	(0.17)
	0.011	0.037	-0.027	0.009	0.009	-0.007	-0.049	-0.033
	(0.54)	(2.18)	(1.02)	(0.40)	(0.48)	(0.31)	(1.01)	(0.71)
1987: Jan	0.075	0.012	0.019	0.067	0.024	-0.006	0.043	0.099
	(3.23)	(0.63)	(0.64)	(2.63)	(1.11)	(0.23)	(0.76)	(1.81)
	-0.053	-0.019	0.012	-0.039	-0.015	0.021	0.016	-0.100
	(2.35)	(1.04)	(0.40)	(1.58)	(0.70)	(0.81)	(0.30)	(1.74)
	0.025	0.033	0.000	-0.010	0.005	0.002	-0.011	0.055
	(1.15)	(1.93)	(0.00)	(0.43)	(0.25)	(0.08)	(0.22)	(1.03)
	0.034	-0.027	-0.006	0.034	0.015	-0.015	-0.040	-0.040
	(1.65)	(1.70)	(0.24)	(1.62)	(0.84)	(0.66)	(0.89)	(0.88)
	-0.013	0.020	0.027	0.004	0.030	0.057	0.052	0.094
	(0.67)	(1.35)	(1.20)	(0.21)	(1.84)	(2.82)	(1.33)	(2.31)
	0.048	0.024	-0.004	0.012	-0.009	-0.015	-0.010	-0.045
	(2.70)	(1.81)	(0.19)	(0.70)	(0.59)	(0.84)	(0.31)	(1.34)
	0.028	-0.002	0.044	0.011	0.025	0.025	0.023	-0.032
	(1.54)	(0.15)	(2.14)	(0.63)	(1.59)	(1.38)	(0.69)	(0.92)
	-0.009	0.023	-0.010	-0.011	0.017	0.008	-0.005	0.041
	(0.44)	(1.52)	(0.44)	(0.56)	(0.98)	(0.41)	(0.13)	(1.06)
	0.015	-0.018	-0.021	0.024	-0.025	-0.014	0.024	-0.020
	(0.76)	(1.22)	(0.92)	(1.22)	(1.47)	(0.73)	(0.63)	(0.54)
	0.019	0.027	0.041	-0.003	0.027	0.006	-0.024	0.020
	(1.01)	(1.82)	(1.84)	(0.16)	(1.61)	(0.31)	(0.65)	(0.52)
	0.038	-0.000	-0.015	0.014	0.010	-0.009	0.003	0.072
	(2.02)	(-0.00)	(0.68)	(0.76)	(0.60)	(0.48)	(0.08)	(1.81)
	0.027	0.041	0.005	0.024	-0.026	0.043	0.003	-0.087
	(1.24)	(2.49)	(0.19)	(1.12)	(1.32)	(1.86)	(0.07)	(1.93)
1988: Jan	0.046	0.029	0.045	0.025	0.130	0.005	0.067	0.098
	(1.79)	(1.51)	(1.54)	(1.03)	(5.68)	(0.19)	(1.34)	(1.88)

	-0.027	-0.027	-0.021	-0.068	-0.085	-0.012	-0.100	-0.056
	(1.14)	(1.45)	(0.79)	(3.05)	(3.93)	(0.48)	(2.14)	(1.13)
	0.026	0.024	0.007	0.089	0.060	0.040	0.059	0.025
	(1.18)	(1.36)	(0.27)	(4.15)	(2.93)	(1.69)	(1.29)	(0.51)
	0.031	0.001	0.005	-0.008	-0.019	-0.000	0.017	-0.002
	(1.54)	(0.06)	(0.21)	(0.40)	(1.00)	(-0.00)	(0.40)	(0.04)
	0.022	0.019	0.048	0.020	0.044	0.005	0.011	0.028
	(1.17)	(1.34)	(2.11)	(1.05)	(2.48)	(0.24)	(0.29)	(0.74)
	0.040	0.025	0.006	0.032	0.022	0.054	0.030	0.010
	(2.26)	(1.89)	(0.28)	(1.80)	(1.32)	(3.06)	(0.93)	(0.32)
	0.022	0.040	0.007	0.019	0.019	-0.030	0.045	-0.004
	(1.24)	(2.92)	(0.34)	(1.05)	(1.10)	(1.70)	(1.39)	(0.12)
	0.000	-0.030	-0.006	-0.019	-0.011	0.037	-0.062	0.043
	(0.00)	(1.93)	(0.27)	(0.98)	(0.58)	(1.98)	(1.70)	(1.11)
	-0.002	0.023	0.022	0.030	0.009	0.015	0.003	-0.035
	(0.11)	(1.56)	(1.02)	(1.56)	(0.48)	(0.81)	(0.08)	(0.93)
	0.028	0.009	-0.017	-0.001	0.009	-0.003	-0.012	0.049
	(1.49)	(0.61)	(0.76)	(0.05)	(0.47)	(0.16)	(0.32)	(1.31)
	-0.011	-0.009	0.008	-0.005	0.037	0.029	0.066	0.031
	(0.59)	(0.61)	(0.34)	(0.26)	(1.93)	(1.51)	(1.73)	(0.80)
	0.026	0.055	0.028	0.044	0.005	-0.018	-0.007	-0.106
	(1.34)	(3.43)	(1.11)	(2.06)	(0.25)	(0.83)	(0.17)	(2.59)
1989: Jan	0.026	0.033	0.025	0.048	0.027	0.052	-0.003	0.075
	(1.18)	(1.87)	(0.96)	(2.08)	(1.26)	(2.12)	(0.06)	(1.57)
	0.022	0.005	-0.018	-0.012	-0.016	-0.029	0.044	-0.001
	(1.01)	(0.29)	(0.69)	(0.54)	(0.81)	(1.20)	(0.84)	(0.02)
	0.012	-0.028	-0.029	0.015	0.043	-0.003	-0.031	0.026
	(0.58)	(1.69)	(1.13)	(0.70)	(2.32)	(0.13)	(0.60)	(0.55)
	0.032	0.034	0.078	0.019	0.001	0.022	0.015	-0.014
	(1.62)	(2.14)	(3.10)	(0.93)	(0.06)	(1.02)	(0.34)	(0.31)
	0.000	0.005	-0.010	0.027	0.009	0.029	0.007	0.037
	(0.00)	(0.33)	(0.42)	(1.41)	(0.53)	(1.46)	(0.18)	(0.94)
	0.023	0.038	0.014	0.016	0.037	0.029	0.064	0.012
	(1.32)	(2.73)	(0.65)	(0.93)	(2.33)	(1.59)	(1.90)	(0.36)
	0.034	0.001	0.042	0.058	0.033	0.037	-0.049	0.048
	(1.92)	(0.07)	(1.90)	(3.19)	(1.93)	(2.02)	(1.41)	(1.39)
	-0.012	0.007	-0.007	-0.066	-0.030	-0.022	0.025	-0.013
	(0.61)	(0.44)	(0.28)	(3.21)	(1.62)	(1.04)	(0.63)	(0.34)
	-0.027	-0.008	-0.029	0.057	0.065	0.015	-0.028	0.063
	(1.38)	(0.51)	(1.10)	(2.66)	(3.49)	(0.68)	(0.68)	(1.61)
	0.012	0.017	0.027	0.014	-0.067	-0.025	0.058	-0.025
	(0.62)	(1.02)	(0.98)	(0.62)	(3.43)	(1.06)	(1.32)	(0.58)
	-0.013	0.021	0.032	-0.004	0.032	0.008	-0.030	-0.060
	(0.67)	(1.25)	(1.24)	(0.18)	(1.66)	(0.34)	(0.65)	(1.37)
	-0.008	0.012	-0.025	-0.004	-0.021	0.024	0.002	-0.000
	(0.38)	(0.66)	(0.93)	(0.17)	(1.02)	(0.97)	(0.04)	(-0.00)
1990: Jan	0.094	0.072	0.127	0.036	0.137	0.076	0.120	0.132
	(3.64)	(3.09)	(3.65)	(1.28)	(5.28)	(2.49)	(2.15)	(2.27)
	-0.021	-0.055	-0.060	0.009	-0.085	-0.044	-0.080	-0.043
	(0.89)	(2.60)	(1.89)	(0.37)	(3.68)	(1.51)	(1.53)	(0.77)
	-0.029	0.041	0.038	-0.016	-0.017	0.038	0.024	-0.035
	(1.39)	(2.31)	(1.47)	(0.73)	(0.88)	(1.53)	(0.49)	(0.74)
	0.061	0.003	-0.005	0.024	0.025	-0.001	0.006	0.066
	(2.78)	(0.16)	(0.19)	(1.07)	(1.25)	(0.04)	(0.12)	(1.44)
	-0.019	-0.008	0.021	-0.013	0.017	-0.004	0.023	-0.010
	(0.88)	(0.45)	(0.83)	(0.61)	(0.91)	(0.17)	(0.52)	(0.24)
	0.007	0.017	0.006	0.076	0.001	0.045	0.017	-0.039
	(0.36)	(1.07)	(0.26)	(3.92)	(0.06)	(2.15)	(0.45)	(1.05)
	-0.032	0.015	0.005	-0.036	0.036	-0.016	0.022	0.088
	(1.64)	(0.93)	(0.22)	(1.87)	(2.08)	(0.80)	(0.61)	(2.47)

	0.059	-0.010	0.011	0.032	-0.001	-0.023	-0.009	0.010
	(2.76)	(0.56)	(0.45)	(1.55)	(0.05)	(1.09)	(0.23)	(0.25)
	-0.034	-0.004	0.012	-0.005	-0.018	0.017	-0.018	-0.003
	(1.63)	(0.22)	(0.50)	(0.23)	(0.95)	(0.79)	(0.47)	(0.08)
	0.025	0.023	-0.044	-0.024	0.000	-0.020	0.006	0.007
	(1.19)	(1.30)	(1.79)	(1.09)	(0.00)	(0.89)	(0.15)	(0.18)
	-0.008	-0.021	0.014	0.029	-0.010	0.012	0.006	-0.056
	(0.38)	(1.19)	(0.59)	(1.38)	(0.53)	(0.54)	(0.15)	(1.44)
	-0.002	0.076	0.008	0.030	0.030	0.056	0.045	0.045
	(0.09)	(4.23)	(0.31)	(1.35)	(1.52)	(2.45)	(1.05)	(1.17)
1991: Jan	0.081	0.016	0.098	0.085	0.123	0.029	0.012	0.096
	(4.06)	(0.91)	(3.65)	(3.79)	(6.12)	(1.21)	(0.26)	(2.23)
	-0.044	-0.064	-0.042	-0.085	-0.096	-0.036	0.015	-0.097
	(2.71)	(3.95)	(1.59)	(4.15)	(5.42)	(1.52)	(0.33)	(2.17)
	-0.023	0.033	-0.011	0.037	0.016	-0.009	-0.072	0.072
	(1.32)	(1.93)	(0.40)	(1.70)	(0.84)	(0.37)	(1.61)	(1.56)
	0.049	-0.012	-0.012	0.008	0.007	-0.002	0.040	-0.020
	(3.07)	(0.76)	(0.45)	(0.38)	(0.37)	(0.09)	(0.93)	(0.46)
	-0.012	0.045	0.043	-0.009	0.002	0.028	0.007	-0.009
	(0.81)	(2.90)	(1.69)	(0.47)	(0.12)	(1.38)	(0.18)	(0.24)
	0.001	-0.029	-0.004	0.027	0.010	-0.007	0.030	-0.013
	(0.07)	(1.85)	(0.17)	(1.45)	(0.62)	(0.35)	(0.80)	(0.36)
	0.015	0.011	0.007	0.005	-0.019	0.022	0.011	0.053
	(0.92)	(0.68)	(0.30)	(0.25)	(1.10)	(1.05)	(0.28)	(1.37)
	-0.036	-0.019	-0.014	0.013	0.017	-0.010	-0.013	-0.016
	(1.91)	(1.01)	(0.51)	(0.61)	(0.91)	(0.44)	(0.31)	(0.37)
	0.009	0.013	-0.033	-0.008	-0.024	0.005	-0.019	-0.056
	(0.48)	(0.68)	(1.20)	(0.37)	(1.31)	(0.22)	(0.47)	(1.29)
	-0.028	-0.026	0.074	-0.019	0.024	0.006	0.082	0.047
	(1.52)	(1.43)	(2.77)	(0.88)	(1.26)	(0.25)	(1.97)	(1.06)
	0.011	0.028	-0.034	0.026	-0.032	-0.009	-0.148	-0.029
	(0.58)	(1.53)	(1.22)	(1.19)	(1.62)	(0.36)	(3.29)	(0.65)
	0.010	-0.042	0.007	-0.095	-0.061	-0.029	0.096	-0.030
	(0.40)	(1.85)	(0.22)	(3.57)	(2.66)	(0.95)	(1.81)	(0.56)
1992: Jan	-0.070	0.004	-0.032	0.096	0.056	0.006	0.043	0.076
	(2.49)	(0.16)	(0.93)	(3.13)	(2.22)	(0.17)	(0.72)	(1.21)
	0.026	-0.030	0.053	-0.053	-0.022	-0.029	-0.142	-0.065
	(0.98)	(1.20)	(1.50)	(1.79)	(0.91)	(0.84)	(2.29)	(1.06)
	-0.003	0.011	-0.051	-0.004	-0.012	-0.038	0.033	0.032
	(0.12)	(0.44)	(1.47)	(0.14)	(0.49)	(1.15)	(0.54)	(0.55)
	-0.027	-0.006	-0.001	0.000	0.010	0.064	0.045	-0.073
	(1.19)	(0.26)	(0.03)	(0.00)	(0.43)	(2.03)	(0.79)	(1.30)
	-0.021	0.008	0.022	-0.001	-0.031	-0.045	-0.025	0.028
	(0.94)	(0.36)	(0.68)	(0.04)	(1.44)	(1.51)	(0.49)	(0.54)
	-0.021	0.000	-0.005	-0.026	0.016	0.023	0.014	0.025
	(0.93)	(0.00)	(0.17)	(1.07)	(0.77)	(0.81)	(0.29)	(0.54)
	-0.024	-0.026	-0.003	0.017	-0.024	0.008	-0.048	-0.018
	(1.05)	(1.14)	(0.10)	(0.68)	(1.08)	(0.29)	(0.99)	(0.36)
	-0.021	-0.038	-0.068	-0.045	-0.020	-0.059	0.026	-0.039
	(0.85)	(1.48)	(2.12)	(1.64)	(0.81)	(1.94)	(0.51)	(0.69)
	-0.038	-0.007	0.086	0.012	0.010	0.002	-0.019	0.020
	(1.43)	(0.28)	(2.58)	(0.42)	(0.41)	(0.06)	(0.35)	(0.34)
	-0.011	-0.023	-0.048	-0.046	-0.084	-0.030	0.038	-0.052
	(0.38)	(0.85)	(1.34)	(1.52)	(3.11)	(0.85)	(0.65)	(0.79)
	-0.032	-0.040	-0.039	-0.014	-0.002	0.027	-0.148	0.002
	(1.13)	(1.43)	(1.08)	(0.45)	(0.07)	(0.72)	(2.35)	(0.03)
	-0.045	0.020	-0.068	-0.055	0.014	-0.088	0.182	-0.043
	(1.45)	(0.60)	(1.70)	(1.43)	(0.46)	(2.07)	(2.36)	(0.58)
1993: Jan	0.054	-0.038	0.059	0.019	-0.037	0.062	-0.164	-0.003
	(1.62)	(1.05)	(1.37)	(0.44)	(1.11)	(1.26)	(2.08)	(0.03)

	-0.075	0.033	-0.030	0.035	-0.001	-0.010	-0.100	0.080
	(2.45)	(0.98)	(0.71)	(0.91)	(0.03)	(0.22)	(1.34)	(0.92)
	-0.002	-0.037	-0.011	-0.026	0.032	-0.061	0.151	-0.084
	(0.07)	(1.17)	(0.27)	(0.77)	(1.08)	(1.53)	(2.22)	(1.01)
	0.059	0.038	0.028	-0.001	-0.025	0.046	-0.081	0.040
	(2.40)	(1.30)	(0.76)	(0.03)	(0.95)	(1.24)	(1.35)	(0.55)
	-0.053	-0.035	0.010	0.027	0.056	0.044	0.038	0.052
	(2.22)	(1.30)	(0.28)	(0.94)	(2.32)	(1.25)	(0.72)	(0.88)
	0.046	0.040	0.028	-0.055	-0.052	-0.054	0.013	-0.054
	(2.00)	(1.54)	(0.82)	(2.07)	(2.31)	(1.56)	(0.26)	(0.99)
	-0.024	-0.005	-0.025	0.024	0.048	0.049	-0.002	0.037
	(0.95)	(0.17)	(0.54)	(0.78)	(1.93)	(1.28)	(0.03)	(0.59)
1993: Aug	-0.004	-0.014	-0.043	-0.024	0.050	0.051	-0.019	-0.306
	(0.06)	(0.21)	(0.38)	(0.28)	(0.69)	(0.51)	(0.11)	(1.33)

Note: Equation also includes a large number of variables reflecting the quality and amenity of individual dwellings. See Englund, et al., 1998, Table IV.

Appendix Table B.1

VAR Model with Five Assets
Quarterly Data: 1982 I through 1993 II

	R.E. Stocks		*Gen Stocks*		*T-bill*		*Bond*		*House*	
R.E. Stocks, t-1	-0.0267	(0.0862)	-0.0463	(0.1689)	-0.0327	(1.2673)	0.0403	(0.4458)	0.0146	(0.3634)
R.E. Stocks, t-2	0.5703	(1.9616)	0.3356	(1.3057)	0.0046	(0.1894)	0.0657	(0.7743)	0.1391	(3.7004)
R.E. Stocks, t-3	-0.3907	(1.0727)	-0.2790	(0.8665)	0.0387	(1.2738)	-0.2069	(1.9456)	0.1242	(2.6382)
R.E. Stocks, t-4	-0.6479	(1.4259)	-0.0088	(0.0220)	0.0761	(2.0105)	0.3178	(2.3954)	0.0799	(1.3603)
Gen Stocks, t	0.4664	(1.2062)	0.4874	(1.4261)	0.0091	(0.2828)	0.0085	(0.0753)	-0.0518	(1.0363)
Gen Stocks, t-2	-0.5491	(1.5524)	-0.4666	(1.4924)	-0.0027	(0.0931)	-0.1163	(1.1264)	-0.1168	(2.5545)
Gen Stocks, t-3	0.6769	(1.6935)	0.4907	(1.3886)	-0.0776	(2.3312)	0.1605	(1.3748)	-0.1113	(2.1540)
Gen Stocks, t-4	0.2166	(0.4874)	-0.2736	(0.6964)	-0.0812	(2.1924)	-0.4212	(3.2463)	-0.0823	(1.4327)
T-bill, t-1	0.3049	(0.1272)	-0.3843	(0.1813)	-0.3595	(1.7999)	-1.2086	(1.7266)	-0.7212	(2.3266)
T-bill, t-2	1.1395	(0.4652)	0.4621	(0.2134)	0.2426	(1.1885)	1.3475	(1.8841)	-0.4096	(1.2934)
T-bill, t-3	6.8644	(2.3700)	4.7512	(1.8557)	-0.1337	(0.5538)	0.1577	(0.1865)	0.2099	(0.5605)
T-bill, t-4	2.1984	(0.8143)	-2.1389	(0.8962)	-0.0268	(0.1193)	-0.6922	(0.8782)	0.1286	(0.3685)
Bond, t-1	-0.7503	(1.1301)	0.0195	(0.0331)	0.0200	(0.3614)	0.2224	(1.1473)	0.1601	(1.8655)
Bond, t-2	-0.7113	(1.1696)	-0.2588	(0.4814)	0.0368	(0.7265)	-0.1413	(0.7956)	0.1368	(1.7391)
Bond, t-3	-0.2158	(0.3258)	-0.1469	(0.2508)	0.1143	(2.0708)	0.2255	(1.1660)	0.1066	(1.2448)
Bond, t-4	-0.5097	(0.7675)	0.2202	(0.3750)	0.1309	(2.3648)	-0.1781	(0.9186)	-0.0028	(0.0320)
House, t-1	2.7190	(1.4996)	0.4812	(0.3002)	0.0316	(0.2093)	0.3578	(0.6759)	-0.0532	(0.2269)
House, t-2	2.7806	(1.6294)	1.0475	(0.6944)	-0.1942	(1.3658)	-0.5979	(1.2001)	0.3071	(1.3921)
House, t-3	-1.5334	(0.9683)	-0.5320	(0.3800)	-0.1297	(0.9829)	-0.6392	(1.3824)	-0.0296	(0.1448)
House, t-4	-2.6471	(1.7865)	-1.0180	(0.7772)	0.0792	(0.6412)	0.2703	(0.6247)	0.1095	(0.5717)
Constant	-0.1041	(1.6038)	-0.0037	(0.0644)	0.0170	(3.1446)	0.0371	(1.9544)	0.0155	(1.8441)
R^2 adjusted	0.2099		-0.0173		0.1991		0.3008		0.7192	
Q-statistics	1.6900		0.6286		1.2679		1.5033		1.1645	
$F_{\text{real estate}}$	1.5514	(0.2181)	0.6224	(0.6508)	1.9151	(0.1392)	3.6806	(0.0173)	5.4469	(0.0027)
F_{stocks}	1.4109	(0.2593)	1.4352	(0.2517)	2.3909	(0.0778)	3.6590	(0.0177)	3.2926	(0.0268)
$F_{\text{t-bills}}$	1.8780	(0.1457)	1.2327	(0.3224)	1.6750	(0.1872)	1.6508	(0.1929)	1.8899	(0.1436)
F_{bonds}	0.7199	(0.5864)	0.1114	(0.9774)	2.6823	(0.0548)	0.9308	(0.4621)	1.5894	(0.2081)
F_{houses}	1.2523	(0.3148)	0.2218	(0.9237)	1.4851	(0.2367)	1.5324	(0.2233)	2.0219	(0.1220)

This table shows the slope coefficients of a VAR model along with adjusted R^2, Box-Pierce Q-statistics and Granger-causality test results, expressed as F-statistics. The values in the parenthesis with the estimated parameters are t-statistics and the values in the parenthesis with Granger-causality tests are p-values.

Estimated Variance-Covariance Matrix of Residuals

	R.E. Stocks	*Gen Stocks*	*T-bill*	*Bond*	*House*
R.E. Stocks	0.0103826	0.0074665	-0.0001042	0.0009854	0.0003317
Gen Stocks	0.0074665	0.0081131	-0.0000167	0.0012473	0.0001372
T-bill	-0.0001042	-0.0000167	0.0000721	0.0000882	0.0000391
Bond	0.0009854	0.0012473	0.0000882	0.0008851	0.0001490
House	0.0003317	0.0001372	0.0000391	0.0001490	0.0001736

Appendix B

In this appendix, we indicate how the variance-covariance matrix of asset returns is estimated for different time periods using the 164 months of home sales data available. It is convenient

To extract $\mathbf{\Gamma}_k$, first note that

$$\text{(A.3)} \qquad \mathbf{\Sigma} = E\left(\xi_t \xi_t'\right) = \begin{bmatrix} \mathbf{\Gamma}_0 & \mathbf{\Gamma}_1 & \mathbf{\Gamma}_2 & \mathbf{\Gamma}_3 \\ \mathbf{\Gamma}_1' & \mathbf{\Gamma}_0 & \mathbf{\Gamma}_1 & \mathbf{\Gamma}_2 \\ \mathbf{\Gamma}_2' & \mathbf{\Gamma}_1' & \mathbf{\Gamma}_0 & \mathbf{\Gamma}_1 \\ \mathbf{\Gamma}_3' & \mathbf{\Gamma}_2' & \mathbf{\Gamma}_1' & \mathbf{\Gamma}_0 \end{bmatrix}.$$

Note also that

$$\text{(A.4)} \qquad \mathbf{\Sigma} = E\left(\xi_t \xi_t'\right) = E\left[\left(\mathbf{F}\,\xi_{t-1} + \mathbf{v}_t\right)\left(\mathbf{F}\,\xi_{t-1} + \mathbf{v}_t\right)'\right] = \mathbf{F\Sigma F}' + \mathbf{Q}.$$

Therefore,

$$\text{(A.5)} \qquad vec(\mathbf{\Sigma}) = \left[\mathbf{I} - (\mathbf{F} \otimes \mathbf{F})\right]^{-1} vec(\mathbf{Q}).$$

From this, we can recover $\mathbf{\Gamma}_0$ thru $\mathbf{\Gamma}_3$. The remaining elements of $\mathbf{\Gamma}_k$ may be recovered by recursion,

$$\text{(A.6)} \qquad \mathbf{\Gamma}_k = \mathbf{\Phi\Gamma}_{k-1} + \mathbf{\Phi\Gamma}_{k-2} + \mathbf{\Phi\Gamma}_{k-3} + \mathbf{\Phi\Gamma}_{k-4}.$$

Now, consider adding an individual house, whose return follows an AR(1) process, to the the above system. Note that

$$\text{(A.7)} \qquad r_t^H - \mu^H = \sum_j \sum_{i=1}^{4} \beta_i^{H,j}\left(r_{t-i}^j - \mu^j\right) + \varepsilon_t^H \text{ where } r_t^H \text{ is the return on the housing index.}$$

Let r_t^h be the log return on an individual house. Since

$$\text{(A.8)} \qquad r_t^h - \mu^h = r_t^H - \mu^H + \nu_t - \nu_{t-1}, \text{ where}$$

$$\text{(A.9)} \qquad \nu_t = \rho\nu_{t-1} + \eta_t,$$

then the return on individual house can be written as

$$\text{(A.10)} \qquad r_t^h - \mu^h = \sum_j \sum_{i=1}^{4} \beta_i^{H,j}\left(r_{t-i}^j - \mu^j\right) + \nu_t - \nu_{t-1} + \varepsilon_t^H$$

$$= \sum_j \sum_{i=1}^{4} \beta_i^{H,j}\left(r_{t-i}^j - \mu^j\right) + (\rho - 1)\nu_{t-1} + \eta_t + \varepsilon_t^H \quad .$$

Note that (A.10) is in VAR form with ν_{t-1} as a new variable. That is, we can simply augment (A.1) with (A.8) and (A.10), such that

(A.11)
$$\begin{bmatrix} r_t^R-\mu^R \\ r_t^E-\mu^E \\ r_t^N-\mu^N \\ r_t^L-\mu^L \\ r_t^H-\mu^H \\ r_t^h-\mu^H \\ \nu_t \end{bmatrix} = \begin{bmatrix} \beta_1^{R,R} & \beta_1^{R,E} & \beta_1^{R,N} & \beta_1^{R,L} & \beta_1^{R,H} & 0 & 0 \\ \beta_1^{E,R} & \beta_1^{E,E} & \beta_1^{E,N} & \beta_1^{E,L} & \beta_1^{E,H} & 0 & 0 \\ \beta_1^{N,R} & \beta_1^{N,E} & \beta_1^{N,N} & \beta_1^{N,L} & \beta_1^{N,H} & 0 & 0 \\ \beta_1^{L,R} & \beta_1^{L,E} & \beta_1^{L,N} & \beta_1^{L,L} & \beta_1^{L,H} & 0 & 0 \\ \beta_1^{H,R} & \beta_1^{H,E} & \beta_1^{H,N} & \beta_1^{H,L} & \beta_1^{H,H} & 0 & 0 \\ \beta_1^{H,R} & \beta_1^{H,E} & \beta_1^{H,N} & \beta_1^{H,L} & \beta_1^{H,H} & 0 & \rho-1 \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho \end{bmatrix} \begin{bmatrix} r_{t-1}^R-\mu^R \\ r_{t-1}^E-\mu^E \\ r_{t-1}^N-\mu^N \\ r_{t-1}^L-\mu^L \\ r_{t-1}^H-\mu^H \\ r_{t-1}^h-\mu^H \\ \nu_{t-1} \end{bmatrix}$$

$$\tilde{\mathbf{r}}_t-\boldsymbol{\mu} = \tilde{\boldsymbol{\Phi}}_1\left(\tilde{\mathbf{r}}_{t-1}-\tilde{\boldsymbol{\mu}}\right)$$

$$+\sum_{i=2}^{4}\begin{bmatrix} \beta_i^{R,R} & \beta_i^{R,E} & \beta_i^{R,N} & \beta_i^{R,L} & \beta_i^{R,H} & 0 & 0 \\ \beta_i^{E,R} & \beta_i^{E,E} & \beta_i^{E,N} & \beta_i^{E,L} & \beta_i^{E,H} & 0 & 0 \\ \beta_i^{N,R} & \beta_i^{N,E} & \beta_i^{N,N} & \beta_i^{N,L} & \beta_i^{N,H} & 0 & 0 \\ \beta_i^{L,R} & \beta_i^{L,E} & \beta_i^{L,N} & \beta_i^{L,L} & \beta_i^{L,H} & 0 & 0 \\ \beta_i^{H,R} & \beta_i^{H,E} & \beta_i^{H,N} & \beta_i^{H,L} & \beta_i^{H,H} & 0 & 0 \\ \beta_i^{H,R} & \beta_i^{H,E} & \beta_i^{H,N} & \beta_i^{H,L} & \beta_i^{H,H} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} r_{t-i}^R-\mu^R \\ r_{t-i}^E-\mu^E \\ r_{t-i}^N-\mu^N \\ r_{t-i}^L-\mu^L \\ r_{t-i}^H-\mu^H \\ r_{t-i}^h-\mu^H \\ \nu_{t-i} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^R \\ \varepsilon_t^E \\ \varepsilon_t^N \\ \varepsilon_t^L \\ \varepsilon_t^H \\ \eta_t+\varepsilon_t^H \\ \eta_t \end{bmatrix}$$

$$+\sum_{i=2}^{4}\tilde{\boldsymbol{\Phi}}_i\left(\tilde{\mathbf{r}}_{t-i}-\tilde{\boldsymbol{\mu}}\right) + \tilde{\boldsymbol{\varepsilon}}_t$$

where

$$E\left(\tilde{\boldsymbol{\varepsilon}}_t\tilde{\boldsymbol{\varepsilon}}_t'\right)=\tilde{\boldsymbol{\Omega}}=\begin{bmatrix} \sigma_R^2 & \sigma_{R,E} & \sigma_{R,N} & \sigma_{R,L} & \sigma_{R,H} & \sigma_{R,H} & 0 \\ \sigma_{R,E} & \sigma_E^2 & \sigma_{E,N} & \sigma_{E,L} & \sigma_{E,H} & \sigma_{E,H} & 0 \\ \sigma_{R,N} & \sigma_{E,N} & \sigma_N^2 & \sigma_{N,L} & \sigma_{N,H} & \sigma_{N,H} & 0 \\ \sigma_{R,L} & \sigma_{E,L} & \sigma_{N,L} & \sigma_L^2 & \sigma_{L,H} & \sigma_{L,H} & 0 \\ \sigma_{R,H} & \sigma_{E,H} & \sigma_{N,H} & \sigma_{L,H} & \sigma_H^2 & \sigma_H^2 & 0 \\ \sigma_{R,H} & \sigma_{E,H} & \sigma_{N,H} & \sigma_{L,H} & \sigma_H^2 & \sigma_H^2+\sigma_\eta^2 & \sigma_\eta^2 \\ 0 & 0 & 0 & 0 & 0 & \sigma_\eta^2 & \sigma_\eta^2 \end{bmatrix}$$

Thus, to compute the variance-covariance matrix of n period returns, we only need compute

(A.12) $$vec\left(\tilde{\boldsymbol{\Sigma}}\right)=\left[\mathbf{I}-\left(\tilde{\mathbf{F}}\otimes\tilde{\mathbf{F}}\right)\right]^{-1}vec\left(\tilde{\mathbf{Q}}\right),$$

and we can extract $\left\{\tilde{\boldsymbol{\Gamma}}_0,\tilde{\boldsymbol{\Gamma}}_1,\tilde{\boldsymbol{\Gamma}}_3,\cdots,\tilde{\boldsymbol{\Gamma}}_T\right\}$ from $\tilde{\boldsymbol{\Sigma}}$ as in the previous example.