

Notes on the Intrinsic Valuation Models and B-K-M Chapter 18

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Overview on Intrinsic Asset Valuation

An asset is a promise to a stream of future payoffs. Unlike a commodity that you consume, eg, an orange that delivers utility now, the demand for an asset depends on the value of the future payoff stream. Buying an asset means sacrificing current consumption for future payoffs that can be converted to consumption. All financial assets provide the same basic service—they transfer consumption over time. But the asset that appreciates the most delivers the most future consumption for today's sacrifice. Therefore choosing the right asset is very important. And asset choice depends on valuing the asset. These notes look at the intrinsic value of an asset mostly in terms of Gordon's formula.

The intrinsic value of an asset is the theoretical value. The market price may differ from the intrinsic value. Eventually, however, the market price converges to the intrinsic value. Notice this implies a decision rule: if an asset's intrinsic value is greater than the price, then buy it.

1. **Economic Theory Definition: the intrinsic value of any asset equals its expected discounted payoffs.**
 - a. Making the theoretical definition operational requires defining the
 - i. payoffs, and the
 - ii. discount factor (rate) (B-K-M call this the capitalization rate)

Applied Economics looks at how closely theoretical models approximate reality. These notes focus on making the theoretical intrinsic valuation definition operational. The simple formulas in B-K-M Chapter 18 capture the essence of a complicated reality. Get to know them intimately. I adopt B-K-M's notation.

Efficient Markets (B-K-M Chapter 12) looks at how well theoretical intrinsic valuation models approximate reality—or how quickly actual prices converge to theoretical values.

Some Preliminaries: Returns to Price and Back

The Return

The return (which is observable), r , on a stock (any asset really) is defined as¹,

$$r_1 \equiv \frac{D_1 + (P_1 - P_0)}{P_0} \equiv \frac{D_1}{P_0} + \frac{(P_1 - P_0)}{P_0} \equiv \frac{D_1}{P_0} + \frac{P_1}{P_0} - 1 \quad (1.1)$$

the flow yield, D_1/P_0 , where D_1 is the flow payoff—eg, the dividend next period, and P_0 is the asset price this period (so D_1/P_0 is the dividend yield) plus the capital gain, $(P_1 - P_0)/P_0$, return.

B-K-M 18.2 Expected holding period²return

Of course, the future values are not known so B-K-M define the *expected* return,

¹ The return—by convention, not logic—does not include the current flow payoff. If you buy an asset, eg, a peach orchard in the fall, then you don't get the payoffs until the next harvest. This definition values assets "ex-dividend".

² The return is a one-period return. The holding period return usually refers to the return over several periods. I don't know why B-K-M use the term "holding period" here.

$$Er_1 \equiv \frac{ED_1 + (EP_1 - P_0)}{P_0} \equiv \frac{ED_1}{P_0} + \frac{(EP_1 - P_0)}{P_0} \equiv \frac{ED_1}{P_0} + \frac{EP_1}{P_0} - 1 \quad (1.2)$$

as the *expected* flow return, ED_1 / P_0 , plus the *expected* capital gain return, $(EP_1 - P_0) / P_0$.

Intrinsic Value vrs Market Price

The market prices can deviate temporarily from their intrinsic value but should converge to their intrinsic value (B-K-M use different symbols to distinguish between intrinsic value, V , and market price, P .) When the market price converges to its intrinsic value the return converges to the intrinsic return, k .

Rearrange equation (1.2) and substitute V for the price and k for the rate of return. The idea is that these are equilibrium, or long run, values. Now write the intrinsic value of the asset as the expected flow payoff, D , plus the intrinsic value³, V , next period discounted with the appropriate capitalization rate⁴, k ,

$$V_0 = E\left(\frac{D_1 + V_1}{K_1}\right); K_1 \equiv 1 + k_1 \quad (1.3)$$

Now use the same formula for V_1 and plug it in (1.3) to push the value one period farther in the future,

$$V_0 = E\left(\frac{D_1}{K_1}\right) + E\left(\frac{1}{K_1}\right)\left(\frac{D_2}{K_2}\right) + E\left(\frac{1}{K_1}\right)\left(\frac{V_2}{K_2}\right) \quad (1.4)$$

The current asset value equals the discounted expected dividend payment one period in the future plus the discounted expected dividend two periods plus the discount intrinsic value two periods in the future.

Substituting for V_2 and then V_3 and then V_4 and then... leads to a general (and generally intractable) formula,

$$V_0 = E\left(\frac{D_1}{K_1}\right) + E\left(\frac{D_2}{K_1 K_2}\right) + E\left(\frac{D_3}{K_1 K_2 K_3}\right) + \dots \quad (1.5)$$

for the intrinsic value of the asset as the expected discounted value of the future flow payoffs. Notice that the flow payoffs and the discount rate vary over time in this formula, and in the real world. Making the formula operation requires restrictions.

Gordon's Formula (Constant dividend growth model B-K-M 18.3)

Gordon's formula (Myron Gordon 1926) makes intrinsic valuation equation tractable (B-K-M Example 18.3 and PS1)

³ B-K-M use the asset price, P , in equation 18.1. But they assume it equals the intrinsic value, V .

⁴ The appropriate capitalization rate includes a risk premium. We have not studied how to determine the appropriate risk premium. For now take the appropriate capitalization rate as exogenous. B-K-M sometime choose the capitalization rate, k , eg on p621, using the CAPM. Accept that they did it correctly.

$$V = \frac{D_1}{k - g} \quad (1.6)$$

The intrinsic value of the asset, V , equals the value of the flow payoff next period, D_1 , divided by the discount rate, k , minus the growth rate of the flow payoff, g . Gordon assumed that:

1. the payoff growth rate and the discount rate are constant and known,
2. the horizon is infinite

Facts, of course, violate all of Gordon's assumptions. Nevertheless he gave us a powerful tool to organize our thinking. Analysts today use more complicated versions of Gordon's formula to value assets.

Let's look at the pieces of Gordon's formula to get a feel for what's going on.

1. Dividends grow at the constant rate, g . This is a huge simplification, now instead of forecasting the dividend stream, D_1, D_2, D_3, \dots one forecasts a single number, the growth rate. Then,

$$D_1 = (1 + g)D_0, D_2 = (1 + g)^2 D_0, \dots, D_j = D_0(1 + g)^j$$

If the dividend growth rate increases, then the asset's intrinsic value increases, $\frac{dV}{dg} > 0$, as you show in PS2.

2. The capitalization rate is constant, k , so $k_1 = k_2 = \dots k_j = k$. If the capitalization rate increases, then the asset's value decreases, $\frac{dV}{dk} < 0$. see PS2. The capitalization rate has two components: the risk free rate—which discounts for time—plus a risk premium.
3. Gordon's formula implies the intrinsic value grows at the dividend (flow payoff) growth rate, g , see PS2.

$$V_1 = V_0(1 + g)$$

4. Rearranging Gordon's formula gives

$$k = \frac{D_1}{V} + g \quad (1.7)$$

the capitalization rate equals the dividend yield plus growth rate. Since the intrinsic value grows at rate g , g is the capital gain return.

Elmo suggests that you play with these to develop some intuition for Gordon's formula.

Growth Opportunities, B-K-M equation 18.6

Companies frequently do not pay their entire current earnings, E , out as dividends to the shareholders. Instead they invest part of the earnings in themselves. If the return on equity, ROE , exceeds the required capitalization rate ($ROE > k$), then the shareholders gain. Define the spread as the difference between the return on equity and the capitalization rate, $s \equiv ROE - k$. When the spread is positive the firm can increase shareholder value (the price of the stock) by investing some of their earnings in themselves.

B-K-M define the “plowback” ratio, b , as the fraction of earnings invested in the project. The growth rate of earnings is the plowback ratio times the return on equity, $g = b * ROE$. B-K-M, equation (18.6), write the price of the firm as,

$$P_0 = \frac{E_1}{k} + PVGO \quad (1.8)$$

the capitalized value of earnings plus the present value of the growth opportunity.

In PS2 you show that $PVGO > 0$ when the spread is positive and $PVGO < 0$ when the spread is negative..Substituting the plowback ratio times ROE for growth in Gordon’s formula gives,

$$P_0 = \frac{D_1}{k - g} = \frac{(1-b)E_1}{k - b * ROE} \quad (1.9)$$

the price as a function of the plowback ratio. In equation (1.9) it is hard to see what increasing the plowback rate does. There are offsetting forces. Increasing the plowback ratio, b , decreases the initial dividend payoff and reduces the value of the firm. But increasing the value of the plowback ratio raises the growth of earnings and dividends. Rewriting Gordon’s formula in terms of the spread makes the relationship more transparent. Use the fact that

$$ROE \equiv k + (ROE - k) \equiv k + s$$

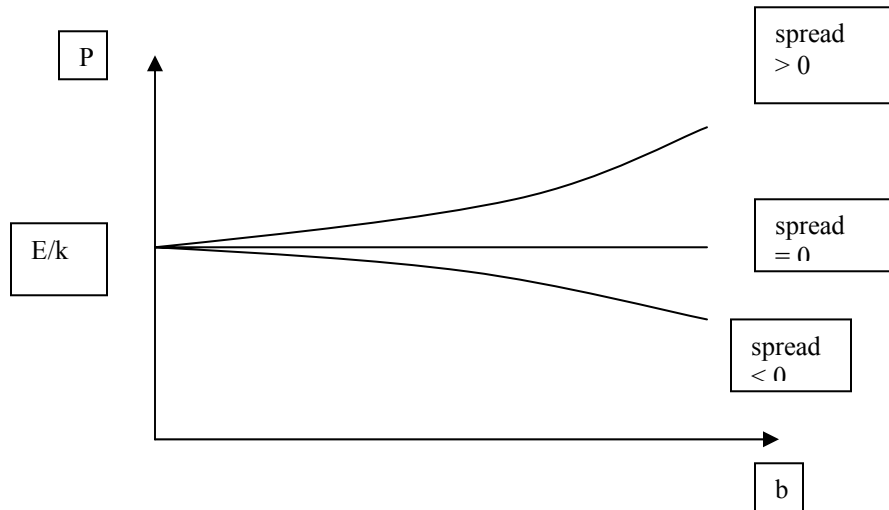
the return on equity equals the capitalization rate plus the spread. Substituting for ROE in equation (1.9) gives

$$P_0 = \frac{(1-b)E_1}{k - b * (k + s)} = \frac{(1-b)E_1}{(1-b)k - b * s} \quad (1.10)$$

The price equals the dividend capitalized by a weighted average of the capitalization rate k and the spread, s . If the spread is positive, then the firm has better investment opportunities than those represented by the capitalization rate. It increases shareholder value by investing in those projects.

The figure illustrates the affect on the current asset price of increasing b for a positive, zero, and negative spread.

Growth Opportunities



When the spread is negative the firm's ROE is less than the required capitalization rate. Investing in these projects lowers the equity price. Why would a firm with a negative *PVGO* represent a good takeover target? (PS2).

B-K-M, equation 18.6, define the present value of growth opportunities, *PVGO*, as the price minus the capitalized value of earnings. Subtracting the capitalized value of earnings, E_1/k , from equation (1.9) gives *PVGO* in terms of the spread,

$$P_0 - \frac{E_1}{k} \equiv PVGO = \frac{(1-b)E_1}{(1-b)k - b*s} - \frac{E_1}{k} \quad (1.11)$$

The *PVGO* is positive when the spread is positive and negative when the spread is negative. For any *b* the vertical distance from the zero spread line in the figure measures *PVGO*.

Price-Earnings Ratio

The dividend-price ratio is a reasonable approximation of equilibrium returns for firms that are not growing,

$$k = \frac{D_1}{V} + g$$

Price-earnings ratio gives a more convenient expression that reveals growth opportunities. Manipulating equation (1.8) (B-K-M 18.6) gives,

$$\frac{P_0}{E_1} = \frac{1}{k} \left[1 + \frac{PVGO}{E/k} \right] \quad (1.12)$$

the price earnings ratio as the capitalized value of one plus the ratio PVGO to the capitalized value of earnings (B-K-M 18.7). When the spread is zero the PVGO is zero and the price earnings ratio equals $1/k$. In equilibrium firms with high growth opportunities have high price earnings ratios and vice-versa. This means that earnings-price ratio for firms with growth opportunities is less than the capitalization rate. Elmo says that makes sense. Do you agree?