Lecture 7 - **Static Labor Demand**

References:

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Some additional background reading:

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The building block of labor demand is a production function

$$f(x_1, x_2, ...x_n)$$

where x_j is the input of "factor" j (say hours of work by some skill group). We usually assume that f has constant returns to scale (CRS). This means that in a competitive industry the scale of individual firms is undefined - firms per se are unimportant. What we can measure and analyze is industry-wide demand. As you may recall in the two-input case with CRS the shape of isoquants is summarized by the elasticity of substitution. Technically this is defined as

$$\sigma = -\frac{d\log(\frac{x_1}{x_2})}{d\log(\frac{f_1}{f_2})}$$

where $f(x_1, x_2) = y$ (i.e., the derivative is along an isoquant). Since $\frac{f_1}{f_2}$ is the slope of the isoquant, this is the proportional change of the relative use of the two factors x_1 and x_2 per percent change in the slope of the isoquant (which, under cost-minimization, would be their relative factor prices).

It can be shown that

$$\sigma = \frac{f_1 f_2}{f\ f_{12}}.$$

A simple version of the proof follows. To begin note that with CRS, we have that f_1 and f_2 are both HD0. So

$$\begin{aligned} x_1 f_{11} + x_2 f_{12} &= 0 \\ x_1 f_{21} + x_2 f_{22} &= 0. \end{aligned}$$

Also,

$$f = x_1 f_1 + x_2 f_2.$$

Now define s as the slope of the isoquant at a point:

$$\begin{aligned} s &= \frac{f_1(x_1, x_2)}{f_2(x_1, x_2)} \\ \log s &= \log f_1(x_1, x_2) - \log f_2(x_1, x_2) \\ d \log s &= [\frac{f_{11}}{f_1} - \frac{f_{21}}{f_2}]dx_1 + [\frac{f_{12}}{f_1} - \frac{f_{22}}{f_2}]dx_2 \\ &= [-\frac{x_2}{x_1}\frac{f_{21}}{f_1} - \frac{f_{21}}{f_2}]dx_1 + [\frac{f_{12}}{f_1} + \frac{x_1}{x_2}\frac{f_{12}}{f_2}]dx_2 \\ &= [\frac{-f_2 f_{21} x_2 - f_{21} f_1 x_1}{x_1 f_1 f_2}]dx_1 + [\frac{f_2 f_{12} x_2 + f_{12} f_1 x_1}{x_1 f_1 f_2}]dx_2 \\ &= \frac{-f_{12}}{f_1 f_2}[f_1 x_1 + f_2 x_2]\frac{dx_1}{x_1} + \frac{f_{12}}{f_1 f_2}[f_1 x_1 + f_2 x_2]\frac{dx_2}{x_2} \\ &= \frac{-f_{12} f}{f_1 f_2}[\frac{dx_1}{x_1} - \frac{dx_2}{x_2}] = \frac{-f_{12} f}{f_1 f_2} d\log[\frac{x_1}{x_2}] = \frac{-1}{\sigma} d\log[\frac{x_1}{x_2}]. \end{aligned}$$

Thus

$$\frac{d\log[\frac{x_1}{x_2}]}{d\log s} = -\sigma.$$

The classic examples are Cobb-Douglas ($\sigma = 1$) and the "CES"

$$f(x_1, x_2) = (\alpha x_1^{-\rho} + (1-\alpha)x_2^{-\rho})^{-1/\rho}$$

which has $\sigma = 1/(1+\rho)$.

It is a lot easier in most cases to work with the cost function $C(w_1, w_2, y)$. With CRS this has the form

$$C(w_1, w_2, y) = \mu(w_1, w_2)y$$

where $\mu()$ is the "unit cost" function. We will show that:

$$\sigma = \frac{C_1 C_2}{C\ C_{12}}.$$

To do so, start with Sheppard's Lemma

$$\begin{aligned} x_1 &= C_1(w_1, w_2, y) \\ x_2 &= C_2(w_1, w_2, y) \end{aligned}$$

Now C_j is HD_0 in input prices, so we can write:

$$\begin{aligned} x_1 &= C_1(\frac{w_1}{w_2}, 1, y) = g(\frac{w_1}{w_2}, y) \\ x_2 &= C_2(\frac{w_1}{w_2}, 1, y) = h(\frac{w_1}{w_2}, y) \end{aligned}$$

Also

$$C_{11} = \frac{\partial x_1}{\partial w_1} = \frac{1}{w_2} g_1(\frac{w_1}{w_2}, y), \qquad C_{21} = \frac{\partial x_2}{\partial w_1} = \frac{1}{w_2} h_1(\frac{w_1}{w_2}, y).$$

Now fixing y:

$$\log \frac{x_1}{x_2} = \log g(\frac{w_1}{w_2}, y) - \log h(\frac{w_1}{w_2}, y)$$

so

$$\begin{aligned} \sigma &= -\frac{\left(\frac{w_1}{w_2}\right) d \log \left(\frac{x_1}{x_2}\right)}{d\left(\frac{w_1}{w_2}\right)} = -\frac{w_1}{w_2}\left(\frac{g_1}{g} - \frac{h_1}{h}\right) \\ &= -\frac{w_1}{w_2}\left(\frac{w_2 C_{11}}{g} - \frac{C_{21} w_2}{k}\right) \\ &= -\frac{w_1 C_{11}}{C_1} + \frac{w_1 C_{21}}{C_2} \end{aligned}$$

Now $w_1 C_{11} + w_2 C_{12} = 0$ since C_1 is HD_0. Substituting we get

$$\sigma = \frac{w_1 C_{21}}{C_2} + \frac{w_2 C_{12}}{C_1} = \frac{C_{12}(w_1 C_1 + w_2 C_2)}{C_1 C_2} = \frac{C_{12} C}{C_1 C_2}$$

using the fact that C is HD_1 in input prices.

Another useful fact:

$$\begin{aligned} \frac{\partial x_1}{\partial w_2} &= C_{12} \\ &\Rightarrow \quad \frac{w_2}{x_1}\frac{\partial x_1}{\partial w_2} = \frac{w_2 C_{12}}{x_1} = \frac{C_{12} C}{C_1 C_2} \frac{w_2 C_2 C_1}{C x_1} \\ &= \sigma \ \frac{w_2 x_2}{C} = \sigma s_2 \end{aligned} \tag{1}$$

here $s_2 = \textit{input } 2's$ cost share. Also

$$\begin{aligned} w_1 \frac{\partial x_1}{\partial w_1} + w_2 \frac{\partial x_1}{\partial w_2} &= 0 \\ &\Rightarrow \frac{\partial x_1}{\partial w_1} \frac{w_1}{x_1} = -\sigma s_2 = -(1 - s_1)\sigma \end{aligned}$$

This says that in the 2-input case the output-constant elasticity of demand for an input is the product of $(1 - s)$ and σ.

Finally, with more than 2 inputs we define the partial elasticity of substitution

$$\sigma_{ij} = \frac{C_{ij} C}{C_i C_j}.$$

Note that

$$x_i = C_i(w, y) \Rightarrow \epsilon_{ij} \equiv \frac{\partial x_i}{\partial w_j} \frac{w_j}{x_i} = C_{ij} \frac{w_j}{x_i} = \frac{C_{ij} C}{C_i C_j} \frac{w_j C_i C_j}{x_i C} = \sigma_{ij} s_j$$

which is a multi-factor generalization of (1).

Marshall's Rules

Let's consider a competitive industry with CRS and a cost function $C(w, y) = \mu(w) y$. Industry output is priced at $p = \mu(w)$, and there is a downward sloping demand curve for the industry's output $y = D(p)$. We are going to show the "classic" connection between the elasticity of demand for an input by the industry and three key parameters: η the elasticity of product demand, σ_{ij} the partial elasticities of substitution, and s_j the cost shares.

We start with

$$\begin{aligned} x_i &= y \mu_i(w) \\ &\Rightarrow \log x_i = \log y + \log \mu_i(w) \\ &\Rightarrow \frac{\partial \log x_i}{\partial w_j} = \frac{\partial \log y}{\partial \log p} \frac{d \log p}{\partial w_j} + \frac{\mu_{ij}(w)}{\mu_i(w)} \\ &= -\eta \frac{d \log \mu(w)}{\partial w_j} + \frac{\mu_{ij}(w)}{\mu_i(w)} \\ &= -\eta \frac{\mu_j(w)}{\mu(w)} + \frac{\mu_{ij} y C_j C}{\mu_i y C_j C} \end{aligned}$$

Therefore

$$\begin{aligned} w_j \frac{\partial \log x_i}{\partial w_j} &= \frac{w_j \mu_j(w) y}{\mu(w) y} + \frac{w_j C_{ij} C_j C}{C_i C_j C} \\ &= -\eta \frac{w_j C_j}{C} + \sigma_{ij} \frac{w_j C_j}{C} \\ &= -\eta s_j + \sigma_{ij} s_j \\ &= \epsilon_{ij} - \eta s_j \end{aligned}$$

Now let's consider the own-price effect:

$$\frac{\partial \log x_i}{\partial \log w_i} = \epsilon_{ii} - \eta s_i.$$

Finally, consider the 2-input case, so $\epsilon_{ii} = -(1 - s_i)\sigma$. Then

$$\frac{\partial \log x_i}{\partial \log w_i} = -(\eta s_i + \sigma(1 - s_i))$$

which says that in the 2-input case the own-price demand elasticity is a combination of the final product demand elasticity (a "scale" effect) and the elasticity of substitution (a "substitution" effect).

To understand the scale effect, note that when $p = MC = \mu(w)$

$$\begin{aligned} \frac{\partial \log p}{\partial w_j} &= \frac{\mu_j(w)}{\mu(w)} \\ w_j \frac{\partial \log p}{\partial w_j} &= \frac{w_j \mu_j(w) y}{\mu(w) y} = \frac{w_j x_j}{C} = s_j. \end{aligned}$$

Thus when w_j rises by 1%, industry selling price rises by s_j%, and this chokes off demand by ηs_j%.

The "standard model" of the demand side in labor economics is one in which all firms have CRS and pay the same prices for all factors. In this model firms *per se* do not matter: in fact the number and size of firms is indeterminant. In trade theory and IO there is considerable interest in models with a lot of heterogeneity across firms (e.g., Melitz, 2003). In these models different firms have different levels of productivity. Less productive firms survive because they produce differentiated products which consumers are willing to buy (though less productive firms are smaller). On the labor side, all firms pay the same wages, so the heterogeneity in firms does not matter directly. An important and growing area of work in labor economics focuses on the impact of firms. The starting point for this work is the recognition that "who you work for matters". See the presentation for the "Vancouver School of Economics" in Sept. 2013 (on the class web site) that tries to summarize some of the older ideas and new thrusts in this area.

Some Functional Forms

Cobb Douglas – often used for modeling labor and capital in contexts where the assumption that $\sigma = \sigma_{KL} = 1$ is not too crazy:

$$y = f(K, L) = AL^{\alpha}K^{1-\alpha}$$

Note that labor's share is α which is constant. This used to be approximately correct. However, over the 2000's, labor's share has fallen from the "historical" value of about 65% to about 58%. See Fleck, Glaser and Sprague, "The

Compensation-Productivity Gap": A Visual Essay" Monthly Labor Review January 2011 (Figure 5 is attached). Note that in any CRS 2-input production function

$$\begin{aligned} f(x_1, x_2) &= x_1 f(1, x_2/x_1) = x_1 g(x_2/x_1) \\ &\Rightarrow f(x_1, x_2) x_1 = g(x_2/x_1). \end{aligned}$$

In the C-D case we get

$$y/L = A(K/L)^{1-\alpha}$$

which says that with fixed A "labor productivity" (y/L, the average product of labor) is a concave function of K/L. With C-D we get

$$\frac{\partial y}{\partial L} = \alpha A(K/L)^{1-\alpha}$$

so marginal product is just a constant times average product. These equations are widely used as a starting point for understanding productivity trends.

CES - often used to study high/low skill labor:

$$f(x_1, x_2) = (\alpha x_1^{-\rho} + (1-\alpha) x_2^{-\rho})^{-1/\rho}.$$

Many textbooks (e.g. Silberberg) have a section on the derivation of the CES from the differential equation

$$\frac{d \log(\frac{x_1}{x_2})}{d \log(\frac{f_1}{f_2})} = -\sigma, \quad \text{a constant}$$

a problem that was solved by Arrow et al (Re *Stat*, 1961). With the CES as defined here:

$$\sigma = \frac{1}{1+\rho}$$

CES with $\rho = 0$ is Cobb Douglas. CES with $\rho \to -1$ (*from above*) has $\sigma \to \infty$ which is the linear isoquant case. CES with $\rho \to \infty$ has $\sigma \to 0$, which is Leontief.

Relative Demand and the Relative Wage Structure

The "demand-supply and wage inequality" literature operates with two key assumptions:

(1) at any point in time there is a national production function $y = f(K, L_1, L_2, ...)$ that determines the relative productivity of different skill groups

(2) supply of each skill group is predetermined (and hence exogenous to "current technoology shocks").

The same basic setup is used to address the effects of immigration on the wages of different groups, at both the national and local levels. Under assumptions (1) and (2), the structure of the relative demand for different skill groups entirely determines the relative wage structure. The key questions are:

how are skill groups defined?

how do we parameterize the degree of substitutability between groups?
how does capital fit in?

We will discuss the "standard" theoretical framework for this analysis, which makes 3 assumptions:

1. $y = f(K, h(L_1, L_2...))$ $\quad with \quad$ $f(K, L) = AL^{\alpha}K^{1-\alpha}$
2. the return to capital (r) is exogenous
3. $h(L_1, L_2...)$ has a nested CES structure

Under the first two assumptions, we have

$$\begin{aligned} \frac{\partial y}{\partial K} &= (1-\alpha)AL^{\alpha}K^{-\alpha} = r \\ &\Rightarrow \quad K = L\left(\frac{(1-\alpha)A}{r}\right)^{1/\alpha} \quad \text{and} \quad \frac{y}{K} = \frac{r}{(1-\alpha)} \end{aligned}$$

This means that K adjusts to match the overall supply of "labor units" L, keeping y/K constant, and keeping K/L on a trend path that is driven by the rate of growth of t.f.p. Figures 2 and 3 from Ottaviano and Peri (2011) suggest this is a reasonable assumption at the national level. At the local level (or for "small open economies" that take the price of capital as exogenous) these assumptions are even more plausible. Substituting for K we get

$$y = AL^{\alpha}K^{1-\alpha} = A^{1/\alpha}(\frac{1-\alpha}{r})^{\frac{1-\alpha}{\alpha}}L$$

which is linear in L. Thus,under these assumptions we can ignore capital.[1]

To analyze the effects of relative supply or relative technology changes (i.e., the part of technology embedded in $h()$) we need to specify the labor aggregator function. A good starting point is a 2-group CES model:

$$L = h(L_1, L_2) = (\theta_1 L_1^{\frac{\sigma-1}{\sigma}} + \theta_2 L_1^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}$$

where θ_1 and θ_2 are possibly trending over time.[2] The marginal product of group 1 is

$$h_1(L_1, L_2) = \theta_1 L_1^{\frac{-1}{\sigma}}(\theta_1 L_1^{\frac{\sigma-1}{\sigma}} + \theta_2 L_1^{\frac{\sigma-1}{\sigma}})^{\frac{1}{\sigma-1}} = \theta_1 L_1^{\frac{-1}{\sigma}} L^{\frac{1}{\sigma}}.$$

Likewise

$$h_2(L_1, L_2) = \theta_2 L_2^{\frac{-1}{\sigma}} L^{\frac{1}{\sigma}},$$

[1] There is another (generally older) literature which works with general 3-factor production functions $y = f(K, L_1, L_2)$. In this setting you can also impose the assumption that the marginal product of capital is set to some exogenous r.

[2] Sometimes people write the CES as $h(L_1, L_2) = (\theta_1 L_1^{\rho} + \theta_2 L_1^{\rho})^{\frac{1}{\rho}}$, which is the same as the expression in the text with $\rho = \frac{\sigma-1}{\sigma}$. This implies that $\sigma = \frac{1}{1-\rho}$. Note that $\rho < 1 \iff \sigma > 0$. The limiting case $\rho \to -\infty$ ($\sigma = 0$) is Leontief. The limiting case $\rho \to 1$ ($\sigma \to \infty$) is linear. The case $\rho = 0$ ($\sigma = 1$) is Cobb Douglas.

and assuming $w_1/w_2 = h_1/h_2$ (i.e., MRTS=relative wage) we have:

$$\log \frac{w_1}{w_2} = \log \frac{\theta_1}{\theta_2} - \frac{1}{\sigma} \log \frac{L_1}{L_2}.$$

The slope of the relative demand curve is $-\frac{1}{\sigma}$, which is 0 if the two types are perfect substitutes, and something larger otherwise. This simple model is widely used to discuss "skill biased technical change" (SBTC).

In the "traditional" SBTC literature (e.g., Katz and Murphy, 1992) it is assumed that

$$\log \frac{\theta_{1t}}{\theta_{2t}} = a + bt + e_t$$

leading to a model for the relationship of relative wages to relative supplies:

$$\log \frac{w_{1t}}{w_{2t}} = a + bt - \frac{1}{\sigma} \log \frac{L_{1t}}{L_{2t}} + e_t. \tag{2}$$

Freeman (1976) and Katz and Murphy (1992) estimate models of this form, using 2 "types" of labor - high-school equivalents and college equivalents. Dropouts are assumed to be perfect substitutes for HS graduates with a relative efficiency of (roughly) 70%. Post-graduates are assumed to be perfect substitutes for college graduates with a relative efficiency of (roughly) 125%. People with 1-3 years of college are assumed to represent 1/2 unit of HS labor and 1/2 unit of college labor. (There are different conventions about whether supply should be based on the total numbers of adults in each education group, or total employees. There are also different ways to combine men and women). The "magic number" is $\frac{1}{\sigma} = 0.7$, which implies $\sigma = 1.4$ (See KM, equation 19, page 69). It has turned out to be hard to get a model like (1) to work as well as it did in KM's study (and in Freeman, 1976) when the sample is extended to the 1990s and 2000's. Katz and Goldin (2008) present some estimates that have trend breaks in the last two decades and manage to get estimates in the range of $\frac{1}{\sigma} = 0.7$.

Card and Lemieux (2001) generalize the 2-skills model by introducing a nested CES:

$$\begin{aligned} h(.) &= (\theta_H H^{\frac{\sigma-1}{\sigma}} + \theta_L L^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}} \qquad (3) \\ H &= \left(\sum_j \alpha_j H_j^{\frac{\sigma_A - 1}{\sigma_A}} \right)^{\frac{\sigma_A}{\sigma_A - 1}} \\ L &= \left(\sum_j \beta_j L_j^{\frac{\sigma_A - 1}{\sigma_A}} \right)^{\frac{\sigma_A}{\sigma_A - 1}} \end{aligned}$$

Here H_j is the number of workers in the "high" education group in age group j, and similarly L_j is the number of workers in the "low" education group in age group j. The new parameter σ_A measures the degree of substitutability across age (or experience) groups (which is implicitly set to infinity in the traditional SBTC literature). This model is motivated by the observation (CL, Figures I, II) that in US, UK and Canada the rise in the college high-school wage gap is mainly driven by changes for young workers, and by the realization that since the late 1970s the relative growth rate in educational attainment of consecutive cohorts has stalled in all 3 countries (CL, Figure III). The argument in CL is that the "twisting" of the age profile of relative returns in their Figure II was caused by the slowdown in the growth of relative supply among young workers that is slowly working its way through the age distribution.

Differentiating (2) w.r.t. L_j and H_j and re-arranging terms it is easy to derive a generalization of (1) of the form:

$$\log \frac{w_{jt}^H}{w_{jt}^L} = \log \frac{\theta_{Ht}}{\theta_{Lt}} + \log \frac{\alpha_j}{\beta_j} - \frac{1}{\sigma} \log \frac{H_t}{L_t} - \frac{1}{\sigma_A} \left(\log \frac{H_{jt}}{L_{jt}} - \log \frac{H_t}{L_t} \right) \tag{4}$$

(CL, equation 8b). This says that for age group j the percentage gap in wages between H and L workers in period t depends on $\log \frac{\theta_{Ht}}{\theta_{Lt}}$, the aggregate index of "skill biased" tech-change, on the aggregate relative supply of H and L workers, $\log \frac{H_t}{L_t}$, on an age-group specific relative productivity effect $\log \frac{\alpha_j}{\beta_j}$ (which we are assuming in (3) does not vary over time) and on the deviation between the relative supply of H workers in age group j and the overall relative supply. There are a number of interesting implications of (3):

1. if $\log \frac{H_{jt}}{L_{jt}} - \log \frac{H_t}{L_t}$ is constant over time then the Freeman-Katz-Murphy model is still OK, and provides a valid estimate of $\frac{1}{\sigma}$. Empirically the relative supply gaps were pretty stable over time in the US until the late 1970s which may explain why Freeman's analysis looks so good.

2. the relative supply of educated workers in age group j is (largely) determined by the choices when the people who are currently age j were finishing school. That suggests a simple model like:

$$\log \frac{H_{jt}}{L_{jt}} = \lambda_{t-j} + \phi_j$$

where λ_c is a cohort effect (and we measure age by years since age 20, and index cohorts by the calendar year they reached age 20) and ϕ_j is an age effect that is constant across cohorts and time. Substituting this into (3) yields a model with age, time and cohort effects:

$$\log \frac{w_{jt}^H}{w_{jt}^L} = \log \frac{\theta_{Ht}}{\theta_{Lt}} + \log \frac{\beta_j}{\alpha_j} - \frac{1}{\sigma_A} \phi_j - \left(\frac{1}{\sigma} - \frac{1}{\sigma_A} \right) \log \frac{H_t}{L_t} - \frac{1}{\sigma_A} \lambda_{t-j} \tag{5}$$

Notice that the cohort effects drop out if $\frac{1}{\sigma_A} = 0$: so evidence of cohort effects is simple evidence of imperfect substitution across age groups.

3. Using (2) the relative wage of any 2 age groups (j, j') in the same education class can be written as:

$$\begin{aligned}
\log \frac{w^H_{jt}}{w^H_{j't}} &= \log \frac{\alpha_j}{\alpha_{j'}} - \frac{1}{\sigma_A}\left(\log \frac{H_{jt}}{H_{j't}}\right) \\
\log \frac{w^L_{jt}}{w^L_{j't}} &= \log \frac{\beta_j}{\beta_{j'}} - \frac{1}{\sigma_A}\left(\log \frac{L_{jt}}{L_{j't}}\right).
\end{aligned}$$

Thus the coefficient $\frac{1}{\sigma_A}$ can be estimated by relating the relative wages of different age groups in the same education class to their relative supplies, and a set of relative efficiency parameters. CL get estimates for $\frac{1}{\sigma_A}$ in the range of 0.2 or a little smaller (see also Ottaviano and Peri, 2011). (Note too that it would be pretty easy to have different values for σ_A for the different education groups).

4. How do you estimate a 2-level nested CES? As you can see from (3), this is somewhat complicated by the fact that the aggregate supply indexes H_t and L_t are "model-based": you need to know the parameters α_j , β_j and σ_A to construct these indexes. An easy way is to proceed in two steps. Step 1 focuses on estimating σ_A , using observation 3 above. Notice that once you normalize one of the efficiency parameters for each education group (e.g., $\alpha_1 = \beta_1 = 1$) you also get estimates of α_j and β_j for the other groups. Using these and the estimate of σ_A we construct the "effective" supply indexes:

$$\begin{aligned}
H_t &= \left(\sum_j \alpha_j H_{jt}^{\frac{\sigma_A - 1}{\sigma_A}}\right)^{\frac{\sigma_A}{\sigma_A - 1}} \\
L_t &= \left(\sum_j \beta_j L_{jt}^{\frac{\sigma_A - 1}{\sigma_A}}\right)^{\frac{\sigma_A}{\sigma_A - 1}}.
\end{aligned}$$

If you examine expressions like these you will see that when $\frac{1}{\sigma_A} \approx 0.2$ the exponents inside the parentheses are numbers like 0.8 and the numbers outside are numbers like 1.25. In this range

$$H_t \approx \sum_j \alpha_j H_{jt}, \quad L_t \approx \sum_j \beta_j L_{jt}.$$

So sometimes people "cheat" by using simple supply aggregates rather than "model consistent" aggregates.

In the second step we estimate (3) on age-group/time-period observations, using the estimates of H_t and L_t,and some assumption on the time series process for $\log \frac{\theta_{Ht}}{\theta_{Lt}}$. This yields estimates of $\frac{1}{\sigma}$ and $\frac{1}{\sigma_A}$. The latter can be compared to the estimate obtained in the first step to provide a consistency check of the process. In general, with multiple nests, we can proceed in the same way: start at the lowest level of aggregation and estimate the lowest-level substitution parameter, and the relative efficiency parameters, then construct the supply indexes for the next level and work backwards up the nesting structure.

5. Anderson and Moroney (1994) show that when you have a nested CES, the Allen partial elasticity of substitution[3] between two inputs in the same nest is related to the within-nest substitution effect and the between-nest effect. In the contex of (2) they show that for any two age groups i and j with high education:

$$\sigma_{i,j}^{Allen} = \frac{1}{s_H}(\sigma_A - s_L\sigma)$$

and for any two groups with low education

$$\sigma_{i,j}^{Allen} = \frac{1}{s_L}(\sigma_A - s_H\sigma)$$

where s_L and s_H are the cost shares of the L and H groups. If σ_A is small relative to σ this means that two groups in the same nest can be complements (which cannot happen in a 1-level CES, where all groups are substitutes). In a two-nest model like CL's, σ_A is quite a bit smaller than σ and so people in the same education group in different age classes are Allen-complements, which might make economic sense.

II. Generalizations of the 2-nest model.

a) Multiple education classes.

One immediate issue that arises in thinking about (3) is whether we need more education groups. For example, with K education groups we get:

$$\begin{aligned} h(.) &= (\sum_{k=1}^{K} \theta_k E_k^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}} && (6) \\ E_k &= \left(\sum_j \alpha_j^k E_{kj}^{\frac{\sigma_A-1}{\sigma_A}}\right)^{\frac{\sigma_A}{\sigma_A-1}} \end{aligned}$$

where E_k represents the supply of people in education group k. Let's consider 2 groups $k=1$ and $k=2$. Then a variant of (3) will imply a model for the relative wages of people of age group j in education groups 1 and 2 of the form:

$$\log\frac{w_{jt}^2}{w_{jt}^1} = \log\frac{\theta_{2t}}{\theta_{1t}} + \log\frac{\alpha_j^2}{\alpha_j^1} - \left(\frac{1}{\sigma} - \frac{1}{\sigma_A}\right)\log\frac{E_{2t}}{E_{1t}} - \frac{1}{\sigma_A}\log\frac{E_{2jt}}{E_{1jt}} \quad (7)$$

Suppose however that we've made a mistake and groups 1 and 2 are really perfect substitutes. Then the relative wages of people of age group j in groups

[3] Recall $\sigma_{i,j}^{Allen} \equiv \frac{C_{ij}C}{C_iC_j}$. For a regular CES the Allen elasticity for two different inputs is just the elasticity of substitution.

1 and 2 should be constant:

$$\log \frac{w_{jt}^2}{w_{jt}^1} = k$$

(potentially we could allow k to vary by age group). This provides a way to think about how to define the right grouping structure: we group together workers whose relative wages are constant over time (or across local markets, if we are focusing on spatial variation in wages).

An interesting application is to the classification of lower-education groups. One view is that dropouts and high school graduates are 2 different skill groups. An alternative (widely used in the SBTC literature) is that dropouts and HS grads are perfect substitutes. The latter implies that the HS graduation premium is constant (over time and across markets) - a prediction that seems remarkably true. See Figure 6 from "Is the New Immigration Really So Bad?", Ottaviano and Peri (forthcoming, Table 5), and Goldin and Katz (2008, chapter 8). This has important implications for interpreting the effect of immigration, since many immigrants (50% or more) have very low education. If they compete with a broader skill group that includes HS graduates their effect (especially on natives with less than a high school education) is substantially diffused.

b) Immigration

A second important application of the relative supply-demand apparatus is to the analysis of the effects of immigration. George Borjas (QJE, 2003) used a variant of the CL model with 4 education groups (dropouts, HS grads, some college, BA+) and argued that immigration has had a pretty big effect on the wages of the least-educated natives. (He estimates a model like (5), then does some simulations, accounting for the presence of immigrants in various education and age cells. He focuses on simulations in which the capital stock is fixed and imposes an estimate of σ across four education groups – assumptions that arguably over-state the effects of immigration). More recent work, including Ottaviano and Peri (forthcoming) has differed from Borjas in 3 ways:

1. in the simulations capital is allowed to vary endogenously

2. in the estimation, careful attention is paid to the number and definition of education groups

3. a third substitution parameter – between immigrants and natives with the same age and education – is introduced and estimated.

Ottaviano and Peri extend the 2-level model to a 3-level model:

$$
\begin{aligned}
h(.) &= (\sum_{k=1}^{K} \theta_k E_k^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}} \\
E_k &= \left(\sum_j \alpha_j^k E_{kj}^{\frac{\sigma_A-1}{\sigma_A}} \right)^{\frac{\sigma_A}{\sigma_A-1}} \\
E_{kj} &= \left(\sum_{n=1}^{2} \lambda_n^{kj} E_{kj,n}^{\frac{\sigma_n-1}{\sigma_n}} \right)^{\frac{\sigma_n}{\sigma_n-1}} \qquad (8)
\end{aligned}
$$

Here $E_{kj,n}$ is the number of natives ($n = 1$) or immigrants ($n = 2$) in education-age cell kj. Using this structure is easy to show that the relative wage of immigrants versus natives in a particular cell is given by:

$$
\log \frac{w_{kj2,t}}{w_{kj1,t}} = \log \frac{\lambda_2^{kj}}{\lambda_1^{kj}} - \frac{1}{\sigma_n} \log \frac{E_{kj,2t}}{E_{kj,1t}}
$$

Models that ignore the final nest are implicitly assuming $\sigma_n = \infty$. Ottaviano and Peri (Table 2) provide estimates of $\frac{1}{\sigma_n}$ in the range of 0 to -.09. They get larger estimates for low-educated groups (-.07 to -.09), and smaller estimates for highly educated groups (close to 0 for college grads). One can easily think of explanations for this pattern. While $\frac{1}{\sigma_n}$ is small, it turns out to matter for simulating the net effects of immigration, since even a modest value for $\frac{1}{\sigma_n}$ implies that more of the impact of immigrants in a given skill (age/education) group is concentrated among immigrants, and less "spills over" to natives.

Ottaviano and Peri give a general version of a nested CES model with N nests $n = 1...N$ (their equation 6), and show the associated generalization of equation (3). They also show the general expressions for the implied effects of a change in the supply of 1 type of workers on the level of wages for workers of other types. When the nests are ordered so the intra-nest substitution elasticities are strictly increasing (so the top nest has the smallest value of σ_n) they show that an increase in the supply of one type at the bottom of the nesting structure has a negative effect on all groups who are in the same type in nest 1, and a positive effect on all groups who are in the opposite type in nest 1. (The "top nest" dominates).

Exercise: derive Ottaviano and Peri's expressions (9) and (10).

II. Modeling Labor Demand with a Traded Sector

The analysis of the effects of relative supply on relative demand is substantially different when an economy has the possibility of "exporting" some (or all) of its excess labor in some skill group. A full analysis of the various trade models would take many lectures - it is worthwhile to read through Johnson and Stafford's Handbook paper at some point.

The classic Hecksher-Olin (HO) model assumes that each industry uses a different combination of input factors, and that the number of traded industries is at least as big as the number of input factors. In this setting there is a region of relative factor endowments (the cone of diversification) such that changes in relative supply of different skill groups have no effect on wages but are simply accomodated by shifts in the size of different sectors. Card and Lewis try to evaluate the importance of inter-industry shifts in "absorbing" different fractions of high school dropouts across different urban labor markets using the following setup. Let N_c^d represent the number of dropout workers in city c, and let N_c represent the total number of workers in the city. Then the share of dropout workers in city c is:

$$\begin{aligned} s_c^d &= \frac{N_c^d}{N_c} \\ &= \frac{1}{N_c}\sum_i N_{ic}^d \\ &= \sum_i \frac{N_{ic}^d}{N_{ic}} \times \frac{N_{ic}}{N_c} \\ &= \sum_i \lambda_{ic} s_{ic}^d \end{aligned}$$

where N_{ic}^d is the number of dropouts employed in industry i in city c, N_{ic} is total employment in industry i and city c, $\lambda_{ic}$is industry $i's$ employment share in city c, and s_{ic}^d is the "dropout intensity" of industy i in city c. In the HO case $s_{ic}^d = s_i^d$: dropout intensity is constant across cities and all that varies is the share of the different industries. Its not hard to form the decomposition:

$$\begin{aligned} s_c^d - s^d &= \sum_i (\lambda_{ic} - \lambda_i) s_i^d + \sum_i (s_{ici}^d - s_i^d)\lambda_i + \sum_i (\lambda_{ic} - \lambda_i)(s_{ici}^d - s_i^d) \\ &= B_c + W_c + I_c \end{aligned}$$

where B_c is the "between industry" component of adjustment, W_c is the "within industry" component of adjustment, and I_c is the "interaction effect". Card-Lewis estimate these components for each city using 2000 census data. Then they consider regression models of the form

$$\begin{aligned} B_c &= a^B + b^B(s_c^d - s^d) + e_c^B \\ W_c &= a^W + b^W(s_c^d - s^d) + e_c^W \\ I_c &= a^I + b^I(s_c^d - s^d) + e_c^I \end{aligned}$$

By construction $b^B + b^W + b^I = 1$. So we can think of b^B as the "share" of the total absorption of the excess fraction of dropout workers in city c that is attributable to between-industry shifts. Its also useful to plot B_c, W_c, I_c against $s_c^d - s^d$. See figures 11-13 from Card-Lewis. The estimates across 150 larger

MSA's are:

$$\begin{aligned} b^B &= .22 \\ b^W &= .76 \\ b^I &= .02 \end{aligned}$$

Card-Lewis also show that out of the 0.22 total between-industry component 0.09 comes from agriculture, 0.05 from textiles and apparel, and 0.03 from low-skill services. (You may be surprised to learn that agriculture is important for MSA-level employment. However, many MSA's are counties with a substantial agricultural presence (like Sonoma County and other counties in CA and TX). Arguably the variation in dropout shares accounted by agriculture represents reverse causality: because there is more agriculture in some areas, low-skilled workers are drawn to the areas.

Other Models

Kuhn and Wooten (1991) consider a model with 3 factors (think of these as 2 types of labor and capital) and 3 goods, 2 of which are traded and one of which is non-traded. We'll outline a simplified version of their model with:

2 sectors: one traded, one untraded

2 types of labor

capital, which is freely mobile at an exogenous price.

In this model the tradeable sector can adjust to partially export away excess supplies of one of the types of labor.

Supply of both types of labor is perfectly inelastic: the total supply of unskilled labor is N_u, while the total supply of skilled labor is N_s. In addition to labor, industry 1 and industry 2 use capital, which is available at an exogenous price r. Both industries are assumed to have constant returns to scale and to be perfectly competitive. The cost functions of the two industries depend on the wages of unskilled and skilled labor, w_u and w_s, and on the price of capital:

$$\begin{aligned} C^1(w_u, w_s, r, y_1) &= y_1 c^1(w_u, w_s, r) \\ C^2(w_u, w_s, r, y_2) &= y_2 c^2(w_u, w_s, r) \end{aligned}$$

where c^1 and c^2 are the unit cost functions. Finally, it is assumed that both unskilled and skilled workers have Cobb-Douglas preferences, and that unskilled workers spend a fraction α_u of their income on the local good, while skilled workers spend a fraction α_s on the local good.

Equilibrium in the labor market requires:

$$\begin{aligned} y_1 c_u^1(w_u, w_s, r) + y_2 c_u^2(w_u, w_s, r) &= N_u \\ y_1 c_s^1(w_u, w_s, r) + y_2 c_s^2(w_u, w_s, r) &= N_s. \end{aligned}$$

Both sectors are competitive so equilibrium prices are p_1 and p_2 where:

$$\begin{aligned} c^1(w_u, w_s, r) &= p_1 \\ c^2(w_u, w_s, r) &= p_2 \end{aligned}$$

Sector 2 is traded so p_2 is fixed. Sector 1 is local so with Cobb-Douglas demands from the local workers[4] we require:

$$y_1 = \alpha_u w_u N_u / p_1 + \alpha_s w_s N_s / p_1.$$

We have 5 equations in 5 unknowns $(w_u, w_s, y_1, y_2, p_1)$, with exogenous variables N_u, N_s, p_2, r. One nice feature of this model is "scale invariance": starting from an initial equilibrium, if the supplies of unskilled and skilled labor are both increased by x percent, then a new equilibrium is established at the original wages and prices, with y_1 and y_2 both increased by x percent. An implication of scale invariance is that relative wages and the relative size of the two sectors in the local economy are only affected by changes in the *relative* supplies of labor.

To analyze the effects of a shift in labor supply, begin by differentiating the labor market equilibrium conditions. After some manipulation, the resulting equations can be written as

$$\begin{aligned} dlogN_u &= \lambda_{1u} dlogy_1 + \lambda_{2u} dlogy_2 + e_{uu} dlogw_u + e_{us} dlogw_s & \text{(10a)} \\ dlogN_s &= \lambda_{1s} dlogy_1 + \lambda_{2s} dlogy_2 + e_{su} dlogw_u + e_{ss} dlogw_s & \text{(10b)} \end{aligned}$$

where $dlogx = dx/x$ is the log differential of x, the coefficients $\lambda_{1u}, \lambda_{2u}, \lambda_{1s}, \lambda_{2s}$ represent the fractions of unskilled or skilled workers initially employed in sector 1 or 2 (with $\lambda_{1u} + \lambda_{2u} = 1; \lambda_{1s} + \lambda_{2s} = 1$), and the coefficients $e_{uu}, e_{us}, e_{su}, e_{ss}$ satisfy:

$$\begin{aligned} e_{uu} &= \lambda_{1u} e^1_{uu} + \lambda_{2u} e^2_{uu} \\ e_{us} &= \lambda_{1u} e^1_{us} + \lambda_{2u} e^2_{us} \\ e_{su} &= \lambda_{1s} e^1_{su} + \lambda_{2s} e^2_{su} \\ e_{ss} &= \lambda_{1s} e^1_{ss} + \lambda_{2s} e^2_{ss} \end{aligned}$$

where e^i_{jg} represents the output-constant elasticity of demand for labor of skill group j with respect to the wage of group g in sector i. (Thus, e_{uu} for example, is the *effective* elasticity of demand for unskilled labor w.r.t. its own wage – a weighted average of the elasticities in the 2 sectors).

These equations in turn can be solved for the proportional changes in output:

$$\begin{aligned} dlogy_1 &= \varphi_{1u} dlogN_u + \varphi_{1s} dlogN_s + \zeta_{1u} dlogw_u + \zeta_{1s} dlogw_s & \text{(11a)} \\ dlogy_2 &= \varphi_{2u} dlogN_u + \varphi_{2s} dlogN_s + \zeta_{2u} dlogw_u + \zeta_{2s} dlogw_s & \text{(11b)} \end{aligned}$$

[4] Note that we are ignoring any other forms of income (in particular capital income)

where

$$
\begin{aligned}
\varphi_{1u} &= \lambda_{2s}/(\lambda_{1u} - \lambda_{1s}) \\
\varphi_{1s} &= -\lambda_{2u}/(\lambda_{1u} - \lambda_{1s}) \\
\zeta_{1u} &= (-\lambda_{2s}e_{uu} + \lambda_{2u}e_{su})/(\lambda_{1u} - \lambda_{1s}) \\
\zeta_{1s} &= (-\lambda_{2s}e_{us} + \lambda_{2u}e_{ss})/(\lambda_{1u} - \lambda_{1s})
\end{aligned}
$$

with parallel expressions for $\varphi_{1u}, \varphi_{2u}, \zeta_{1u}, \zeta_{2u}$.[5] The coefficients $(\varphi_{1u}, \varphi_{1s}, \varphi_{2u}, \varphi_{2s})$ represent the so-called "Rybcznski" effects of changes in factor endowments on sectoral outputs (for more on this, read Kuhn and Wooten). These are the effects that would be observed if wages were unaffected by shifts in labor supply – as would occur if both sectors were traded and we were inside the cone of diversification. If the export sector (sector 2) is relatively skill-intensive, for example, then $\lambda_{1u} > \lambda_{1s}$ and therefore $\varphi_{1u} > 0$ and $\varphi_{1s} < 0$. Ignoring wage adjustments, an increase in N_u causes an increase in output in the sector that is more intensive in unskilled labor, whereas an increase in N_s causes a reduction in output in that sector. Also, note that $\varphi_{1s} = 1 - \varphi_{1u}$, reflecting the scale invariance property of the model.

The next step is to differentiate the marginal cost equations, yielding the standard equations relating the share-weighted changes in input prices to the changes in output prices:

$$
\begin{aligned}
\theta_{1u} dlogw_u + \theta_{1s} dlogw_s &= dlogp_1 && \text{(12a)} \\
\theta_{2u} dlogw_u + \theta_{2s} dlogw_s &= 0, && \text{(12b)}
\end{aligned}
$$

where $\theta_{1u} = w_u N_{1u}/p_1 y_1$ is unskilled labor's share in sector 1, etc. Finally, differentiating the untraded good's equilibrium condition leads to:

$$
dlogy_1 = S_{1u}[dlogN_u + dlogw_u] + S_{1s}[dlogN_s + dlogw_s] - dlogp_1, \qquad (13)
$$

where S_{1u} is the share of output from sector 1 consumed by unskilled labor, and $S_{2u} = 1 - S_{1u}$ is the share consumed by skilled labor.

Combining (10a), (11a), (11b), and (12) leads to an equation that can be solved for the change in the unskilled wage as a function of the changes in the supplies of unskilled and skilled labor:

$$
dlogw_u = \pi_{uu} dlogN_u + \pi_{us} dlogN_s
$$

where

$$
\begin{aligned}
\pi_{uu} &= (S_{1u} - \varphi_{1u})/M \\
\pi_{us} &= (S_{1s} - \varphi_{1s})/M \\
M &= [\theta_{1u} - S_{1u} + \zeta_{1u}] - (\theta_{2u}/\theta_{2s}) \times [\theta_{1s} - S_{1s} + \zeta_{1s}].
\end{aligned}
$$

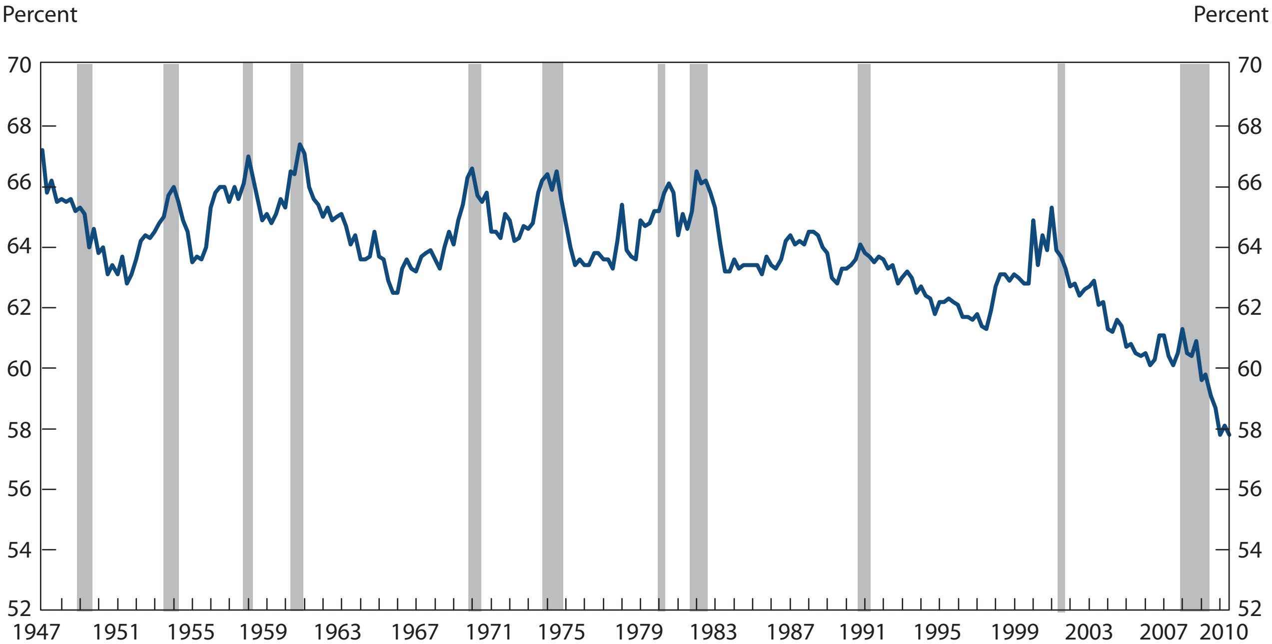
[5] These expressions are derived by writing equations (12) in matrix form: $dlogN = \Lambda d\log y + E d\log w$, and solving $dlogy = \Lambda^{-1} dlogN - \Lambda^{-1} Edlogw$, where Λ is the matrix of λ_{ij} terms ($i = 1, 2; j = u, s$) and E is the matrix of e_{jg} terms ($j = u, s; g = u, s$). The determinant of Λ is $\lambda_{1u}\lambda_{2s} - \lambda_{2u}\lambda_{1s} = \lambda_{1u} - \lambda_{1s}$.

The effects of shifts in relative labor supply on the wages of skilled workers can be derived using equation (11b). Thus:

$$dlogw_s = \pi_{su} dlogN_u + \pi_{ss} dlogN_s$$

where $\pi_{su} = -(\theta_{2u}/\theta_{2s})\pi_{uu}$ and $\pi_{ss} = -(\theta_{2u}/\theta_{2s})\pi_{us}$. These two can be substituted into the system to derive effects on output and employment in the two sectors. Table 1, at the end of the lecture, shows the simulated effect of an increase in unskilled labor supply under 2 simple choices for the technologies of the 2 sectors.

5. Labor share of nonfarm business sector output, first quarter 1947–third quarter 2010

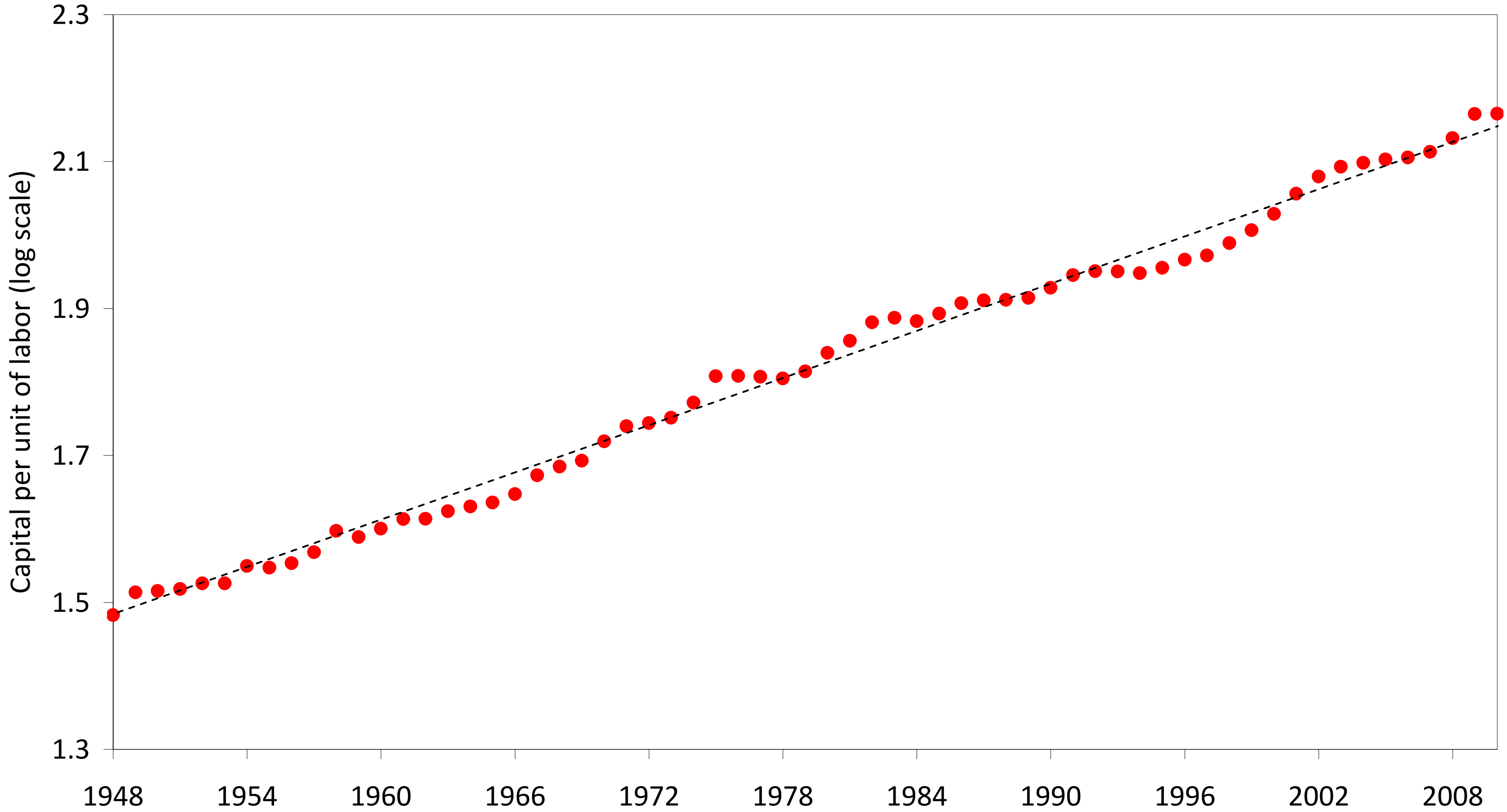


NOTE: The shaded bars denote National Bureau of Economic Research (NBER)-designated recessions.

- Labor share is the portion of output that employers spend on labor costs (wages, salaries, and benefits) valued in each year's prices. Nonlabor share—the remaining portion of output—includes returns to capital, such as profits, net interest, depreciation, and indirect taxes.

- Labor share averaged 64.3 percent from 1947 to 2000. Labor share has declined over the past decade, falling to its lowest point in the third quarter of 2010, 57.8 percent. The change in labor share from one period to the next has become a major factor contributing to the compensation–productivity gap in the nonfarm business sector.

The Long Run Trend in Capital per Unit of Labor



Source: BLS Multifactor Productivity Tables (http://www.bls.gov/mfp)

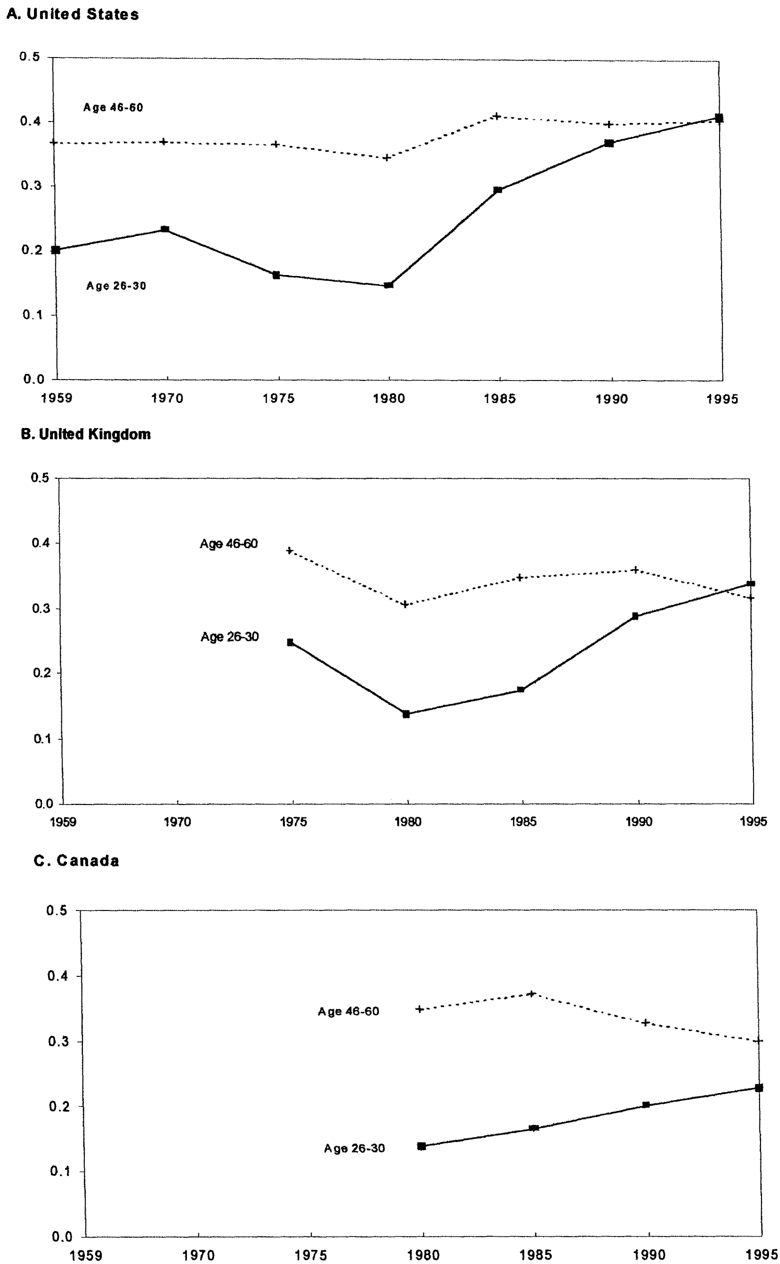


FIGURE I
Estimated College-High School Wage Differentials for Younger and Older Men

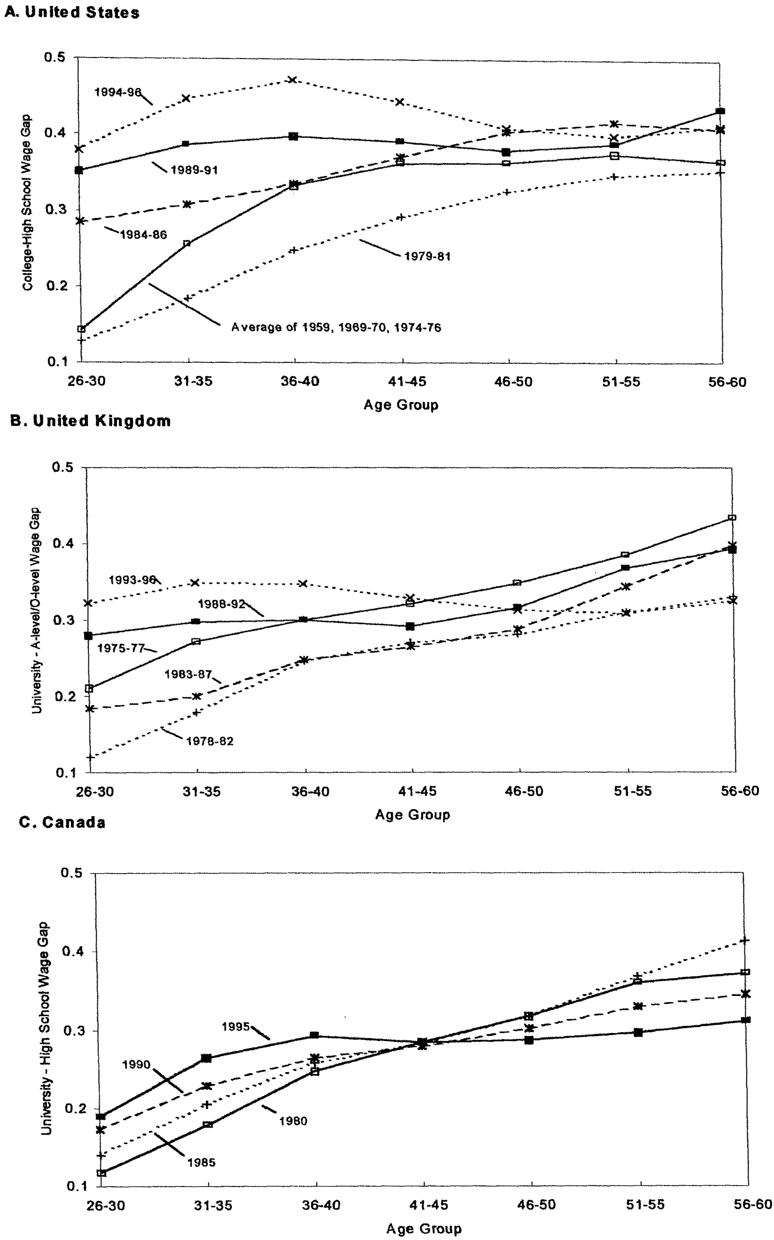


FIGURE II
Age Profiles of the College-High School Wage Gap

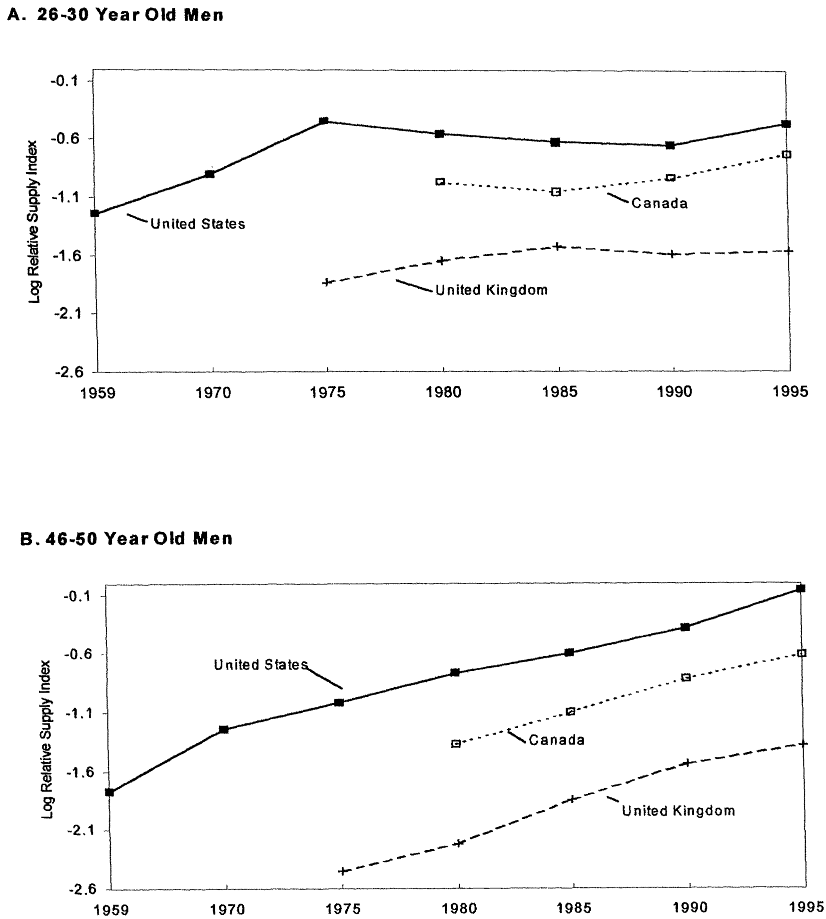


FIGURE III
Age-Group Specific Relative Supplies of College-Educated Labor

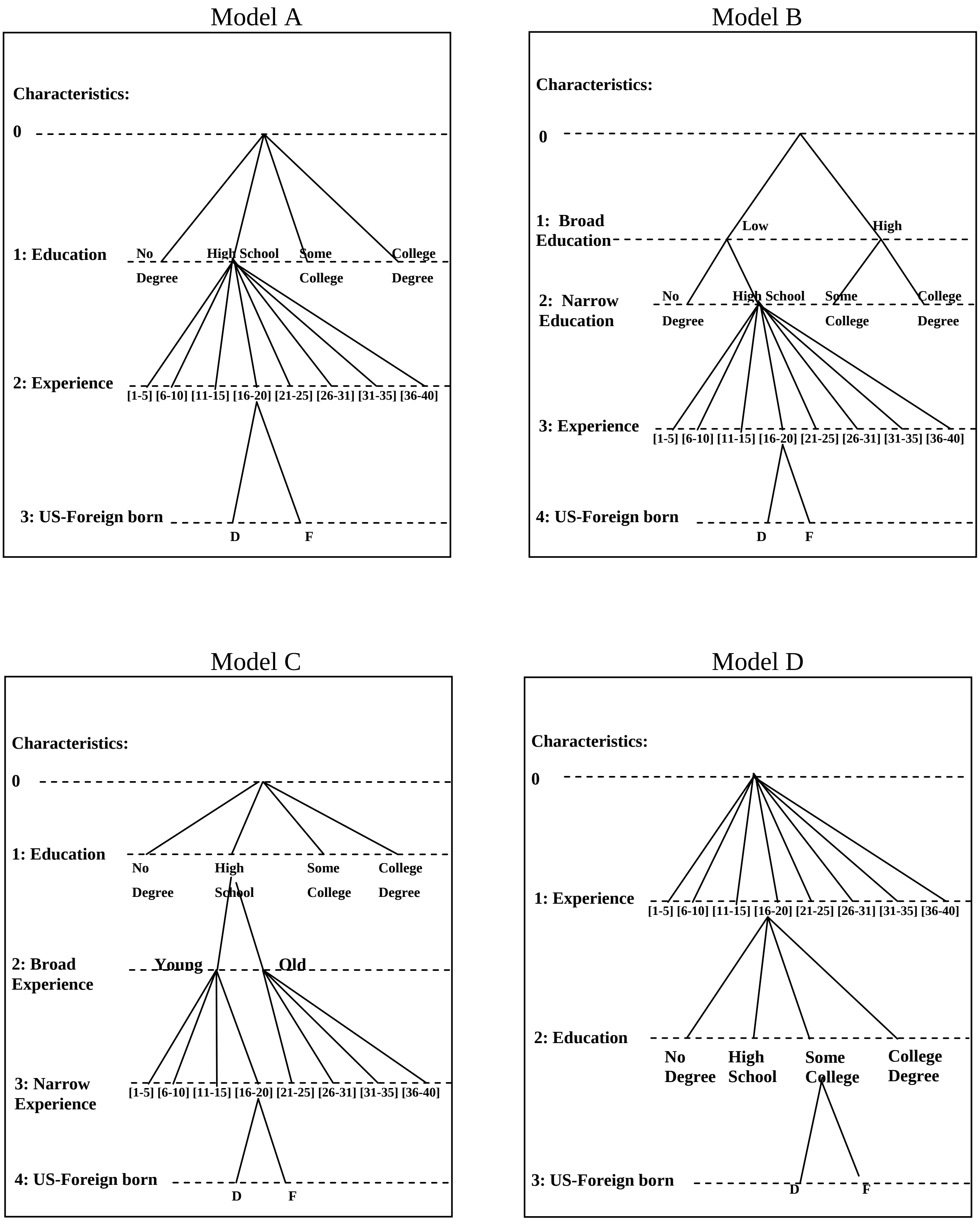
Kingdom before leveling off. The parallel movements in the United States and Canada are especially striking, and suggest that whatever forces led to the slowdown in the intercohort trend in educational attainment were common to the two nations.[17]

An important feature of Figure IV is that the timing of the

17. The 1945–1949 cohort in the United States seems to have slightly higher educational attainment than would be predicted given earlier and later cohorts and the pattern in Canada. This may be an effect of draft avoidance behavior by men in this cohort, who entered college to avoid service during the Vietnam war.

Figures

Figure 1: Alternative nesting models



Tables

Table 1:
Immigration and Changes in Native Wages: Education-Experience groups, 1990-2006

Column 1: Education	Column 2: Experience	Column 3: Percentage change in hours worked in the group due to new immigrants 1990-2006	Column 4: Percentage change in weekly wages, Natives, 1990-2006
No High School Degree (ND)	1 to 5 years	8.5%	0.7%
	6 to 10 years	21.0%	-1.5%
	11 to 15 years	25.9%	0.6%
	16 to 20 years	31.0%	1.6%
	21 to 25 years	35.7%	1.3%
	26 to 30 years	28.9%	-1.6%
	31 to 35 years	21.9%	-8.8%
	36 to 40 years	14.3%	-10.1%
	All Experience groups	**23.6%**	**-3.1%**
High School Degree (HSD)	1 to 5 years	6.7%	-5.3%
	6 to 10 years	7.7%	-1.6%
	11 to 15 years	8.7%	-1.4%
	16 to 20 years	12.1%	1.8%
	21 to 25 years	13.0%	0.6%
	26 to 30 years	11.8%	-0.9%
	31 to 35 years	11.0%	-2.0%
	36 to 40 years	9.3%	-4.0%
	All Experience groups	**10.0%**	**-1.2%**
Low Education (ND+HSD)	**All Experience groups**	**13.2%**	**-1.5%**
Some College Education (SCO)	1 to 5 years	2.6%	-5.4%
	6 to 10 years	2.6%	-2.0%
	11 to 15 years	3.9%	0.1%
	16 to 20 years	6.2%	0.6%
	21 to 25 years	8.4%	-2.5%
	26 to 30 years	12.0%	-3.1%
	31 to 35 years	12.3%	-3.8%
	36 to 40 years	12.7%	-3.0%
	All Experience groups	**6.0%**	**-1.9%**
College Degree (COD)	1 to 5 years	6.8%	0.4%
	6 to 10 years	12.2%	6.5%
	11 to 15 years	13.7%	14.2%
	16 to 20 years	12.2%	17.3%
	21 to 25 years	17.5%	9.1%
	26 to 30 years	24.4%	4.3%
	31 to 35 years	26.1%	1.7%
	36 to 40 years		
	All Experience groups	**14.6%**	**9.3%**
High Education (SCO+COD)	**All Experience groups**	**10.0%**	**4.5%**

Table 2
Estimates of the coefficient ($-1/\sigma_N$)
National Census and ACS, U.S. data 1960-2006

Specification	**(1) No Fixed Effects**	**(2) With FE**	**(3) Not weighted with FE**	**(4) No Fixed Effects**	**(5) With FE**	**(6) Not weighted with FE**
Wage Sample:	**All workers, weighted by hours**			**Full time workers only**		
Estimates of ($-1/\sigma_N$)						
Men	-0.053***	-0.033**	-0.045***	-0.063**	-0.048***	-0.059***
	(0.008)	(0.013)	(0.013)	(0.005)	(0.010)	(0.012)
Women	-0.037***	-0.058***	-0.067***	-0.050***	-0.066***	-0.071***
	(0.009)	(0.017)	(0.016)	(0.007)	(0.014)	(0.012)
Pooled Men and Women	-0.032***	-0.024*	-0.026**	-0.044***	-0.037***	-0.038**
	(0.008)	(0.015)	(0.15)	(0.006)	(0.012)	(0.013)
Men, Labor supply measured as employment	-0.057**	0.027**	0.030**	-0.066***	-0.040**	-0.041**
	(0.007)	(0.014)	(0.015)	(0.006)	(0.012)	(0.014)
Separate estimates of ($-1/\sigma_N$) by Education Group						
Men, No degree	-0.073***	-0.070***	-0.070***	-0.085***	-0.084**	-0.081**
	(0.007)	(0.010)	(0.009)	(0.004)	(0.006)	(0.007)
Men, High School Graduates	-0.089***	-0.090***	-0.093***	-0.097***	-0.099***	-0.100***
	(0.016)	(0.020)	(0.018)	(0.013)	(0.015)	(0.015)
Men, Some College education	-0.071**	-0.060	-0.070*	-0.077**	-0.068*	-0.075**
	(0.024)	(0.035)	(0.034)	(0.023)	(0.033)	(0.034)
Men; College Graduates	-0.017	0.006	0.019	-0.024	-0.009	-0.0150
	(0.026)	(0.042)	(0.030)	(0.027)	(0.041)	(0.029)
Separate estimates of ($-1/\sigma_N$) by Experience Group						
Men, 0-10 years of experience	-0.012	-0.14***	-0.15**	-0.037**	-0.151***	-0.157***
	(0.018)	(0.028)	(0.030)	(0.014)	(0.020)	(0.031)
Men, 11-20 years of experience	-0.044**	-0.061***	-0.066**	-0.050***	-0.068***	-0.073***
	(0.011)	(0.014)	(0.013)	(0.011)	(0.014)	(0.014)
Men, 21-30 years of experience	-0.073**	-0.052**	-0.058**	-0.077***	-0.059**	-0.066***
	(0.008)	(0.022)	(0.017)	(0.007)	(0.022)	(0.018)
Men, 31-40 years of experience	-0.094**	-0.065**	-0.063**	-0.096***	-0.064***	0.060**
	(0.013)	(0.014)	(0.016)	(0.013)	(0.015)	(0.018)

Note: Each cell reports the estimate of the parameter $-1/\sigma_N$. from specification (12) in the text. Method of estimation is Least Squares. In parenthesis we report the heteroskedasticity-robust standard errors, clustered over the 32 education-experience groups. In specification 1, 2, 4 and 5 we weight each cell by its employment. FE (fixed Effects) include Education by Experience plus time effects in Rows one to four, Experience fixed effects are included in rows 5 to 8 and Education fixed Effects are in rows 9-12..***= significant at 1% level; **=significant at 5% level; *= significant at 10% level.

Table 3
Estimates of $(-1/\sigma_{EXP})$
(National Census and ACS U.S. data 1960-2006)

Structure of the nest	**Model A and B**	**Model C**		**Model D**
	(1)	**(2)**	**(3)**	**(4)**
Estimated coefficient:	$(-1/\sigma_{EXP})$	$(-1/\sigma_{EXP})$	$(-1/\sigma_{Y\text{-}O})$	$(-1/\sigma_{EXP})$
Men	-0.16***	-0.19**	-0.31*	-0.30***
Labor Supply is Hours worked	(0.05)	(0.08)	(0.15)	(0.06)
Women	-0.05	0.08*	-0.14	-0.01
Labor Supply is Hours worked	(0.05)	(0.045)	(0.12)	(0.06)
Pooled Men and Women	-0.14***	-0.17**	-0.28**	-0.23***
Labor Supply is Hours worked	(0.04)	(0.06)	(0.12)	(0.05)
Men	-0.13***	-0.18**	-0.26*	-0.22***
Labor Supply is Employment	(0.05)	(0.08)	(0.12)	(0.06)
Cells:	Education-experience-year	Education-experience-year	Education-Young/Old-year	Experience-year
Effects Included	Education by Year and Education by Experience	Education-Young-Year, Education-Old-Year and Education by Experience	Education- Year and Education-Young/Old	Experience effects and year effects
Observations	192	192	96	48

Note: Each cell reports the estimates from a different regression that implements equation (7) in the text for the appropriate characteristics and using the appropriate aggregate and fixed effects. The method of estimation is 2SLS using immigrant workers' hours as instrument for total workers' hours. Cells are weighted by their employment. Standard errors are heteroskedasticity robust and clustered at the education-experience level for columns 1 and 2, at the education-young/old level for column 3 and at the experience level for column 4. *, **, *** = significant at the 10, 5 and 1% level.

Table 4
Estimates of $(-1/\sigma_{EDU})$
(National Census and ACS, U.S. data 1960-2006)

	Model A		Model D	
Specification:	**(1) With education-specific FE and trends**	**(2) With education-specific trends only**	**(3) With experience-year FE**	**(4) With experience-year, education-experience and education-year FE**
Men Labor Supply is Hours worked	-0.16 (0.12)	-0.28** (0.10)	-0.22* (0.12)	-0.04 (0.03)
Women Labor Supply is Hours worked	-0.16 (0.15)	-0.34** (0.14)	-0.25** (0.11)	-0.02 (0.04)
Pooled Men and Women Labor Supply is Hours worked	-0.15 (0.10)	-0.30** (0.11)	-0.23** (0.11)	-0.02 (0.03)
Men Labor Supply is employment	-0.17 (0.10)	-0.43** (0.16)	-0.28** (0.09)	-0.03 (0.03)
Cells	Education-Year	Education-Year	Education-Experience-years	Education-Experience-years
Fixed Effects Included:	Education-specific effects, Education-specific trends and Year effects	Education-specific trends and Year effects	Experience by year only	Experience by year, Education by year and education by Experience
Number of observations	24	24	192	192

Note: Each cell reports the estimates from a different regression that implements (7) in the text using the appropriate wage as dependent variable and labor aggregate as explanatory variable and the appropriate fixed effects. The method of estimation is 2SLS using immigrant workers as instrument for total workers in the relative skill group. Cells are weighted by their employment. Standard errors are heteroskedasticity robust and clustered at the education level for columns 1 and (2), and at the education-experience level for column 3 and 4.
*, **, *** = significant at the 10, 5 and 1% level.

Table 5

Elasticity of substitution between Broad and Narrow Education groups

CPS data 1962-2006, Pooled Men and Women

	Model B			
	(1) **$-1/\sigma_{H\text{-}L}$**	**(2)** **$-1/\sigma_{EDU,L}$**	**(3)** **$-1/\sigma_{EDU,H}$**	**Observations**
"Some College" split between L_{HIGH} and L_{LOW}	-0.54*** (0.06) [0.07]	-0.029 (0.018) [0.021]	-0.16* (0.08) [0.10]	44
"Some College" in L_{HIGH}	-0.32*** (0.06) [0.08]	-0.029 (0.018) [0.021]	-0.16* (0.08) [0.10]	44
Employment as a Measure of Labor Supply	-0.66*** (0.07) [0.09]	-0.039 (0.020) [0.024]	-0.08 (0.09) [0.11]	44
1970-2006	-0.52*** (0.06) [0.08]	0.021 (0.028) [0.025]	-0.13 (0.08) [0.09]	36

Note: Each cell is the estimate from a separate regression on yearly CPS data. In the first column we estimate the relative wage elasticity of the group of workers with a high school degree or less relative to those with some college or more. Method and construction of the relative supply (hours worked) and relative average weekly wages are described in the text in Section 4.2.2. In the first row we split workers with some college education between H and L. In the second row we include them in group H, following the CES nesting in our model. In the second column we consider only the groups of workers with no degree and those with a high school degree (the dependent variable is relative wages and the explanatory is relative hours worked). In the third column we consider only workers with some college education and workers with a college degree or more (the dependent variable is relative wages and the explanatory is relative hours worked). In brackets are the standard errors and in square brackets the Newey-West autocorrelation-robust standard errors.

***= significant at 1% level; **=significant at 5% level; *= significant at 10% level.

Table 6
Calculated Long-Run Wage Effects of Immigration, 1990-2006

Nesting Structures:	Model A/C			Model D		Model B			
Specifications:	**(1)** $\sigma_N=\infty$	**(2)** **Estimated** σ_N	**(3)** **Education specific** σ_N	**(4)** **Estimated** σ_N	**(5)** **Education specific** σ_N	**(6)** **Estimated** σ_N	**(7)** **Education specific** σ_N	**(8)** **Katz-Murphy** $\sigma_{HIGH\text{-}LOW}$	**(9)** σ_{EXP}**=10**
Parameters:									
$\sigma_{HIGH\text{-}LOW}$	**2.5**	**2.5**	**2.5**	**2.5**	**2.5**	**2**	**2**	**1.41**	**2**
$\sigma_{EDU,HIGH}$	**2.5**	**2.5**	**2.5**	**2.5**	**2.5**	**10**	**10**	**10**	**10**
$\sigma_{EDU,LOW}$	**2.5**	**2.5**	**2.5**	**2.5**	**2.5**	**10**	**10**	**10**	**10**
σ_{EXP}	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**10**
$(\sigma_N)_H$	∞	**20**	**33**	**20**	**33**	**20**	**33**	**33**	**33**
$(\sigma_N)_L$	∞	**20**	**12.5**	**20**	**12.5**	**20**	**12.5**	**12.5**	**12.5**

% Real Wage Change of US-Born Workers Due to Immigration, 1990-2006

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Less than HS	-4.1%	-3.1%	-2.3%	-2.6%	-1.9%	-0.1%	0.6%	0.5%	0.6%
HS graduates	0.9%	1.4%	1.6%	1.4%	1.7%	0.5%	0.8%	0.7%	0.8%
Some CO	2.2%	2.5%	2.4%	2.4%	2.3%	1.0%	0.8%	0.9%	0.8%
CO graduates	-1.4%	-0.6%	-1.0%	-0.7%	-1.0%	0.5%	0.2%	0.2%	0.2%
Average US-born	**0.0%**	**0.7%**	**0.6%**	**0.7%**	**0.6%**	**0.6%**	**0.6%**	**0.6%**	**0.6%**

% Real Wage Change of Foreign-Born Workers Due to Immigration, 1990-2006

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Less than HS	-4.1%	-8.4%	-11.0%	-8.5%	-11.0%	-5.5%	-8.0%	-8.1%	-8.0%
HS graduates	0.9%	-6.1%	-10.2%	-6.0%	-10.1%	-6.9%	-11.0%	-11.1%	-11.0%
Some CO	2.2%	-2.4%	-0.6%	-2.5%	-1.0%	-4.0%	-2.2%	-2.1%	-2.2%
CO graduates	-1.4%	-9.2%	-6.3%	-9.3%	-6.7%	-8.1%	-5.0%	-5.0%	-5.0%
Average Foreign-born	**0.0%**	**-7.0%**	**-6.6%**	**-7.0%**	**-6.9%**	**-6.5%**	**-6.1%**	**-6.1%**	**-6.1%**
Overall average	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**

Note: The percentage wage changes for each education group are obtained averaging the wage change of each education-experience group (calculated using the formulas for the appropriate nesting structure and the coefficient listed in the first 6 rows). Those percentage changes are weighted by the wage share in the education group. The US-born and Foreign-born average changes are obtained weighting changes of each education group by its share in the 1990 wage bill of the group. The overall average wage change adds the change of US- and foreign-born weighted for the relative wage shares in 1990 and it is always equal to 0 due to the long-run assumption that the capital-labor ratio adjusts to maintain constant returns to capital.

Table 1: Simulated Effects of Increase in the Supply of Unskilled Labor

	Baseline Scenario	Baseline Settings Except: σ(u,s)=3	σ(u,s)=6	θ(2,u)=.30 θ(2,s)=.35	θ(2,u)=.50 θ(2,s)=.20
1. Own-elasticity of demand for unskilled labor (average of two sectors)	-0.70	-1.54	-2.80	-0.66	-0.53
2. Elasticity of unskilled wage w.r.t. increase in unskilled labor π(u,u)	-0.69	-0.29	-0.15	-0.54	-0.28
3. Elasticity of relative wage w.r.t. increase in unskilled labor π(u,u) - π(s,u)	-1.00	-0.42	-0.22	-1.00	-1.00
4. Elasticity of output of local sector w.r.t. increase in unskilled labor	0.46	0.41	0.40	0.49	0.54
5. Elasticity of output of export sector w.r.t. increase in unskilled labor	0.31	0.35	0.37	0.46	0.71
6. Derivative of unskilled employment share in local sector w.r.t. increase in unskilled labor	0.24	0.26	0.26	0.25	0.25
7. Derivative of unskilled employment share in export sector w.r.t. increase in unskilled labor	0.24	0.23	0.23	0.25	0.19

Note: see text. Baseline scenario has Cobb-Douglas technologies in both sectors (all cross substitution elasticities=1) expenditure shares on local good equal to 0.5, unskilled share in local sector=0.4, skilled share in local sector=0.4, unskilled share in export sector=0.2, and skilled share in export sector=0.45.