

### Economics 250a: Problem Set 1.

1. There are 2 goods, 1 and 2, initially selling at prices  $(p_1^0, p_2^0)$ . A consumer with income  $I^0$  buys  $x_1^0$  units of good 1 and  $x_2^0$  units of good 2. Now suppose the consumer is offered membership in a "discount club" that sells good 1 at price  $(1 - \lambda)p_1^0$  (for  $0 < \lambda < 1$ ). How much would the consumer be willing to pay for a membership?

Hint: show that if a membership costs  $M$ , the consumer will buy if  $I^0 - M \geq e((1 - \lambda)p_1^0, p_2^0, u^0)$ . Now approximate  $e$ .

2. Consider a consumer who can buy goods 1,2 and has some income  $I^0$ . Suppose the price of good 1 rises from  $p_1^0$  to  $p_1'$ , while the price of good 2 (a composite commodity) is fixed at  $p_2^0$ . Define the equivalent and compensating variations for the price change as:

$$\begin{aligned} EV &= e(p_1', p_2^0, u') - e(p_1^0, p_2^0, u') \\ CV &= e(p_1', p_2^0, u^0) - e(p_1^0, p_2^0, u^0) \end{aligned}$$

where  $u^0$  is the level of utility achieved at the original prices  $(p_1^0, p_2^0)$  and  $u'$  is the level of utility achieved at the new prices  $(p_1', p_2^0)$ .

a) Show that  $EV = e(p_1^0, p_2^0, u^0) - e(p_1', p_2^0, u')$  and that  $CV = e(p_1', p_2^0, u^0) - e(p_1^0, p_2^0, u^0)$  and illustrate these on a graph.

b) Use part (a) and a graph to show that if there is no income effect in demand for good 1,  $EV=CV$ .

Hint: if there is no income effect, indifference curves in  $(x_1, x_2)$  space are 'vertically parallel.

c) Define the change in consumer surplus for the price change as

$$\Delta CS = \int_{p_1^0}^{p_1'} x_1(p_1, p_2^0, I^0) dp_1$$

where  $x_1(p_1, p_2, I)$  is the ordinary (uncompensated) demand for good 1. Show that if good 1 is not inferior then  $|CV| > |\Delta CS| > |EV|$ .

Hint: use the fact that  $x_1^c(p_1, p_2, u) = \frac{\partial e(p_1, p_2, u)}{\partial p_1}$  to express the EV and CV in terms of integrals of the compensated demand functions.

d) Using a second order approximation, develop expressions for EV and CV in terms of observable prices and quantities and the compensated elasticity of demand for good 1.

e) Suppose that the increase in prices of good 1 is due to the imposition of a tax  $d\tau$ . Define the excess burden of the tax as the difference between the EV and the tax collections of the government. Give an approximate expression for this.

3. A husband and wife each have their own incomes, and have to decide how to allocate their funds between their own consumption, and expenses for their child. The husband has utility function

$$U^1(c_1, k) = c_1^\alpha k^{1-\alpha},$$

where  $c_1$  is his private consumption and  $k$  is total spending on the child. The wife has a utility function

$$U^2(c_2, k) = c_2^\alpha k^{1-\alpha},$$

where  $c_2$  is her private consumption. The husband has income  $y_1$ , and contributes an amount  $k_1$  to the child's expenses. The wife has income  $y_2$ , and contributes  $k_2$ . Hence,  $k = k_1 + k_2$ ,  $c_1 = y_1 - k_1$ , and  $c_2 = y_2 - k_2$ .

a) Suppose that the spouses choose their contributions to child expenses, taking each other's choices as given. Find 1's optimal choice for  $k_1$ , taking  $k_2$  as given, and 2's optimal choice for  $k_2$ , taking  $k_1$  as given. Solve for the Cournot equilibrium and find total expenditures on the child  $k^*$ . Show how this changes in response to income changes of the two spouses.

b) Find the Pareto-optimal amount of expenditure on the child for the family,  $k^{**}$ . Show that  $k^{**} > k^*$ . Explain why.

c) Sometimes people try to distinguish models of family choice by looking at how income shocks to men and women affect the amounts a family spends on kids. Can you see how to model that idea in this framework? What has to be true for the source of the income shock to matter?

Note: there is a large literature on trying to compare the implications of Nash-bargained household behavior and alternatives, including models that achieve Pareto efficiency, and models that assume that partners act to maximize a single utility function. One useful overview is Martin Browning, Pierre-Andre Chiappori, and Valerie Lechene, "Collective and Unitary Models: A Clarification". Oxford U unpublished paper, 2004.