Economics 250a: Problem Set 1.

1. There are 2 goods, 1 and 2, initially selling at prices (p_1^0, p_2^0). A consumer with income I^0 buys x_1^0 units of good 1 and x_2^0 units of good 2. Now suppose the consumer is offered membership in a "discount club" that sells good 1 at price $(1-\lambda)p_1^0$ (for $0 < \lambda < 1$). How much would the consumer be willing to pay for a membership?

Hint: show that if a membership costs M, the consumer will buy if $I^0 - M \geqslant e((1-\lambda)p_1^0, p_2^0, u^0)$. Now expand e and express the answer in terms of initial expenditure on good 1, the discount factor, and the compensated elasticity of demand for 1....

2. Consider an $n-person$ economy, where each consumer has identical homothetic preferences. Let I^j denote the income of consumer j. Prove that aggregate demand is "as if" there were a single consumer with the same preferences and the total income of all consumers.

3. Consider a consumer who can buy goods 1,2 and has some income I^0. Suppose the price of good 1 rises from p_1^0 to p_1' , while the price of good 2 (a composite commodity) is fixed at $.p_2^0$. Define the equivalent and compensating variations for the price change as:

$$\begin{aligned} EV &= e(p_1', p_2^0, u') - e(p_1^0, p_2^0, u') \\ CV &= e(p_1', p_2^0, u^0) - e(p_1^0, p_2^0, u^0) \end{aligned}$$

where u^0 is the level of utility achieved at the original prices (p_1^0, p_2^0) and u' is the level of utility achieved at the new prices (p_1', p_2^0)..

a) Show that $EV = e(p_1^0, p_2^0, u^0) - e(p_1^0, p_2^0, u')$ and that $CV = e(p_1', p_2^0, u^0) - e(p_1', p_2^0, u')$ and illustrate these on a graph.

b) Use part (a) and a graph to show that if there is no income effect in demand for good 1, EV=CV.

Hint: if there is no income effect, indifference curves in (x_1, x_2) space are 'vertically parallel.

c) Define the change in consumer surplus for the price change as

$$\Delta CS = \int_{p_1^0}^{p_1'} x_1(p_1, p_2^0, I^0) dp_1$$

where $x_1(p_1, p_2, I)$ is the ordinary (uncompensated) demand for good 1. Show that if good 1 is not inferior then $|CV| > |\Delta CS| > |EV|$.

Hint: use the fact that $x_1^c(p_1, p_2, u) = \frac{\partial e(p_1, p_2, u)}{\partial p_1}$ to express the EV and CV in terms of integrals of the compensated demand functions.

d) Using a second order approximation, develop expressions for EV and CV in terms of observable prices and quantities and the compensated elasticity of demand for good 1.

4. A husband and wife each have their own incomes, and have to decide how to allocate their funds between their own consumption, and expenses for their

child. The husband has utility function

$$U^1(c_1, k) = c_1^{\alpha} k^{1-\alpha},$$

where c_1 is his private consumption and k is total spending on the child. The wife has a utility function

$$U^2(c_2, k) = c_2^{\alpha} k^{1-\alpha},$$

where c_2 is her private consumption. The husband has income y_1, and contributes an amount k_1 to the child's expenses. The wife has income y_2, and contributes k_2. Hence, $k = k_1 + k_2, c_1 = y_1 - k_1$, and $c_2 = y_2 - k_2$.

a) Suppose that the spouses choose their contributions to child expenses, taking each other's choices as given. Find 1's optimal choice for k_1, taking k_2 as given, and 2's optimal choice for k_2, taking k_1 as given. Solve for the Cournot equilibrium and find total expenditures on the child k^* .

b) Find the Pareto-optimal amount of expenditure on the child for the family, k^{**}. Show that $k^{**} > k^*$. Explain why.

c) To think about: what would happen if the $\alpha' s$ in U^1 and U^2 were different?

Note: there is a large literature on trying to compare the implications of Nash-bargained household behavior and alternatives, including models that achieve Pareto efficiency, and models that assume that partners act to maximize a single utility function. One useful overview is Martin Browning, Pierre-Andre Chiappori, and Valerie Lechene, "Collective and Unitary Models: A Clarification". Oxford U unpublished paper, 2004.