Econ 250a
Problem Set 5

1. Consider a simple search model with no on-the-job search, no job destruction, and similar utility while working or searching. As shown in Lecture, the reservation wage $w^*$ satisfies the equation:

$$w^* = b + \frac{\lambda}{r} \int_{w^*}^{\infty} (w - w^*) f(w) dw$$

Assume that the offered wage distribution is discrete, $w \in \{w_1, ....w_n\}$ with d.f.:

$$\begin{aligned}
P(w &= w_1) = \Phi(\frac{w_1 - \mu}{\sigma}), \\
P(w &= w_2) = \Phi(\frac{w_2 - \mu}{\sigma}) - \Phi(\frac{w_1 - \mu}{\sigma}) \\
&\ldots \\
P(w &= w_{n-1}) = \Phi(\frac{w_{n-1} - \mu}{\sigma}) - \Phi(\frac{w_{n-2} - \mu}{\sigma}) \\
P(w &= w_n) = 1 - \Phi(\frac{w_{n-1} - \mu}{\sigma}),
\end{aligned}$$

where $\Phi$ is the standard normal d.f., and $(\mu, \sigma)$ are parameters. Assume that time in measured in months, and that $b = 1000$, $r = 0.02$. Set up a numerical procedure to find the optimal reservation wage, using a grid with $n = 200$, and setting $w_1 = 0$ and $w_n = 8000$. Assuming $\lambda = 0.1$ and $\mu = 1800$, find $w^*$ for $\sigma = 100, 500, 1000, 1500, 2000$. Find the expected duration of job search $d$ for each $\sigma$. Graph the relationships between $\sigma$, $w^*$ and $d$.

*Extra credit*: repeat the above, assuming $\log w \tilde{} N(\mu, \sigma)$, with $\mu = 7.5$ and $\sigma = 0.025, 0.05, 0.10, 0.20, 0.40$.

2. Consider the simplified model with on the job search described in Lecture 9, where wage offers are distributed on the interval $[0, \overline{w}]$ according to a given d.f. $F(w)$, and the two value functions are:

$$U(w) = \frac{w - c}{r + \delta} + \frac{\delta}{r + \delta} V + \frac{\lambda(1 - \delta)}{r + \delta} \int_{w}^{\overline{w}} (U(\widetilde{w}) - U(w)) f(\widetilde{w}) d\widetilde{w} \qquad (1)$$

and:

$$V = \frac{b}{r} + \frac{\lambda(1 - \delta)}{r} \int_{w^*}^{\overline{w}} (U(\widetilde{w}) - V) f(\widetilde{w}) d\widetilde{w} \qquad (2)$$

where $w^* = b + c$ is the reservation wage.

a) Find the derivative $U'(w)$. What is $U'(\overline{w})$? What is $U'(w^*)$? Draw a picture of $U(w)$ and $V$.

b) Suppose $b = 800$ (per month), $c = 0$, $\lambda = 0.4$, $r = 0.02$, and $\delta = 0.25$, and $F(w) = N(1200, 400)$. Assume that with this distribution $\overline{w} = 2500$ (which is almost true). Develop a numberical procedure to solve for $U(w)$ and $V$.

3. Read Card-Chetty-Weber, QJE 2008. The model is discrete time, with variable search intensity and endogenous asset accumulation. Each worker has a fixed wage $w$; all jobs last indefinitely (no job destruction); and utility is additively separable in consumption and search effort. Some details:

$$\begin{gathered}
\text{discount rate} = \delta; \quad \text{interest rate } = r \quad \text{(non-random)} \\
\text{flow utility} = u(c_t) - \psi(s_t); \quad c_t = consumption; \quad s_t = search\ effort \\
\text{Beginning-of-period:} \quad \text{start with assets } A_t, \text{ choose } s_t \\
\text{If successful } (prob = s_t) \text{ start working, receive } w, \\
\text{if not successful } (prob = \ \ 1 - s_t), \text{ receive benefit } b, \\
End - of - period : choose\ c_t^e \ \ or \ \ c_t^u \ \ \text{depending on seach outcome}
\end{gathered}$$

Value function at the end of period $t$ for an individual who finds a job:

$$V_t(A_t) = \max_{A_{t+1} \geq L} u(A_t - A_{t+1}/(1+r) + w_t) + \frac{1}{1+\delta} V_{t+1}(A_{t+1}),$$

Value function at end of period $t$ for an individual who does not find a job:

$$U_t(A_t) = \max_{A_{t+1} \geq L} u(A_t - A_{t+1}/(1+r) + b) + \frac{1}{1+\delta} J_{t+1}(A_{t+1})$$

Beginning-of-period value function:$J_t(A_t) = \max_{s_t} s_t V_t(A_t) + (1 - s_t) U_t(A_t) - \psi(s_t)$.

a) Find the first order condition for optimal search intensity, $s_t^*$. Show that the marginal utility of effort depends on the gap $V_t(A_t) - U_t(A_t)$.

b) Using your answer in (a) and the derivatives of $V_t$ and $U_t$, find $\partial s_t^* / \partial A_t$, $\partial s_t^* / \partial w_t$, and $\partial s_t^* / \partial b_t$ and discuss the conditions under which $\partial s_t^* / \partial A_t = 0$.

c) Suppose that benefits $b$ are available indefinitely, that $u(c_t) = \log c_t$, that $\psi(s_t) = a s_t^2$, and that $\delta = r$. Suppose in addition that $L = 0$ (i.e., assets have to be positive). Can you derive an algorithm to compute $U_t(A_t)$ and the optimal search intensity choice $s_t^*(A_t)$?

HINTS: find $V_t(A_t) = V(A_t)$. Now think about how to compute $U_t(A_t) = U(A_t)$.