Expected Interest Rate Parity or Forward Discount Puzzle

Presentations October 24
Notation

$S_t$  
spot exchange rate, price of foreign currency (#$/yen)

$F_t$  
forward rate (#$/yen for t+1) contracted today

$1 + I_t$  
gross interest rate

$1 + I^*_t$  
gross foreign interest rate

$s_t$  
$\ln(S_t)$

$f$  
$\ln(F)$

$i$  
$\ln(1+I)$

$i^*$  
$\ln(1+I^*)$
Argentina

- Inflation = 12% (official) = 25% black mkt
- Interest rate = 19%
- Exchange rate \( \% \Delta \text{peso} / \$ = 54\% \)
Covered Interest Rate Parity

\[ 1 + I_t \equiv \frac{1}{S_t} (1 + I^*_t) F_t, \]  
or in logs

\[ i_t - i^*_t \approx f_t - s_t \]
Theory: Expected Interest rate parity

\[ i_t - i_t^* = E_t s_{t+1} - s_t \]

\[ \Rightarrow \]

\[ f_t = E_t s_{t+1} \]
Arbitrage profit vrs Expected profit

• Arbitrage Profit => no risk
  – eg, violation of covered interest rate parity

• Expected Profit => risky
  – eg, violation of expected interest rate parity

• Examples
Arbitrage Profit

• covered interest rate parity: \( i - i^* = f - s \)

• Suppose

\[
i - i^* = -5\\%
\]

\[
f - s = \ln(1) - \ln(1) = 0
\]

• borrow at \( i \), exchange at \( S = 1 \)

• invest at \( i^* \), enter forward at \( F = 1 \)

• Result 5% riskless profit for each 1$

• Eliminate Arbitrage Profit: \( f - s = i - i^* \)
Expected Profit

• Expected interest rate parity: $E_s(t+1) - s = i - i^*$

• Suppose $i - i^* = -5\%$

\[
E_{S_{t+1}} - s = \ln(1) - \ln(1) = 0
\]

• borrow at $i$, exchange at $S = 1$

• invest at $i^*$, wait until $t+1$ and exchange at $S(t+1)$

• Result: profit if $(s_{t+1} - s_t) \leq (i - i^*)$
How to get to equilibrium?

- $E_t s_{t+1}$ given
- Spot price adjusts to clear market
- borrow at $i$ => $i \uparrow$ and buy yen => $s_t \uparrow$
- then, buy $i^*$ => $i^* \downarrow$
- How much do prices have to change?
- Until $f = E_t s_{t+1} = \ln(1) = (i-i^*) + s = 0$
- No arbitrage links: $i - i^* = f - s$
Test the Hypothesis

• Substitute definition in model

\[ i_t - i_t^* = E_t s_{t+1} - s_t \]
\[ s_{t+1} \equiv E_t s_{t+1} + e_{t+1} \Rightarrow \]
\[ s_{t+1} - s_t = a + b(i_t - i_t^*) + e_{t+1}, or \]
\[ s_{t+1} - s_t = a + b(f_t - s_t) + e_{t+1} \]

\[ H_o : a = 0 \text{ and } b = 1 \]
Dependent Variable: DLNS  
Method: Least Squares  
Date: 09/29/03   Time: 17:15  
Sample(adjusted): 1993:09 2003:08  
Included observations: 120 after adjusting endpoints

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.000202</td>
<td>0.002672</td>
<td>0.075411</td>
<td>0.9400</td>
</tr>
<tr>
<td>F_S</td>
<td>-0.288133</td>
<td>0.422369</td>
<td>-0.682182</td>
<td>0.4965</td>
</tr>
</tbody>
</table>

R-squared: 0.003928   
Mean dependent var: -0.000247   
Adjusted R-squared: -0.004513   
S.D. dependent var: 0.028310   
S.E. of regression: 0.028374   
Akaike info criterion: -4.270195   
Schwarz criterion: -4.223737   
Log likelihood: 258.2117   
F-statistic: 0.465372   
Durbin-Watson stat: 2.007813   
Prob(F-statistic): 0.496461
Theory: $s_{t+1} - s_t = f_t - s_t + e_{t+1}$
Q: Can one make money betting against the theory?

• Use the “carry trade” rules (Cavallo)
  
i > i* lend in home country @ i and borrow in foreign @ i*

  i < i* borrow @ home and lend in foreign

\[
xr = \begin{cases} 
  (i - i^*) - (s_{t+1} - s_t) & ; i > i^* \\
  -[(i - i^*) - (s_{t+1} - s_t)] & ; i < i^* 
\end{cases}
\]
Q: Or with forward rates

- Use the “carry trade” rules (Cavallo)
  \( f > s \) sell the forward contract
  \( f < s \) buy the forward contract

\[
x_r = \begin{cases} 
  f - s_{t+1}; & f > s \\
  s_{t+1} - f; & f < s
\end{cases}
\]
Results: annual average return 4.2%
Is it risky?

• Sharpe ratios (average excess return/standard deviation)
  – Carry trade  12%
  – VWNYSEretd-risk free 12%

• Is the Sharpe ratio the correct measure?
• How would one reduce risk for the “carry trade” strategy?
How Can these Excess Average Profits Persist?

• Friction
  – Transactions costs absorb most of the profit
  – Infrequent Portfolio Rebalancing

• Longer Horizons
  • Chenn, M, (2006), The (partial) Rehabilitation of interest rate parity in the floating rate era,
continued

– Differences in the Risk Premia

• Alvarez, Atkeson, and Kehoe, (2008), *If Exchanges Rates are Random Walks, then Almost Everything We Say about Monetary Policy is Wrong*, Federal Reserve Bank of Minneapolis Quarterly Review
Puzzle Solved?

• Frictions
  – Yes for outsiders, but what happened to dealers? They set the spreads and make the market.

• Differences in the Risk Premia
  – Neat theoretical model that abysmally fails to explain equity returns

• Longer maturity?

• Still a puzzle
Forward Discount Project

• Get monthly data for the US, Japan, and another country. Try Datastream and maybe IFS.

• Test the model, ie, run the regressions,

\[ s_{t+1} - s_t = a + b(f_t - s_t) + e_{t+1} \]

• If you find a violation (which you should), then

• Calculate the profits and Sharpe ratios for a “carry trade” strategy

• Explain the tests and results in 3-5 pages
Next Class

• Individual Projects
  – Rewrite your thesis idea (or a new one) summarizing another relevant article
  – Formulate the idea as a testable hypothesis and list potential data sources. Explain how the data might be used to test your idea.

• Be Prepared to Present a Paper