Expected Interest Rate Parity or Forward Discount Puzzle

Presentations October 25
Notation

\begin{align*}
S_t & \quad \text{spot exchange rate, price of foreign currency (\$/$/yen)} \\
F_t & \quad \text{forward rate ($/$/yen for t+1) contracted today} \\
1+I_t & \quad \text{gross interest rate} \\
1+I*_{t} & \quad \text{gross foreign interest rate} \\
s_t & \quad \ln(S_t) \\
f & \quad \ln(F) \\
i & \quad \ln(1+I) \\
i* & \quad \ln(1+I*)
\end{align*}
Covered Interest Rate Parity

\[ 1 + I_t \equiv \frac{1}{S_t} (1 + I^*_t) F_t, \text{ or in logs} \]

\[ i_t - i^*_t \approx f_t - s_t \]
Theory: Expected Interest rate parity

\[ i_t - i_t^* = E_t s_{t+1} - s_t \]

\[ \Rightarrow \]

\[ f_t = E_t s_{t+1} \]
Arbitrage profit vrs Expected profit

• Arbitrage Profit => no *risk*
  – *eg*, violation of covered interest rate parity

• Expected Profit => *risky*
  – *eg*, violation of expected interest rate parity

• Examples
Arbitrage Profit

• covered interest rate parity: \( i - i^* = f - s \)

• Suppose
  \[
  i - i^* = -5\%
  \]

  \[
  f - s = \ln(1) - \ln(1) = 0
  \]

• borrow at \( i \), exchange at \( S = 1 \)
• invest at \( i^* \), enter forward at \( F = 1 \)
• Result 5% riskless profit for each \( 1\$ \)
• Eliminate Arbitrage Profit: \( f - s = i - i^* \)
Expected Profit

• Expected interest rate parity: \( E_s(t+1) - s = i - i^* \)
• Suppose \( i - i^* = -5\% \)

\[ E_s(t+1) - s = \ln(1) - \ln(1) = 0 \]

• borrow at \( i \), exchange at \( S = 1 \)
• invest at \( i^* \), wait until \( t+1 \) and exchange at \( S(t+1) \)
• Result: profit if \( (s_{t+1} - s_t) \leq (i - i^*) \)
How to get to equilibrium?

- $E_t s_{t+1}$ given
- Spot price adjusts to clear market
- borrow at $i$ => $i \uparrow$ and buy yen => $s_t \uparrow$
- then, buy $i^*$ => $i^* \downarrow$
- How much do prices have to change?
- Until $f = E_t s_{t+1} = \ln(1) = (i-i^*) + s = 0$
- No arbitrage links: $i - i^* = f - s$
Test the Hypothesis

- Substitute definition in model

\[ i_t - i_t^* = E_t s_{t+1} - s_t \]
\[ s_{t+1} \equiv E_t s_{t+1} + e_{t+1} \Rightarrow \]
\[ s_{t+1} - s_t = a + b(i_t - i_t^*) + e_{t+1}, \text{ or} \]
\[ s_{t+1} - s_t = a + b(f_t - s_t) + e_{t+1} \]

\[ H_o : a = 0 \text{ and } b = 1 \]
Australia (s=ln(#AU$/$/))

Dependent Variable: DLNS
Method: Least Squares
Date: 09/29/03   Time: 17:15
Sample(adjusted): 1993:09 2003:08
Included observations: 120 after adjusting endpoints

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.000202</td>
<td>0.002672</td>
<td>0.075411</td>
<td>0.9400</td>
</tr>
<tr>
<td>F_S</td>
<td>-0.288133</td>
<td>0.422369</td>
<td>-0.682182</td>
<td>0.4965</td>
</tr>
</tbody>
</table>

R-squared     0.003928  Mean dependent var -0.000247
Adjusted R-squared -0.004513  S.D. dependent var 0.028310
S.E. of regression 0.028374  Akaike info criterion -4.270195
Sum squared resid 0.094997  Schwarz criterion -4.223737
Log likelihood 258.2117  F-statistic 0.465372
Durbin-Watson stat 2.007813  Prob(F-statistic) 0.496461
Theory: $s_{t+1} - s_t = f_t - s_t + e_{t+1}$
Q: Can one make money betting against the theory?

• Use the “carry trade” rules (Cavallo)
  
i > i* lend in home country @ i and borrow in foreign @ i*

  i < i* borrow @ home and lend in foreign

\[
x_r = \begin{cases} 
(i - i^*) - (s_{t+1} - s_t) & ; i > i^* \\
-[i - i^*) - (s_{t+1} - s_t)] & ; i < i^* 
\end{cases}
\]
Or with the forward rate,

- \( f < s \) buy forward \( \rightarrow i < i^* \),
- \( f > s \) sell forward
- realized payoffs

\[
x_r^f = \begin{cases} 
  s_{t+t} - f & \text{buy} \\
  -(s_{t+1} - f) & \text{sell}
\end{cases} = x_r, \text{ eg for buy strategy,}
\]

\[
x_r^f = s_{t+1} - f = -[(i - i^*) - (s - s_{t+1})] = x_r
\]
Results: annual average return 4.2%
Is it risky?

• Sharpe ratios (average excess return/standard deviation)
  – Carry trade 12%
  – VWNYSEretd-risk free 12%

• Is the Sharpe ratio the correct measure?
• How would one reduce risk for the “carry trade” strategy?
How Can these Excess Average Profits Persist?

• Friction
  – Transactions costs absorb most of the profit
  – Infrequent Portfolio Rebalancing
– Differences in the Risk Premia

• **Alvarez, Atkeson, and Kehoe, (2008), *If Exchanges Rates are Random Walks, then Almost Everything We Say about Monetary Policy is Wrong, Federal Reserve Bank of Minneapolis Quarterly Review***
Puzzle Solved?

• Frictions
  – Yes for outsiders, but what happened to dealers? They set the spreads and make the market.

• Differences in the Risk Premia
  – Neat theoretical model that abysmally fails to explain equity returns

• My Conclusion
  – Still a puzzle
Forward Discount Project

- Get monthly data for the US, Japan, and another country. Try Datastream and maybe IFS.
- Test the model, ie, run the regressions,
  \[ s_{t+1} - s_t = a + b(f_t - s_t) + e_{t+1} \]
- If you find a violation (which you should), then
- Calculate the profits and Sharpe ratios for a "carry trade" strategy
- Explain the tests and results in 3-5 pages
Next Class

• Individual Projects
  – Rewrite your thesis idea (or a new one) summarizing another relevant article
  – Formulate the idea as a testable hypothesis and list potential data sources. Explain how the data might be used to test your idea.

• Be Prepared to Present a Paper