Spatial Sorting and Inequality∗

Rebecca Diamond† and Cecile Gaubert ‡

April 11, 2022

Abstract

The spatial segregation of college and non-college educated workers between commuting zones in the U.S. has steadily grown since 1980. We summarize prior work on sorting and location and document new descriptive patterns on how sorting and locations have changed over the past four decades. We find that there has been a shift in the sorting of college-educated workers from cities primarily centered around production in 1980 to cities centered around consumption by 2017. We develop a spatial equilibrium model to understand these patterns, and highlight key places where further research is needed. Our framework helps understand the causes and consequences of changes in spatial sorting, their impact on inequality, and how they respond to, and feed into, the changing nature of cities.

The dramatic increase in the wage gap between college-educated and lower-skill workers over the past four decades has been accompanied by a substantial increase in the geographic sorting of workers by skill. We review the literature that studies the causes of these changes in spatial sorting and their consequences on inequality, and policy.

In terms of scope, our analysis is focused on studying sorting between cities in the U.S., leaving aside related questions that the literature has been tackling, and in particular the small but burgeoning literature that studies within-city sorting.1 We focus on sorting by education level, and specifically on the location choices of two worker groups: those with a 4-year college degree versus those without, following previous work such as Moretti (2013) and Diamond (2016).2 We refer to these groups as high-skill and low-skill workers.

1We thank Matt Tauzer for excellent research assistance. Diamond acknowledges support from the National Science Foundation (CAREER Grant 1848036). Cecile Gaubert acknowledges support from NSF CAREER grant 1941917.

†Stanford and NBER.
‡UC Berkeley, NBER and CEPR.

1This literature has a close connection in methodology and questions with our object of study. We refer the interested reader to papers analyzing trends in neighborhood change and gentrification in U.S. cities (Guerrieri et al., 2013; Su, 2019; Couture and Handbury, 2020; Baum-Snow and Hartley, 2020; Couture et al., orth; Almagro and Domínguez-Iino, 2021; Hoelzlein, 2020) changes in transportation infrastructure (Tsivanidis, 2019), public school choice (Bayer et al., 2007) and their impacts on sorting.

2Some other studies have focused on sorting by income level and shown that residential income segregation in the United States has been continuously rising since the 1980s (Reardon and Bischoff, 2011; Reardon et al., 2018; Gaubert et al., 2021). We prefer to focus on skill sorting because income is shaped, in part, by one’s place of residence.
In Section 1, we document stylized facts related to changes in spatial skill sorting from 1980 to today. We show in particular that the high skilled have shifted from sorting into cities primarily centered around production in 1980 to cities centered around consumption by 2017. Section 2 develops a spatial equilibrium model with heterogeneous agents to think through these descriptive patterns. It highlights the important feedback loops that exist between changes in location choices of skill groups, and endogenous changes in location characteristics (such as wage, rents and amenities). We use this template to organize our review of the existing literature, and flag where more research is needed. Section 3 discusses the implications of spatial sorting for the measurement of inequality and for policy. Section 4 concludes.

1 Measuring spatial sorting and inequality

There is a wide variety of statistics that quantify segregation and sorting of different groups across geographic areas. We focus on the exposure gap index to measure how high- and low-skill workers tend to live in areas with systematically different characteristics, such as average wages, housing costs or indicators of quality-of-life. The exposure gap at time $t$ for characteristic $Y$ is defined as:

$$Exposure_t = \sum_j \frac{H_{jt}Y_{jt}}{H_{kt}} - \sum_j \frac{L_{jt}Y_{jt}}{L_{kt}},$$

where $H_{jt}$ and $L_{jt}$ are the number of high- and low-skill workers living in location $j$, and $Y_{jt}$ is some characteristic of location $j$. Conceptually, these exposure gaps tell us, first, how different the average location experienced by high-skill workers is from the average location of low-skill workers. Intuitively, they therefore shed light on how sorting may contribute to inequality, in terms of income and quality of life - an analysis we complement in Section 2 with corresponding theoretically-consistent measures of well-being inequality. Second, when $Y_{jt}$ is the high-skill share of location $j$ ($Y_{jt} = \frac{H_{jt}}{H_{jt} + L_{jt}}$) the exposure gap constitutes a measure of segregation itself.

We follow Diamond (2016) and focus on full-time full-year employed workers between the ages of 25 to 55 to study worker location sorting. We use the 1980 and 2000 5% samples of micro data from the Decennial Censuses (Ruggles et al., 2021). To track the most recent evolutions in location choices, we use the 2015-2019 5-year pooled American Community Survey sample and label this as year 2017, the average year of the 5-year ACS data. We define a city based on the 1990 commuting zones. The Census and ACS public-use data report households’ place of residence at the “public-use micro area” (PUMA) level. We translate these to 1990 commuting zones based on each PUMA’s population overlap with each commuting zone.\(^3\)

1.1 Spatial Skill Sorting: 1980-2017

We begin by documenting the level and change in spatial skill sorting from 1980 to 2017. We measure the exposure gap of college vs. non-college workers to the local high-skill share, a measure

\(^3\)We use crosswalks provided by David and Dorn (2013).
of spatial skill segregation as explained above. Column 1 of Panel A in Table 1 shows that in 1980, the average college graduate lived in a commuting zone (CZ) with a high-skill share 1.9 percentage points (pp) higher than the average non-college graduate. This gap had increased to 3.1 pp by 2000, and 3.9 pp by 2017 (Columns 4 and 7). Therefore, high and low-skill workers have been and are still moving away from each other. However, interestingly, the speed of divergence has slowed substantially over the two last decades. Of course, these measures may be mechanically driven by the nationwide growth in high-skill share. However, repeating the analysis holding fixed this nationwide share fixed at its 1980 level (but allowing sorting patterns to change) paints a similar picture of deceleration in spatial segregation at the commuting zone level. Indeed, with a fixed aggregate share of high skilled, the exposure gap would have increased to 2.6 pp in 2000 and 2.9 pp in 2017. While it is clear that segregation is increasing, the economic magnitude of this index is a bit hard to interpret. To help interpret these magnitudes, we investigate how the average high skill worker’s CZ differs from the average low skill worker’s CZ along a variety of dimensions.

Spatial skill sorting has been increasing. We turn to documenting how this sorting and its change over time contributes to differences in earnings, housing costs and quality of life experienced by college and non college workers.

1.2 Geographic differences in earnings

1.2.1 Measuring exposure gaps in earnings

A key reason why high- and low-skill workers may choose different commuting zones is local labor market conditions: commuting zones that pay high wages for high-skill labor need not be the same places that pay the best wages for lower-skill labor. To measure CZ wages, we run a regression using the Census/ACS micro data on log earnings where we control for a quartic in age, race dummies, and gender. CZ-skill group fixed effects proxy for local wages. Exposure indexes based on these measures are reported in Panel B of Table 1, which we now comment.

In 1980, the average college worker lived in a CZ that paid high-skill workers 2.6 percent more than what they would earn where the average non-college worker lived. This accords with the intuition that high-skill workers choose to locate in commuting zones that pay them well. However, interestingly, these same locations preferred by the high skill also paid non-college workers 2.6 percent more than locations preferred by low-skilled workers, which is more surprising as it shows that the low-skill chose to live in commuting zones that offered them lower wages. Overall, the different locations choices of high- and low-skill workers at the time did not seem to reflect the comparative advantages of CZs in high- versus low-skill labor.

As sorting intensifies from 1980 to 2000, the earnings premium of high-skill locations increase and a comparative advantage wage gap opens up. By 2000, high-skill workers lived in CZs that paid them 4.8 percent more than the CZs chosen by low-skill workers. These places still also paid low-skill workers more, but only by 3.9 percent. The high-skill wage premium was therefore 0.9 pp

---

4These wages are not adjusted for any differences in local prices or purchasing power.
higher in the average high-skill location. To tease out how much of these changes are driven by places changing over time versus migration patterns, we hold fixed the location choices of workers in 1980, but allow wages to evolve as observed in the data from 1980 to 2000. The corresponding exposure gap captures what we call a place effect (Column 2 of Table 1). The remainder to explain the total change in exposure (reported in Column 4) is due to differences in net migrations between skill group. We call it the sorting effect (reported in Column 3). We find an important role played by place effects, compared to sorting effects: place effects drives two thirds of the growth in the college wage exposure gap, and half of the growth in the non-college wage exposure gap.

From 2000 to 2017, the earnings premium of high-skill locations increases further to 5.4 percent for college workers and 3.0 percent for non-college workers, so that the comparative advantage wage gap spikes to 2.4 pp. This increasingly wide skilled wage premium in high-skill CZs echoes the findings of Autor (2019). He finds that historically dense (and thus high-skill) cities paid high wages to middle- and high-skill labor in 1980, but that the urban wage premium to middle-skill work has eroded and is essentially non-existent today, while, in contrast, the urban wage premium to high-skill work has continued to intensify. We find that this increase in the skilled wage premium in high-skill CZs is entirely due to place effects and not driven by differential migration between college and non-college workers. Specifically, locations historically chosen by the high skill saw their college wages increase by 0.8 pp while their non-college wages decreased by 0.7 pp. Migration actually contributed to slightly narrowing the exposure wages gaps (-0.02 pp). While small in magnitude, this pattern stands in stark contrast to the 1980-2000 period where high-skill workers were migrating to places that paid them especially well. Overall, in the past 20 years, high-skill workers have been differentially migrating to places that pay a high wage, but less so than where they lived in 2000, so that migration tied to labor market conditions appears to be waning. Investigating this change in migration patterns is a ripe place for future research.

1.2.2 Place effect or sorting on unobserved ability?

A key question in measuring differences in local labor markets across space is whether the observed wage differences across space represent the true causal effect of place on earnings. Alternatively, there could be sorting of workers based on unobserved ability measures that confound measurement of earnings differentials across space: CZ’s that appear to pay high wages for a given skill group might actually just hire especially high ability workers. Glaeser and Mare (2001) first investigated this question using survey data from the NLSY and PSID and analyzing wages of movers. Their findings suggest that places did impact earnings substantially, but that these earnings effects accrued slowly over time. More recent work using administrative data has built on this study. Using French administrative panel earnings data, Combes et al. (2008) show that 40-50% of the observed differences in mean wages across space are due to worker sorting. They also find that place effects due to agglomeration are important. Using Spanish administrative data, De La Roca and Puga (2016) find that worker sorting on unobserved initial ability plays essentially no role in cross-city earnings differentials. Instead, they find important differences in human capital acquisi-
tion across cities, where large, high-wage cities enable workers to accumulate skills that they can
can take with them to other cities if they were to move. These differences in city-acquired human
capital explain about half of the cross-sectional differences in mean earnings. Dauth et al. (2018)
performs similar analysis using German data, but focuses more on the importance of worker-firm
match effects and how they vary by city size. They find that worker characteristics (observed and
unobserved) explain about 40 percent of the cross-sectional variance in wages across cities, and that
large cities allow workers and firms to match better. Most recently, Card et al. (2021) uses U.S.
administrative data to study worker moves. A key advance of their work is to study the impact
of place on earnings separately by workers’ education level. They find that, for low-skill (resp.
high-skill) workers, sorting on ability explains 33 (resp. 53) percent of the cross-sectional variation
in CZ earnings. High-skill workers are much more sorted by ability into high-wage CZs. Indeed,
the higher-skilled wage premium found in large cities seems to be entirely due workers sorting on
unobserved ability.

A key question for future research would be to understand how sorting on unobserved ability
has changed over time. Baum-Snow and Pavan (2013) show that the positive relationship between
city size and wage inequality only developed after 1990. Card et al. (2021) uses data from 2010 to
2018, while the initial work by Glaeser and Mare (2001) used data mostly from before 1990. How
much of the growth in the positive relationship between city size and wage inequality is due to
worker sorting versus place effects? The exposure analysis in Section 1.2 suggests a slow down in
skill sorting on labor market earnings over the past two decades, relative to the 1980-2000 period.
At the same time, the literature focused on measuring unobserved worker skills shows a very high
level of ability sorting within the college-educated group. Has sorting within skill-group become the
more dominant force, compared to between-group sorting? Reconciling the literature focused on
changes in sorting and wage premia across space with the literature focused on unobserved ability
sorting in the cross-section is still a very open research topic.

1.3 Geographic differences in local prices

High-skill workers are increasingly located in high-paying CZ’s compared to low-skill workers,
contributing to the increase in nationwide wage inequality. At the same time, Moretti (2013) shows
that these locations also tend to have high housing costs, a force that mitigates the increase in
nationwide real wage inequality. We therefore continue our empirical analysis by zooming in on
changes in exposure gaps to housing affordability between college and non-college workers. To
measure housing affordability, we use the Census/ACS micro data on log monthly gross rents (for
renters) and log housing values (for owners) and regress them on a CZ-fixed effect controlling for
year built, number of units in the structures and number of bedrooms. The estimated a CZ-fixed
effect is our measure of local housing costs.

Panel B of Table 1 shows that in 1980, the average high-skill worker lived in a CZ that cost 4.5
percent more in rent and 7.3 percent more in home values than the average location of low-skill
workers. Therefore, already in 1980, sorting was much more apparent in housing costs than in
wages, consistent with the notion that housing costs place a disproportionate burden on lower-skill, lower-income workers. By 2000, these differences had increased: the average high-skill worker lived in a CZ that cost 7.7 percent more in rent and 10.6 percent more in housing value than the average CZ of low-skilled workers. Similar to what we saw for wages, changes in place effects play a dominant role in this change (about 60 to 70 percent) compared to net migration. In the 2000-2017 period, the exposure gap for housing costs increases further (+ 0.3 pp in rents, and + 1.5pp in housing values), but again here the rate of growth slows substantially. Strikingly, more than 100% of the growth in the exposure gap to housing costs is due to place effects: had workers remained in the 1980 locations, the rent and housing value exposure gaps would have increased even more, by 0.8 and 2.0 pp respectively. Therefore, the changing location choices of the two skill groups between 2000 and 2017 have tended to narrow the housing affordability gap between groups. In contrast to the 1980-2000 period, where college workers were disproportionately migrating to expensive cities on net, college workers are now disproportionately migrating to relatively more affordable CZs, compared to non-college workers. We are not aware of any work exploring this sharp change in migration patterns.

Housing costs, available in Census data, are only a component of household expenditure. More generally, Diamond and Moretti (2021) study how consumption and expenditure by skill varies across space, using detailed bank account and credit card data. They find that housing prices constitute larger share of the consumption bundle of lower income households and that local prices of other goods are higher in high housing price cities. The results in Card et al. (2021) suggest that high wage CZs have such high house prices that they more than offset higher nominal wages, leading to lower real earnings in high-house-price CZs.

1.4 Geographic differences in local amenities

A last important difference across geographic locations is the local amenities that they provide, which directly impact quality of life. Diamond (2016) highlighted that local amenities influence location choices, especially for the high skill. We measure here change in the exposure gap of college and non-college workers to a range of local amenities. We start with public amenities, which one has access to by simply being present (and paying taxes) in the city. Our first measure is the Air Quality Index (where a higher value of AQI indicates worse air). In 1980, exposure to the 90th percentile of a CZ’s annual AQI was 1.8 points higher for the average high-skill worker, relative to the average low-skill worker (Panel C of Table 1). That is, the urban areas disproportionately chosen by college workers had worse air. This negative amenity gap is fully eroded by 2000 where the AQI gap falls to -0.004. About 80% of that improvement was due to place effects, while 25% was due to migration. By 2017, college graduates lived in CZs with better air than low-skill workers (with an exposure gap of -0.94), and more than 100 percent of this widening air quality gap was due to migration. The migration of high-skill workers to high-air quality places has increased and accelerated substantially, in contrast to their sorting on wage and rent. Data measuring flood risk...
paints a similar picture.\textsuperscript{5} In 1980, college workers lived in CZs with a 1.6 pp lower probability of an annual flood. Due to migration, this changed to a 2.5 pp lower probability by 2000 (resp. 2.7 pp by 2017). These results suggest that college workers are increasingly sorted on environmental amenities. Another interesting local amenity is local crime rates.\textsuperscript{6} In 1980, college workers lived in CZs with 7.3 percent higher property crime rates per capita and 10.4 percent higher violent crime rates per capita. These statistics paint the picture of urban centers as production hubs that come with urban disamenities. In 1980, college workers lived in larger cities with higher housing costs that paid them well, however, these production advantages did not come along with the amenity advantage we associate with large cities today. Workers had to endure worse air and higher crime there. By 2017, exposure gaps to property crime had turned to a negative 1.9\% (and a small positive 1.5\% for violent crime). Taken together with our environmental quality results, these results suggest that we see a transformation away from “production cities” towards “production and consumer cities” offering both high wages and better amenities (albeit at higher housing costs).

Commute times are an exception to these patterns. Indeed, we measure that college workers lived in CZs with 4.2 percent longer median commutes in 1980, a gap that increased to 5.1 percent in 2000 and 5.5 percent by 2017. Since longer commutes are considered a disamenity, this is the only amenity we find that college graduates are increasingly negatively sorted on.

To further analyze the shift of high-skilled cities becoming hubs of consumption, we investigate sorting on consumption amenities, such as the variety of restaurants and other privately provided local services.\textsuperscript{7} In 1980, college workers lived in CZs that had on average 1.1 percent more restaurants than the average low-skill CZs. This increased slightly by 2000, to 1.4 percent. By 2017, the exposure gap to restaurants per capita had grown to 4.0 percent. We find very similar patterns in exposure gaps to the log number of gyms per capita (growing from 4.7 percent to 10.7 percent over 1980-2017) and log salons per capita (growing from 1.5 percent to 12.9 percent over 1980-2017). In all cases, the biggest driver of these exposure gap changes are changes in place effects between 2000 and 2017, although migration also contribute to them. Log clothing store per capita also exhibits growth in exposure, but more so during the 1980-2000 period than the 2000 to 2017 period. Overall, it appears that establishments selling local services, more so than goods, are increasingly highly concentrated in today’s high-skilled consumer cities.

Taking stock, we find that across both consumption and public amenities, high-skill workers continue to migrate to better amenity CZs from 2000 to 2017 more so than low-skill workers, unlike our findings on wages and housing costs. Amenities and quality-of-life could be becoming increasingly important as the nationwide high-skill wage premium continues to rise.

In the next section, we discuss how a model can help structure the possible causes and consequences of the changes in jointly determined wage, housing costs, amenities, and location choices over time, and use this template to review the corresponding literature.

\textsuperscript{5}Our flood risk data come from Flood Factor, and does not vary over time. Thus, changes in the exposure index can only be driven by migration.

\textsuperscript{6}Our crime data come for county level reports of crime from the FBI.

\textsuperscript{7}Consumption amenity data come from County Business Patterns.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Segregation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Share College Graduate</td>
<td>0.019</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.039</td>
</tr>
<tr>
<td>Share College Grad, No Aggr. Growth</td>
<td>0.019</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.029</td>
</tr>
<tr>
<td>B. Wages and Housing Costs</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log College Wage</td>
<td>0.026</td>
<td>0.013</td>
<td>0.008</td>
<td>0.048</td>
<td>0.008</td>
<td>-0.002</td>
<td>0.054</td>
</tr>
<tr>
<td>Log Non-College Wage</td>
<td>0.026</td>
<td>0.007</td>
<td>0.006</td>
<td>0.039</td>
<td>-0.007</td>
<td>-0.002</td>
<td>0.030</td>
</tr>
<tr>
<td>Log Wage Gap</td>
<td>0.000</td>
<td>0.006</td>
<td>0.002</td>
<td>0.009</td>
<td>0.015</td>
<td>0.000</td>
<td>0.024</td>
</tr>
<tr>
<td>Log Rent</td>
<td>0.045</td>
<td>0.021</td>
<td>0.010</td>
<td>0.077</td>
<td>0.008</td>
<td>-0.004</td>
<td>0.080</td>
</tr>
<tr>
<td>Log Home Value</td>
<td>0.073</td>
<td>0.019</td>
<td>0.014</td>
<td>0.106</td>
<td>0.020</td>
<td>-0.006</td>
<td>0.121</td>
</tr>
<tr>
<td>C. Public Amenities</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AQI-90th Percentile</td>
<td>1.805</td>
<td>-1.453</td>
<td>-0.356</td>
<td>-0.004</td>
<td>0.035</td>
<td>-0.968</td>
<td>-0.936</td>
</tr>
<tr>
<td>Flood Risk</td>
<td>-0.016</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.027</td>
</tr>
<tr>
<td>Log Property Crimes per Capita</td>
<td>0.073</td>
<td>-0.068</td>
<td>-0.006</td>
<td>-0.001</td>
<td>-0.007</td>
<td>-0.010</td>
<td>-0.019</td>
</tr>
<tr>
<td>Log Violent Crimes per Capita</td>
<td>0.104</td>
<td>-0.061</td>
<td>0.008</td>
<td>0.051</td>
<td>-0.023</td>
<td>-0.013</td>
<td>0.015</td>
</tr>
<tr>
<td>Log Median Commute Time</td>
<td>0.042</td>
<td>-0.003</td>
<td>0.011</td>
<td>0.051</td>
<td>0.005</td>
<td>-0.001</td>
<td>0.055</td>
</tr>
<tr>
<td>D. Consumption Amenities</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log Restaurants per Capita</td>
<td>0.011</td>
<td>0.000</td>
<td>0.003</td>
<td>0.014</td>
<td>0.023</td>
<td>0.003</td>
<td>0.040</td>
</tr>
<tr>
<td>Log Gyms per Capita</td>
<td>0.047</td>
<td>0.001</td>
<td>0.018</td>
<td>0.066</td>
<td>0.037</td>
<td>0.004</td>
<td>0.107</td>
</tr>
<tr>
<td>Log Salons per Capita</td>
<td>0.015</td>
<td>0.035</td>
<td>0.021</td>
<td>0.071</td>
<td>0.050</td>
<td>0.008</td>
<td>0.129</td>
</tr>
<tr>
<td>Log Clothing Stores per Capita</td>
<td>0.000</td>
<td>0.022</td>
<td>0.008</td>
<td>0.030</td>
<td>0.016</td>
<td>-0.006</td>
<td>0.040</td>
</tr>
</tbody>
</table>

Notes: Data on location choices, wages, rents, housing values, commute times come from the 1980 and 2000 5% samples of the US Census. These data in 2017 come from the 2015-2019 5-year pooled ACS data. AQI data come for the EPA. Flood risk data come from Flood factor. Crime data from the county-level Uniform Crime Reports provided by the FBI. Consumption amenity data come from County Business Patterns. Sample of college and non-college workers are restricted to age 25-55 full-time, full-year workers.
2 Change in Skill Sorting: Framework

2.1 Setup

To organize our thoughts on the causes and consequences of spatial sorting, we lay out a framework where heterogeneous workers sort across locations within a country. Like in quantitative spatial equilibrium models reviewed in Redding and Rossi-Hansberg (2017), worker demand for locations is modeled as a discrete choice and the characteristics of locations are endogenous. Unlike the bulk of this literature which is based on homogeneous agents, we model heterogeneous groups of agents, who may value location characteristics differently. On the production side, rather than modeling imperfect trade between locations, we consider an economy that is more stylized spatially, with two types of goods: a homogeneous manufactured good that is freely traded across space, and housing, a local non-traded good.

2.1.1 Preferences

Consider a spatial equilibrium framework with two skill groups (unskilled and skilled) $\theta = U, S$, who choose where to live among locations $i \in [1, ..., N]$. Aggregate skill supply for each group, $L^\theta$, is exogenously given and each worker supplies one unit of labor for wage $w^\theta_i$ in location $i$. The utility of worker $\omega$, who is of type $\theta$ and lives in location $i$, is:

$$u^{\theta}_i(\omega) = \max_{c,h} \log U^{\theta}(A_i, c, h) + \epsilon^{\theta}_i(\omega)$$

such that: $c + r_i h = w^\theta_i$

where $\log U^{\theta}(.)$ is the representative utility of a worker of type $\theta$, $c$ is consumption of the freely traded good and is taken as the numeraire, while $h$ stands for housing, with price $r_i$ in location $i$, and $A_i$ is a vector of amenities in location $i$. Finally, $\epsilon^{\theta}_i(\omega)$ is a worker-specific preference shock for living in location $i$. This shock is i.i.d. across workers within a group and across locations.

The literature has made different choices of utility functions $U^{\theta}(.)$. The first type of choices pertains to the consumption of $c$ and $h$. A strand of the literature follows quantitative spatial models and uses Cobb-Douglas preferences over traded and non-traded goods. Another strand chooses non-homothetic preferences for $U^{\theta}(.)$ with housing modeled as a necessity. We make a middle-of-the-road assumption in which, in each group $\theta$, workers have Cobb-Douglas preferences over the traded and non-traded goods, but allow for the housing expenditure share $\alpha^{\theta}$ to be group specific, and typically higher for the unskilled. We refer to Gaubert and Robert-Nicoud (2022) for a full analysis with non-homothetic preferences. The assumption we make could capture true preference heterogeneity between groups. It is also a reduced-form way of capturing qualitatively forces that are due to non-homotheticity, and therefore allows us to speak to the main forces that may drive sorting, including income effects, with a streamlined exposition.

Second, we assume that amenities are separable from consumption. This assumption is shared by essentially all papers reviewed here, although this choice is arguably made in part for convenience. We allow for amenities in location $i$ to be valued differently by the two groups, as captured by a
group-specific amenity level $A^\theta_i$.

Third, preference shocks are typically chosen to be EV distributed. Papers in the tradition of urban, labor economics or IO tend to use logit shocks, with normalized variance $\frac{\sigma^2}{6}$ shifted by a factor $\frac{1}{\kappa^\theta}$, which together with Cobb-Douglas utility lead to the following indirect utility of worker $\theta$ in location $i$:

$$v^\theta_i(\omega) = \log A^\theta_i + \log w^\theta_i - \alpha^\theta \log r_i + \frac{1}{\kappa^\theta} \epsilon^\theta_i(\omega).$$

Equivalently, papers in the tradition of trade and economic geography typically choose Frechet shocks for $\epsilon^\theta_i(\omega)$ with scale parameter $\kappa^\theta > 1$ that enter utility in a multiplicatively separable way (rather than additively).\(^8\) In this case, the indirect utility of worker $\theta$ in location $i$ is:

$$v^\theta_i(\omega) = \frac{A^\theta_i w^\theta_i}{r^\alpha^\theta} \epsilon^\theta_i(\omega).$$

In either case, location choices in group $\theta$ can be summarized with $\lambda^\theta_i$, the share of $\theta$-workers who choose location $i$:

$$\lambda^\theta_i = \frac{L^\theta_i}{L^\theta} = \frac{\left(\frac{A^\theta_i w^\theta_i r^\alpha^\theta}{r^\alpha^\theta} \right)^{\kappa^\theta}}{\sum_{j=1}^{N} \left(\frac{A^\theta_j w^\theta_j r^\alpha^\theta}{r^\alpha^\theta} \right)^{\kappa^\theta}}, \quad (1)$$

The parameter $\kappa^\theta$ captures the elasticity of population shares with respect to amenity-adjusted real wages, and is therefore a measure of mobility of group $\theta$, which we allow to be group-specific. Expected utility for a worker in group $\theta$ across locations is:

$$W^\theta = \delta^\theta \left[ \sum_{k=1}^{N} \left(\frac{A^\theta_k w^\theta_k r^\alpha^\theta}{r^\alpha^\theta} \right)^{\kappa^\theta} \right]^\frac{1}{\kappa^\theta}, \quad (2)$$

with $\delta^\theta = \Gamma \left(\frac{\kappa^\theta - 1}{\kappa^\theta}\right)$ and $\Gamma(.)$ is the gamma function in the Frechet case. The same expression (up to the constant $\delta^\theta$) captures the log of expected utility in the additive logit case, expressed in log wage units.\(^9\) The two models are therefore intimately related. We proceed with the Frechet formulation and multiplicative notations below.

2.1.2 Supply of goods, amenities and housing

**Traded good** To close the model, we first write down the labor demand side of the economy. In location $i$, output is produced by perfectly competitive firms. They combine skilled and unskilled

---

\(^8\)The Frechet distribution is $G(\epsilon) = e^{-\epsilon^{-\kappa}}$.

\(^9\)Specifically, $\frac{E(U^\theta)}{dU^\theta/d\log w} = \log W^\theta + C$ where $C$ is a constant.
labor who are imperfect substitutes in production:

\[ Y_i = \left[ \left( z_i^U \right)^{\frac{1}{\rho}} \left( L_i^U \right)^{\frac{\rho-1}{\rho}} + \left( z_i^S \right)^{\frac{1}{\rho}} \left( L_i^S \right)^{\frac{\rho-1}{\rho}} \right]^{\rho \over \rho-1}. \tag{3} \]

In the constant returns to scale CES production function (3), \( \rho \geq 1 \) is the elasticity of substitution between skills, and \( z_i^\theta \) are location- and skill- specific productivity shifters. These shifters can be in part exogenous, reflecting fundamental differences between locations, such as natural resources. They can also be in part endogenous, reflecting externalities. That productivity is subject to local spillovers reflects traditional agglomeration forces dating back at least to Marshall (1890). Specifically, we assume that, for \( \theta = \{U, S\} \) and \( \forall i \):

\[ z_i^\theta = z^\theta \left( \bar{Z}_i, L_i^U, L_i^S \right), \tag{4} \]

where \( \bar{Z}_i \) is the exogenous productivity component of city \( i \). Local productivity spillovers are allowed here to depend not just on city size or density, but also on its composition \( (L_i^U, L_i^S) \). In addition, these agglomeration effects may differ by skill, as captured by a group-specific spillover function \( z^\theta(.) \). Given (3), relative labor demand in location \( i \) is:

\[ \log \left( \frac{L_i^S}{L_i^U} \right) = \log \left( \frac{z_i^S}{z_i^U} \right) - \rho \log \left( \frac{w_i^S}{w_i^U} \right). \tag{5} \]

Furthermore, competition across cities ensure that the unit cost of production in all cities is 1, the common price of the freely traded good:

\[ \sum_\theta z_i^\theta \left( w_i^\theta \right)^{1-\rho} = 1 \quad \forall i. \]

**Amenities** Similar to productivity, amenities \( A_i^\theta \) are assumed to be driven by both exogenous differences (e.g., climate or scenery) and endogenous differences between cities, that is:

\[ A_i^\theta = A^\theta \left( \bar{A}_i, L_i^U, L_i^S \right), \tag{6} \]

where \( \bar{A}_i \) is the exogenous amenity component of city \( i \). Endogenous amenities capture elements of quality-of-life that change when the size or composition of cities change (e.g., local pollution, quality of schools, crime, presence of entertainment options, variety of restaurants). When the function \( A^\theta(.) \) differs across type \( \theta \), different groups have systematically different preferences or valuation of locational amenities.

---

\(^{10}\)For instance, Moretti (2004) estimates human capital spillovers that driven by the local skill share, \( \frac{L_i^S}{L_i^U} \).
Finally, we assume that housing is supplied by atomistic absentee landowners and that the aggregate housing supply function in city $i$ is:

$$H_i = \bar{H}_i r_i^{\eta_i}. \quad (7)$$

The housing supply elasticity $\eta_i$ is allowed to be city specific. It captures in a reduced-form way forces that help or hinder the expansion of the housing stock in a given city. As Saiz (2010) documents, housing supply tends to be shaped both by exogenous forces (e.g., geographical constraints to expansion, like mountains or a waterway delimitating the city), and endogenous forces (such as local land-use or housing regulations). Consistent with the dominant approach in the literature, we nevertheless take $\eta_i$ as a parameter in the model.

A spatial equilibrium of this economy is a set of location choices $\{\lambda_{i,\theta}\}$, prices $\{w_{i,\theta}^0, r_i\}$, and amenities and productivity shifters $\{z_{i,\theta}^0, A_{i,\theta}^0\}$ such that workers and firms optimize, traded good firms make no profits and markets clear. Since amenities and productivity shifters $\{z_{i,\theta}^0, A_{i,\theta}^0\}$ typically depend on the equilibrium distribution of economic activity, these local spillovers act as feedback loops that may amplify or dampen concentration and sorting. A question that remains largely unanswered is: under what conditions is the sorting equilibrium unique? This question is important since quantitative frameworks can be used to compute model-based counterfactuals and predict the effect of a shock or a policy on the spatial equilibrium; this exercise is well-defined when the equilibrium is unique. The quantitative spatial literature has established sufficient conditions for uniqueness for a range of models with homogeneous workers. Establishing such conditions in the presence of two groups is complicated by the fact that spillovers to one group depend the other group’s distribution. Against this backdrop, the question of equilibrium uniqueness has been typically treated numerically, rather than formally, in the sorting literature. An exception is Fajgelbaum and Gaubert (2020) who derive conditions for uniqueness of a market equilibrium corrected by efficient taxes, when spillovers take a Cobb-Douglas form between two skill groups. More research is needed on this technical but important issue.

### 2.2 Drivers of sorting

We now discuss conditions under which spatial sorting arises in equilibrium. We call “spatial sorting” the fact that the skilled and unskilled groups make different locations choices, i.e., there exists locations $i$ and $j$ such that, denoting $\Delta X \equiv X_i - X_j$ for any variable $X$:

$$\Delta \log \left( \frac{L_i^S}{L_j^U} \right) \neq 0.$$
Given locations choices (1), and combining labor supply (8) and labor demand (5), relative spatial labor supply is given by:

$$\Delta \log \left( \frac{L^S}{L^U} \right) = \frac{\bar{\kappa}^S}{\rho} \Delta \log \left( \frac{z^S}{z^U} \right) + \bar{\kappa}^S \Delta \log \left( \frac{A^S}{A^U} \right) + \bar{\kappa}^S \left( \alpha^U - \alpha^S \right) \Delta \log r + \frac{\bar{\kappa}^S}{\kappa^U} \left( 1 - \frac{\kappa^U}{\kappa^S} \right) \Delta \log L^U,$$

(8)

where we have denoted $\bar{\kappa}^S = \frac{\kappa^S \rho}{\kappa^S + \rho}$. Conceptually, one can therefore distinguish four sources of sourcing in this framework: we will say that sorting is shaped by comparative advantage in production when $\Delta \log \left( \frac{z^S}{z^U} \right) \neq 0$, by amenities when $\Delta \log \left( \frac{A^S}{A^U} \right) \neq 0$, by housing prices when $\alpha^S \neq \alpha^U$ and by heterogeneous mobility across groups when $\kappa^U \neq \kappa^S$. It is important to note that all of these four forces are endogenous to the sorting equilibrium, as we discuss in detail below. In turn, changes in sorting occur when either of the four forces $\Delta z, \Delta A, \Delta \alpha$, and $\Delta \kappa$, as defined in equation 8, change over time. We examine these sources of sorting in turn, although they are cumulative (and inter-related) in practice.

2.2.1 Comparative advantage in production

When does comparative advantage in production directly drive sorting? This happens only when the first term in (8) is non-zero. A first takeaway is therefore that when productivity is (multiplicatively) separable between a location shifter $Z_i$ (perhaps subject to agglomeration effects) and nationwide group productivity $z^\theta$, so that $z^\theta_i = Z_i \bar{z}^\theta$, the productivity advantage of a location is skill-neutral, hence does not drive sorting directly. Likewise, changes in the Hicks-neutral productivity of location $i$, or a nationwide skill-biased technical change (i.e. changes in $\frac{z^S}{z^U}$) can only drive changes in sorting indirectly, through equilibrium rents or heterogeneous mobility across groups, a case we return to below.

We now assume, in contrast, that some skill group has comparative advantage in production in some location over another, so that $\Delta \log \left( \frac{z^S}{z^U} \right) \neq 0$. This comparative advantage could stem from exogenous differences between places - e.g., skilled workers could have a comparative advantage in locations specialized in services. In addition, it could stem from different skill groups benefiting differentially from agglomeration effects - e.g., skilled workers could benefit more from knowledge spillovers in dense cities. The literature has proposed different parameterizations for these agglomeration effects. For instance, $z^\theta_i$ might depend on the local skill share, population, and/or or population of each group separately. For simplicity, we assume that local productivity depends on population, which is the most classic way to parameterize agglomeration effects, but with a different intensity $\gamma^\theta_P$ for different skill groups, that is:

$$z^\theta_i = \bar{z}^\theta \left( L_i^U + L_i^S \right)^{\gamma^\theta_P}.$$

(9)
In this expression, $z^\theta_i$ are exogenous location-group productivity shifters. Equilibrium sorting is then pinned down by:

$$\Delta \log \left( \frac{L^S}{L^U} \right) = \frac{\tilde{\kappa}^S}{\rho} \Delta \log \left( \frac{z^S}{z^U} \right) + \frac{\tilde{\kappa}^S}{\rho} \left( \gamma^S_P - \gamma^U_P \right) \Delta \log L + \Delta_A + \Delta_\alpha + \Delta_\kappa.$$ 

Changes in sorting due to productivity corresponds to the first two terms. They may occur because of changes in exogenous comparative advantage $\Delta \log \left( \frac{z^S}{z^U} \right)$. Again here, productivity shocks would have to be city- and skill-biased to generate changes in sorting. Second, they may occur because of changes in relative city sizes $\Delta \log L$, or because of changes in relative agglomeration forces $\gamma^S_P - \gamma^U_P$.

### 2.2.2 Amenities

The term “amenities” is typically used to encompass a wide range of services, local public goods, and environmental conditions that impact residents’ quality-of-life (Glaeser et al. (2001)). We parameterize utility derived from amenities as a Cobb-Douglas aggregator of a vector of amenities $\{A_{ki}\}_k$ in location $i$, with skill-group specific preference parameters $\gamma^\theta_{kA}$. This allows both skill groups to have different preferences over each city’s amenity bundle:

$$A^\theta_i = \prod_k (A_{ki})^{\gamma^\theta_{kA}}. \tag{10}$$

We allow a component of each amenity in the amenity bundle to be endogenous. Following Diamond (2016), we model the endogenous component of amenity as responding to the skill ratio $\frac{L^S_i}{L^U_i}$ of the city, that is:

$$A_{ki} = \tilde{A}_{ki} \left( \frac{L^S_i}{L^U_i} \right)^{\beta_k}, \tag{11}$$

where $\tilde{A}_{ki}$ is the exogenous component of amenity $k$ and $\beta_k$ measures how elastic the supply of amenity $k$ is to the skill ratio. This formulation captures in a reduced-form way the notion that higher skill, hence higher income, individuals might spur the growth of consumption amenities in cities in which they reside, or that they foster reductions in crime and pollution because, for instance, of their influence on the political process. With these formulations, equilibrium sorting is:

$$\Delta \log \left( \frac{L^S}{L^U} \right) = \frac{\tilde{\kappa}^S}{1 - \tilde{\kappa}^S} \left( \tilde{\gamma}^S_A - \tilde{\gamma}^U_A \right) \Delta \log \left( A^S_i \right) A^U_i + \frac{1}{1 - \tilde{\kappa}^S \left( \tilde{\gamma}^S_A - \tilde{\gamma}^U_A \right)} [\Delta z + \Delta_\alpha + \Delta_\kappa],$$

where we have denoted $\tilde{\gamma}^\theta_A = \sum_k (\beta_k (\gamma^\theta_{kA}))$ and $\frac{\tilde{A}^S_i}{\tilde{A}^U_i} = \prod_k \left( \tilde{A}_{ki} \right)^{\gamma^S_{kA} - \gamma^U_{kA}}$. The takeaways are twofold. First, amenities are a source of sorting in themselves only to the extent that the first term is non-zero, i.e., that valuations of the exogenous component of amenities are heterogeneous across groups ($\tilde{\gamma}^S_A \neq \tilde{\gamma}^U_A$). Second, the endogenous provision of amenities ($\beta_k \neq 0$) together with their heterogeneous valuation across skills ($\gamma^S_{kA} - \gamma^U_{kA} \neq 0$) only serve as an amplifier (or
dampener) of other sorting forces. Sorting driven by productivity, housing prices, or heterogeneous mobility, in the bracketed term, are amplified by the feedback loop played by amenity provision, so long as agglomeration forces are not too strong so that the model remains well-behaved \((0 < \tilde{\kappa}^S (\tilde{\gamma}_A^S - \tilde{\gamma}_A^U) < 1)\). Similarly, sorting based on exogenous differences in amenities across places (the first term) are magnified in the presence of such endogenous amenities. It is in theory possible that endogenous amenities could dampen sorting if low-skill workers’ had a stronger preference for the endogenous amenity composite than the high-skill (that is, if \(\tilde{\gamma}_A^S - \tilde{\gamma}_A^U < 0\)), although most empirical evidence finds that endogenous amenities enhance, rather than dampen, skill sorting.

### 2.2.3 Housing prices

We turn to the role of housing prices in driving sorting. To make clear the specific mechanisms at play here, we shut down the other sources of sorting by making the following assumption:

**Assumption 1.** \(\Delta \log \left( \frac{z^S}{\bar{z}^S} \right) = 0\), \(\Delta \log \left( \frac{A^S}{\bar{A}^S} \right) = 0\) and \(\kappa^U = \kappa^S\).

Under Assumption 1, some cities may still be more productive or have higher amenities than others, but in a way that is skill-neutral: there exists city-wide shifters \(Z_i\) and \(A_i\) and nationwide group-specific shifters \(z^\theta\), \(A^\theta\) such that \(z^\theta_i = Z_i z^\theta\) and \(A^\theta_i = A_i A^\theta\), for all locations \(i\).

We see from (8) that the groups who have a higher expenditure share on housing, all else equal, are under-represented in expensive cities as they are disproportionately hurt by the high housing cost there. If housing is a necessity, then \(\alpha^U - \alpha^S > 0\) and skilled workers are over-represented in expensive cities.

Which cities are more expensive in equilibrium? Given the housing supply equation (7) equilibrium housing prices are the implicit solution to:

\[
Z_i^{\frac{1+\kappa}{\rho-1}} A_i^\kappa \bar{H}_i = r_i n \sum_{\theta} \omega^\theta f_i (r_i) r_i^{\kappa \alpha^\theta \frac{1-\rho}{\kappa+\rho-1}} - 1 \tag{12}
\]

where \(f_i (r_i)\) captures that in equilibrium, wages depend on rents. This function can be shown to be equal to 1 when skills are perfect substitutes, and a decreasing function of \(r_i\) otherwise. Given (12), rents \(r_i\) increases with \(Z_i^{\frac{1+\kappa}{\rho-1}} A_i^\kappa\): more productive cities and cities with higher amenities (per unit of land) are more expensive in equilibrium. We denote \(R_i(\cdot)\) the corresponding solution to equation 12. Turning to the implication of housing rents for skill sorting, we obtain:

\[
\Delta \log \left( \frac{L^S}{L^U} \right) = (\alpha^U - \alpha^S) \Delta \log R \left( \frac{Z_i^{\frac{1+\kappa}{\rho-1}} A_i^\kappa}{H} \right), \tag{13}
\]

and high-skill workers sort into more productive and/or more attractive (per unit of land) locations. A first takeaway is that when housing expenditure shares differ across groups, Hicks-neutral city advantage is enough to drive sorting, through its impact on housing prices. That is, even if the productivity advantage of cities is skill-neutral and their amenities are valued identically by
both skill groups, the two groups will still make systematically different location choices because housing prices bear a different weight on their real wages. Sorting is then driven by a form of non-homotheticity in consumption. If, in contrast, preferences are homothetic and identical across groups, housing impacts all households proportionally, and no spatial sorting emerges under Assumption 1. A second takeaway is that the role of housing in mediating spatial sorting forces is stronger, all else equal, in locations with more inelastic housing supply (lower $\eta$). Inelastic supply directly leads to steeper $\mathcal{R}(.)$, hence a steeper response of housing prices to productivity and amenities, and in turn, a steeper response of the skill ratio through (13).

### 2.2.4 Heterogeneous migration elasticities

Finally, a last possible driver of sorting arises when $\kappa^U \neq \kappa^S$. Empirical studies tend to find that higher skilled workers are more mobile than lower skilled workers, so that to fix ideas we will consider the case where $\kappa^S > \kappa^U$. Equation 8 shows that if high skill workers are more mobile, their sorting into attractive cities is reinforced, all else equal, in the sense that places that attract low skill workers (high $\Delta \text{log } L_U$ places) attract high skill workers disproportionately more. To characterize the equilibrium in more detail, we isolate this force of sorting and shut down others with the following assumption:

**Assumption 2.** $\Delta \text{log } (\frac{z^S}{x^S}) = 0$, $\Delta \text{log } (\frac{A^S}{A^U}) = 0$ and $\alpha^U = \alpha^S$.

In this case, it is easy to see that:

$$\Delta \log (L^S) = \left[1 + \frac{\rho}{\kappa^S} + \rho \frac{\kappa^S}{\kappa^U} - 1\right] \Delta \log L^U$$

Under assumption 2, there is no skill-biased amenity or productivity advantage of places. Still, high-skill population increases faster than low-skill population in attractive cities because they are more sensitive to city characteristics. The mobile high skill workers move more to reap higher indirect utility, while less skilled workers respond less and are therefore more spread out in equilibrium. Overall, higher skills are over-represented in places that are attractive for both skill groups. Similar to the case where sorting was driven by housing prices, a skill-neutral advantage of cities (such as a Hicks-Neutral productivity advantage) leads to spatial sorting when groups are heterogeneous in their mobility rates.

### 2.2.5 Urban skill premium and sorting

We have seen how four different forces may shape skill sorting in spatial equilibrium. We now turn to considering how they shape the distribution of the skill wage premium in the cross-section of cities, in equilibrium. Solving out for the equilibrium skill premium and its variation over space leads to:
\[
\Delta \log \left( \frac{w^S}{w^U} \right) = \frac{1}{\kappa^S} \Delta_z - \frac{1}{\rho} \Delta_A - \frac{1}{\rho} \Delta_\alpha - \frac{1}{\rho} \Delta_\kappa \tag{15}
\]

Comparing this expression with the one that summarizes skill sorting \(8\):

\[
\Delta \log \left( \frac{L^S}{L^U} \right) = \Delta_z + \Delta_A + \Delta_\alpha + \Delta_\kappa,
\]

directly yields the following insights. When sorting is driven by skill-biased productivity effects, the skill premium and skill ratios go in the same direction, both driven up by \(\Delta \log \left( \frac{z^S}{z^U} \right)\). Absent other sources of sorting, skill premia and skill ratios are unambiguously positively associated in equilibrium. In contrast, when skill sorting is driven by either of the other forces (skill-biased amenities, housing price effects \((\alpha^U > \alpha^S)\), or heterogeneous migration rates \((\kappa^S > \kappa^U)\)), the skill premium and skill ratios tend to go in the opposite direction in the cross-section. Absent skill-biased productivity differences, skill premia and skill ratios are negatively associated in equilibrium. This is because wages act as a compensating differential in these cases. The high skill are disproportionately attracted to cities that are attractive for reasons other than skill-biased productivity. In these cities, low-skill workers are therefore in relative high demand on the labor market, pushing up their relative wages. In equilibrium, the skill premium is lower where high skill workers are over represented.

2.3 Drivers of sorting: evidence

Having laid out the main potential drivers behind changes in sorting and in the city-skill premium, we now review the literature investigating these channels.

2.3.1 Productivity and sorting

Changes in labor demand are typically put forward as the triggering force behind the increased skill sorting and increased city-skill premium of the recent decades. Diamond (2016) estimates a spatial equilibrium model, with labor demand factors, amenities, rents, and heterogeneous preferences potentially shaping sorting. She finds that changes in return to skills, especially in cities that were initially high-skilled, are an important mechanism behind the Great Divergence. Baum-Snow et al. (2018) analyze the increasing wage inequality across space, with the the skill premium increasing the most in larger cities. They estimate that the primary driver behind the increasingly positive relationship between skill premium and city size is an increase over time in the skill bias of agglomeration economies, that is, an increase in \(\gamma^S_P - \gamma^U_P\) in the language of our model (specification 9). Giannone (2019) emphasizes a break in trend in the spatial distribution of wages in the U.S.: before 1980, wages were converging across U.S. cities, but they have been diverging since. In her the model, the key force behind convergence is that technology diffuses over space, while the key force behind divergence are local skill-biased agglomeration effects, which drive spatial sorting. A key challenge with identifying agglomeration effects, and especially skill-specific agglomeration effects, is that changes in the supply of workers not only impacts agglomeration but also leads to
standard shifts along firms’ labor demand curves. When studying productivity and agglomeration with data aggregated to the skill-group-by-city level, labor demand shifts and agglomeration effects are perfectly co-linear. Indeed many of the papers discussed above have this co-linearity problem and solve it by assuming functional form differences between the two forces. However, with micro data on firms, these two effects can be non-parametrically decoupled and credibly estimated, similar to the strategy taken by Moretti (2004). He estimates plant-level production functions to quantify the slopes of the labor demand curves and then shows that the city-wide skill mix still appears to impact firm pay, over and beyond what the production function estimates imply. The setup in Moretti (2004) could be embedded into a spatial equilibrium model, such as the one above, along with clean variation in city’s skill mixes to provide a new estimate of skill-specific agglomeration forces. Finally, Eckert (2019) takes a different perspective to explain the increasing sorting of high-skill workers in high-skill cities, which does not rely on skill-biased agglomeration effects. In his model, trade between locations is costly, so that local wages are determined not just by local productivity, but also by market access. As communication costs fall following the rise of the internet, markets integrate and locations with a comparative advantage in business services, which are both communication-cost-intensive and skill-intensive, increase their specialization in this sector. This drives up local relative demand for high skills and the local skill premium, while the opposite arises in locations specialized in manufacturing.

Through what channel can agglomeration effects be skill-biased? The model laid out above is silent on the micro-foundations and channels through which the high skill may benefit disproportionately from agglomeration forces. Theoretically, a branch of the literature microfounds heterogeneous agglomeration effects from local interaction between economic agents, both individuals and firms. An early contribution is Berry and Glaeser (2005) who propose a theory for the increasing clustering of skills. In the model, the driving force is that entrepreneurs tend to innovate locally, and in technologies that are biased in favor of their own skill. Growth in local skill then correlates with the initial local skill level. More generally, the spatial labor demand for skilled workers can be concentrated in specific cities because firms that are skill-intensive are located there. That is, the spatial sorting of firms may explain the spatial sorting of skills. Duranton and Puga (2000) develop a theory of the lifecycle of firms where young, innovative firms are located in dense cities where they benefit from knowledge spillovers, and older established firms locate in low density, cheaper areas where production costs are low. To the extent that innovation is a skill-intensive activity, skilled workers will likewise cluster in dense cities. In Gaubert (2018), high-productivity firms benefit disproportionately from agglomeration effects offered by dense cities following the empirical finding in Combes et al. (2012). Hence, they sort disproportionately there. This sorting drives nearly half of the productivity advantage of large cities. To the extent that high-productivity firms are more skill-intensive, skilled workers will also cluster in dense cities. Hendricks (2011) proposes a model in which skilled-workers are complementary in production to business services, and business services feature local increasing returns to scale, generating forces akin to skill-biased agglomeration effects. The model explains why high skill clustering correlates with the concentration of business services.
Finally, a strand of the literature reviewed in detail in Behrens and Robert-Nicoud (2015) propose complete models of systems of cities with a continuum of worker types, and include micro-foundations for sorting, such as complementarities in learning between skill and the quality of the learning environment (Davis and Dingel (2019)), complementarity in production between different skill types (Eeckhout et al. (2014)), or an interplay between agglomeration, skill sorting and the selection of efficient firms (Behrens et al. (2014)).

### 2.3.2 Amenities and sorting

We turn to amenities as a potential force behind recent changes in spatial sorting. First, exogenous amenities may drive sorting: a classic one is weather and climate. Albouy et al. (2016) estimates willingness to pay to live in cool and hot climates, and find that college educated households are willing to pay more than lowerskilled ones avoid excessive heat, while the college educated are relatively more tolerant of cold temperatures. Both groups prefer a temperate climate of 65 degrees. Second, the logic of a feedback loop between sorting and amenity provision has also been put forward as a potential explanation of sorting. In an early contribution on the topic, Shapiro (2006) shows that local skill concentration leads to higher local employment growth, while unskilled concentration does not. Using a model-based approach where amenities are treated as a residual to explain the spatial equilibrium, he finds that part of this effect is driven by quality of life: higher skill regions give rise to richer amenities, making them more attractive. Diamond (2016) shows that endogenous amenities played an important role in amplifying the increasing skill sorting. Amenities are captured by an index that aggregates empirical measures of a variety of amenities like school quality, environmental quality, or retail environment. She estimates that this index responds positively to local skill mix and that college graduates’ location choices are more sensitive to the amenity index level than high school graduates. Finally, she finds that endogenous amenity changes were very important in amplifying the sorting of skilled workers initiated by productivity changes over the 1980-2000 period. Handbury (orth) provides interesting evidence on the role of cities in fostering consumption amenities in a way that is systematically different for different income groups. Using the Nielsen retail price data, she finds that the variety of products offered in wealthy cities is higher than in poorer cities, and especially so for goods preferred by wealthy consumers. In addition, the higher prices of stores in wealthier cities are muted for goods consumed by high-incomes. On net, high-income households enjoy 40 percent higher utility per dollar expenditure in wealthy cities, relative to poor cities and to low-income households. These results are consistent with the theory that amenities respond to the composition of cities, so that preference externalities arise.

### 2.3.3 Housing prices and sorting

**Income elasticity of housing demand** Despite the fundamental role of heterogeneous housing demand in location sorting, there is a wide range of estimates of how the expenditure share on housing (captured by $\alpha^\theta$ in our model) varies across the income distribution. The first chal-
The challenge to identifying the relationship between housing expenditure share and income is that housing choices are sticky (Chetty and Szeidl, 2016), while annual income fluctuates. In years when income is idiosyncratically high, households don’t move and consume more housing, lowering their expenditure share on housing. In contrast, in years when income is low, expenditure shares on housing go up since these households have pre-committed to their housing consumption. Thus, the cross-sectional correlation between annual income and annual housing expenditure is biased towards under-estimating the income elasticity of housing consumption. Using this cross-sectional regression of house prices on annual income, Rosenthal (2014) reports an income elasticity of 0.41 for owner-occupiers and 0.12 for renters.

The second difficulty is that datasets that can measure better permanent income proxies (such as total expenditure) often do not contain the geographic details to measure variation in local housing prices. This makes it impossible to separate out income effects on housing expenditure from price effects. Albouy et al. (2016) make progress here and develop a demand system to estimate the price and income elasticity of housing demand. To overcome the transitory income measurement issues, they estimate relationships between housing, income, and expenditures at the city level, instead of the household level, hoping to smooth out the transitory household-level fluctuations. They use the 2000 Census to measure geographically detailed house prices. They estimate an elasticity of 2/3, close to the one also found in Finlay and Williams (2020). They also estimate that the price elasticity of housing demand is 2/3. This is consistent with the empirical fact that housing expenditure shares are higher in high house price cities. Aguiar and Bils (2015) use CEX data and estimate a higher expenditure elasticity of housing at 0.9. They directly deal with measuring permanent income by proxying it with total expenditure. However, they cannot account for the geographic variation in house prices. Since high-income households are likely to live in high house price cities and housing demand elasticities with respect to price are likely less than one, they likely over estimated the elasticity of housing demand with respect to income. Finally, Davis and Ortalo-Magné (2011) compare median expenditure shares across U.S. cities and find that they are approximately constant, justifying the use of Cobb-Douglas preferences with constant expenditure shares on housing. One possibility to reconcile these findings is that housing expenditures do fall with income, but systematic sorting of higher incomes to expensive cities pushes in the other direction: high housing costs increase their housing expenditure share, when housing is price inelastic. To our knowledge, this argument has not yet been studied formally.

**Housing prices and sorting** The papers we review next are chiefly interested in understanding housing price changes. They point at spatial sorting as an amplification mechanism behind housing price changes. Of course, conversely, house price changes also impact sorting in these papers. Gyourko et al. (2013) aim to shed light on the wide dispersion in house price increases across US communities between 1950 and 2000. They show that aggregate shocks such as aggregate population growth are enough, qualitatively, to generate increased skill sorting and increased house price dispersion across places over time, without invoking any change in preferences nor shocks that
are heterogeneous over space. This mechanism is stronger for attractive locations with inelastic housing supply. Relatedly, Van Nieuwerburgh and Weill (2010) propose a dynamic quantitative model to investigate the empirical fact that housing price dispersion across places has gone up much more than wage dispersion. In the model, higher-skill workers outbid lower-skill workers in high productivity locations. An increase in productivity dispersion across places is taken as the primary shock in the economy. Rents dispersion then adjusts to reflect dispersion of the indifference condition of marginal households between two cities. A reason why rents dispersion increases more than productivity is that sorting increase, hence the indifference condition is pinned down by more and more dispersed ability across cities. Finally, Ganong and Shoag (2017) document the end of income convergence across U.S. states. A driving force in their paper is the increase over time of land-use regulations mandated in high-skill places. It decreases housing supply elasticity and prices out the lower incomes there. This mechanism contributes to slowing down convergence, as places are populated by increasingly different skills.

2.3.4 Heterogeneous migration elasticities and sorting

There is robust evidence that local labor demand shocks lead to higher levels of migration by high-skill workers than by lower skilled ones (Bound and Holzer, 2000). The literature has mixed results on what is driving this phenomenon. Notowidigdo (2020) finds this to be driven by the offsetting effect of means-tested government transfers mitigating the labor demand shock on low-skill workers. He finds that both skill groups actually have the same migration elasticity. Diamond (2016) finds this migration difference to be driven by low-skill workers’ especially strong preference to live in their state of birth. She finds that after controlling for preferences to live in one’s state of birth, low-skill workers actually have higher migration elasticities than high-skill workers. Piyapromdee (2020) finds that taking into account gender and immigrant status is important and finds that only non-immigrant low-skill men are less mobile than non-immigrant high-skill men. Immigrants are more mobile and women, regardless of skill, are less mobile. More work is needed here to draw robust conclusions.

3 Implications

3.1 Measurement of inequality

The past decades have seen an increase in nominal wage inequality nationwide, becoming an important issue in the current policy debate. In parallel, spatial sorting has increased, and the high-skill are increasingly sorted in high-productivity, high-amenity, and expensive cities. How has well-being inequality changed as a result? Do changes in spatial sorting reinforce, or mitigate the welfare effects of nominal wage inequality? We explore here how the framework above and its quantification can shed light on these issues. Consistent with the focus on Section 1, we are concerned here with the welfare implications of changes in across-city sorting over the period 1980-2017 in the U.S.
3.1.1 Model-based measure of welfare inequality

The model of Section 2 lends itself naturally to welfare analysis. Let $W^\theta_t$ denote the representative well-being of group $\theta$ in year $t$, defined in equation 2. In the analysis below, we denote $\hat{x} = \frac{x_{t_2}}{x_{t_1}}$ the proportional change in variable $x$ between two equilibria $t_2$ and $t_1$. In particular, let $\hat{W}^\theta = \frac{W_{2017}^\theta}{W_{1980}^\theta}$ denote changes in well-being for group $\theta$ over our period of analysis. First, given the structure of the model, changes in well-being for group $\theta$ is simply

$$\hat{W}^\theta = \left[ \sum_{k=1}^{N} (\hat{V}^\theta_{i,k})^\kappa \lambda_{i,1980}^\theta \right]^{\frac{1}{\kappa}}$$

so that:

$$\frac{\hat{W}^S}{\hat{W}^U} = \left[ \sum_{k=1}^{N} (\hat{V}^S_{i,k})^\kappa \lambda_{i,1980}^S \right]^{\frac{1}{\kappa}}$$

That is, changes in well-being over time, within a group, is a weighted power mean of the change in the utility index in each city $\hat{V}^\theta_i = \frac{\hat{A}_{i,1980}^\theta}{\hat{w}_{i,1980}^\theta}$, weighted by the initial distribution of population $\lambda_{i,1980}^\theta$ in each location. Therefore, knowledge of initial population distribution by group, migration elasticities $\kappa^\theta$, and proportional changes in amenities, nominal wages and housing costs for each group in each city allows in principle to compute change in well-being inequality over time, $\hat{W}^S/\hat{W}^U$.

If measuring changes in prices of labor and housing is in principle readily doable given typical data, measuring changes in amenities is more challenging. Following Rosen (1979) and Roback (1982), one strand of the literature typically backs out amenities as residuals that help explain the distribution of economic activity over space. Unfortunately, this method only allows to measure local amenities up to a multiplicative shifter, which is necessary to make welfare statements. One way to make progress is to isolate the contribution of changes in endogenous amenities to welfare inequality, using a specific parameterization for it that can be estimated. We follow Diamond (2016) in assuming that amenity supply depends on the skill ratio and is valued by the two skill groups as in (10). We use her estimates for the various parameters of the model.\footnote{We simplify Diamond’s model by not allowing for preference heterogeneity based on race or immigrant status or to allow preferences for living in one’s state or Census division of birth. We use parameter estimates from Column 3 of Panel A in Table 5 for non-black, non-immigrant workers from Diamond (2016). Endogenous amenity supply parameters come from Column 3 of Panel D.}

3.1.2 Decomposition by driver of sorting

To understand how changes in spatial forces have shaped well-being inequality between 1980 and 2017, we decompose changes in well-being inequality into those driven by changes in nominal wages in isolation, we then add rents, and finally endogenous amenities.

First, one can compute what would have been the change in well-being inequality if only nominal wages had changed, but not rents or amenities. In this case, one can apply formula (16) to changes in nominal wages only ($\hat{V}^\theta_i =\hat{w}^\theta_{i,1980}$), so that:
This nominal wage inequality has increased by 16.7 percentage points (pp.) between 1980 and 2000 and by 10.7 pp. between 2000 and 2017. Second, adding the effects of change in rents (hence using now \( V_i^\theta = \hat{w}_i^\theta \hat{r}_i^\alpha \)) leads to a lower change in well-being inequality than the one suggested by nominal wages only: this real wage inequality has increased by 15.2 and 9.6 pp. between 1980-2000 and 2000-2017, respectively. Over both of these time periods, rent increases mitigate about 10 percent of wage inequality increases. This is because over the period, the high-skill live in increasingly expensive locations (Moretti (2013)), as we found in Section 1. Finally, adding the effects of changes in endogenous amenities triggered by changes in sorting (hence using \( \hat{V}_i^\theta = \left( \frac{L_i^S}{L_i^U} \right) \sum_k \delta_k (\gamma^k_A) \frac{\hat{w}_i^\theta}{\hat{r}_i^\alpha} \)) leads to a higher change in well-being inequality, of 17.0 and 12.1 percentage points between 1980-2000 and 2000-2017, respectively.\(^\text{12}\) Consistent with our findings using the exposure index in Section 1, high-skill workers sorted in the high amenity locations more from 2000-2017 than they did during the 1980-2000 period. Taking amenities into account increases overall changes in inequality, particularly so in the later period.

3.2 Change in Skill Sorting: Policy implications

3.2.1 Efficient sorting and Efficient redistribution

In an economy as described in Section 2, the laissez-faire equilibrium is generically inefficient because of the presence of local externalities: the productivity and well-being of each resident depends directly on the location choices of others, in a way that they do not take into account when choosing where to live. The extent of spatial sorting, in particular, will be generically inefficient. A natural question is then: does the laissez-faire equilibrium feature too much or too little spatial segregation? What policies can lead to more efficient sorting? These questions are only a fraction of those considered in the vast literature concerned with place-based policies. We refer the reader to Neumark and Simpson (2015) for a broader discussion of spatial policies. Related questions have also been studied in the literature concerned with within-city sorting and neighborhood effects (Benabou (1993, 1996b,a)). This literature, reviewed in detail in Durlauf (2004), typically finds that spatial sorting tends to compound disparities in human capital building, and ends up being inefficient.

Fajgelbaum and Gaubert (2020) show how sorting can be inefficient using a framework nesting the one presented in Section 2. They establish formulas for the optimal transfers between cities and groups that lead to an efficient allocation. The size of optimal subsidies to a given group-city are

\(^\text{12}\)Following Diamond (2016), we only include the welfare value of the changing endogenous amenities driven by changes in the sorting of high and low-skill workers across commuting zones, not allowing the aggregate supply of college graduates nationwide to contribute to amenity increases in all commuting zones. See Diamond (2016) for more discussion of this issue.
driven by (a) the strength of the within-group spillovers that group generates and (b) the strength
of the spillover that group generates on the other group, compounded by how much group \( \theta \) (who
generates the spillover) is under-represented compared to group \( \theta' \) (who benefits from the spillover)
in a city.\(^{13}\) In their quantification based on the U.S. economy, they find that a counterfactual
efficient allocation features, in some cities, a higher concentration of high-skill, and in others, more
mixing of high-skill with the low-skill group compared to the observed allocation. For the largest
cities, the first effect dominates. For the majority of cities, especially those at the bottom of the
distribution, the second effect dominates. The efficient equilibrium features overall more mixing of
skills than the observed one. These findings are driven by the fact that high-skilled workers have
large within-group externalities, commanding more concentration of high skill together, but at the
same time large cross-group positive externality on less skilled workers, commanding more mixing.
Rossi-Hansberg et al. (2019) enrich the model with heterogeneous sectors that have heterogeneous
local production externalities, which they estimate, but do not model externalities in amenities. In
their setup, aggregate productivity is enhanced by more concentration of the high skill in cognitive
hubs.

Gaubert et al. (2021) focus on a different rationale for place-based policies: rather than studying
the efficiency motives for place-based transfers, triggered by the prevalence of local externalities,
they study the extent to which indexing the redistributive system on place of residence, rather
than on income only, enhances equity. To focus on this point, their spatial equilibrium model
features agents that are heterogeneous in skill, income, and location choice, but abstracts from
local externalities. The government is averse to inequality. They find that when poorer households
are spatially concentrated, transfers indexed on location yield equity gains that can outweigh the
distortion in location choice they generate. Using their model calibrated to the U.S. they find
optimal place-based transfers that are of the same order of magnitude as prominent American
place-based policies. Colas and Hutchinson (2021) study the distortive effects of the nationwide
income tax in the U.S., in a spatial equilibrium with heterogeneously skilled workers. The model
does not feature externalities, so that the no-tax equilibrium is efficient. The findings extend Albouy
(2009) who argues that a progressive income tax leads to an additional deadweight loss when spatial
equilibrium is taken into account, since it allocates workers away from high productivity cities. Here,
the authors argue that, in addition, the income tax helps alleviate inequality more when spatial
equilibrium is taken into account, because of differences in mobility and land ownership across
groups.

### 3.2.2 Housing Policy and Sorting

Government intervention in the housing market is a key tool that can influence where different
types of households locate. Changing the allocation of households to locations not only impacts

\(^{13}\)Formally, within-group spillovers are defined as \( \frac{\partial \log z^\theta_i}{\partial \log L^\theta_i} \) where \( z^\theta_i \) is as defined in (4) for production and \( \frac{\partial \log A^\theta_i}{\partial \log L^\theta_i} \) where \( A^\theta_i \) is as defined in (6) for amenities, while across groups spillovers are \( \frac{\partial \log z^\theta_i}{\partial \log L^\theta'_i} \) and \( \frac{\partial \log A^\theta_i}{\partial \log L^\theta'_i} \) for \( \theta \neq \theta' \).
the lives of those living in subsidized housing, but the entire market through general equilibrium effects, as highlighted by the model above, so that analyzing the impact of such policies is complex. Local governments of high housing cost areas, such as New York and San Francisco, worry that their cities are increasingly unaffordable to middle and low income households and put in place policies to prevent further displacement. Rent control is a popular local government policy to curb displacement, since it forbids rent increases among tenants already living in the city. Economists have long complained about the market inefficiencies of rent control, despite cities often wanting to expand or enact it. Diamond et al. (2019) shows how both sides can be correct. She finds that rent control expansions in San Francisco did help prevent displacement of renters who already lived in San Francisco at the time of rent control expansion. However, the benefits to these initial tenants were eroded away as landlords removed their properties from the rental market or redeveloped them to make them ineligible for rent control. This decrease in rental supply led to higher rents citywide, fully undoing the initial benefits accrued to tenants. This highlights the importance of studying general equilibrium effects along with the direct effects of policy interventions.

An alternative to regulating rent increases is to subsidize the development of properties that must be rented to low-income households at below market rates. When built in high-quality neighborhoods, this subsidized housing can help bring low-income households to neighborhoods offering better opportunities. Chetty et al. (2014) and Chetty et al. (2016) show that moving families with young kids to better neighborhoods led to these kids’ future earnings as adults to be substantially higher: childhood location seems to be a key contributor to adult earnings. Fogli and Guerrieri (2019) embed this mechanism into a dynamic model to study the intergenerational effects of sorting on inequality and quantify how the increased residential segregation since 1980 has amplified wage inequality of the next generation.\(^\text{14}\)

These policies have natural limits, however, as it is unrealistic to move all low-income kids to high opportunity areas. In addition, the housing constructed to house low-income families can have externalities on the receiving neighborhoods themselves. Diamond and McQuade (2019) study the place-based effects of new low-income housing construction and how it varies by neighborhood type. They find that building low-income housing in low-income neighborhoods acts as a catalyst to revitalization, since they correspond to some of the nicest local housing stock. Low-income neighborhoods experience declines in crime, more in-migration of higher income households, and a general increase in demand. In contrast, low-income construction in higher income areas depress prices. This highlights the stark trade-off of helping tenants of low-income housing (by building in a high-opportunity area) with helping the broader low-income population residing in private market housing in a low-income area. To develop optimal housing policies that influence sorting going forward –such as inclusionary zoning, the LIHTC, housing vouchers, land-use regulation, or

\(^{14}\)Taking a different perspective, Bilal and Rossi-Hansberg (2021) highlight the fact that low-quality cities are affordable cities. They enable liquidity constrained household to effectively borrow, in form of low house prices, in exchange for worse long-term outcomes, such as kids’ future earnings. This trade-off of short-term savings at the expense of long-term gains acts as a credit market for those unable to access traditional credit. Access to this credit market can improve the well-being of low-income households, even if their long-run outcomes worsen.
market-based new construction – accounting for both direct effects and indirect general equilibrium effects is crucial.

4 Conclusion

Spatial sorting between commuting zones has been increasing since 1980, although the rate of segregation has slowed in recent years. Spatial sorting of college workers was initially strongly directed at high-wage locations, but is now increasingly directed at high-amenity locations. We develop a framework to help think through the causes and consequences of spatial sorting changes. Importantly, the model embeds feedback loops through which an economic shocks or policy changes impact equilibrium sorting, including their effect on locations’ wages, rents and amenities that may affect migration decisions of high- and low-skill workers differently. These general equilibrium forces are important to take into account when assessing the overall impact of a shock or policy. We expect the literature studying spatial sorting to continue to explode as more papers find ways to combine quasi-experimental research designs with general equilibrium analysis to better understand the causes and consequence of spatial sorting on inequality and contribute to the policy debate.

References


