Economics 204
Fall 2010
Problem Set 2
Due Tue, Aug 3 in Lecture

1. Determine whether the following sets are open, closed, both or neither:
   (a) $\mathbb{Z}$ in the topology on $\mathbb{R}$ induced by the usual metric;
   (b) $\{1/n \mid n \in \mathbb{N}\}$ in the topology on $\mathbb{R}$ induced by the usual metric;
   (c) $\mathbb{Q}$ in the topology on $\mathbb{R}$ induced by the usual metric;
   (d) $\{(x, y) \in \mathbb{R}^2 \mid y \geq x^2\}$;
   (e) $\{(x, y) \in \mathbb{R}^2 \mid |y| > x\}$.
2. Give examples of the following:
   (a) A continuous function $f : S \to \mathbb{R}$, where S is a closed subset of $\mathbb{R}$, that attains neither a maximum nor a minimum on S;
   (b) A continuous function $f : S \to \mathbb{R}$, where S is a closed and unbounded subset of $\mathbb{R}$, that attains both a maximum and a minimum on S;
   (c) A continuous function $f : S \to \mathbb{R}$, where S is a bounded subset of $\mathbb{R}$, that attains neither a maximum nor a minimum on S;
   (d) A continuous function $f : S \to \mathbb{R}$, where S is a bounded but not closed subset of $\mathbb{R}$, that attains both a maximum and a minimum on S;
   (e) A discontinuous function $f : S \to \mathbb{R}$, where S is a closed and bounded subset of $\mathbb{R}$, that attains neither a maximum nor a minimum on S;
   (f) A discontinuous function $f : S \to \mathbb{R}$, where S is a closed and bounded subset of $\mathbb{R}$, that attains both a maximum and a minimum on S.
3. Suppose $\{x_n\}$ is a Cauchy sequence in a metric space X, and some subsequence $\{x_{n_t}\}$ converges to $x \in X$. Prove that $\{x_n\}$ converges to x.
4. Let $E, F \subseteq \mathbb{R}^n$. For $x \in E$, let $g(x) = \inf\{|x - y| : y \in F\}$. Prove that $g : E \to \mathbb{R}$ is continuous.
5. For all subsets A, B of a metric space (X, d) prove:
   (a) A is both open and closed if and only if $\partial A = \varnothing$;
   (b) $\partial A = \partial(X \backslash A)$;
   (c) $\partial\partial A \subseteq \partial A$ (and give an example of a set A, such that this is a strict inclusion);
   (d) $\partial\partial\partial A = \partial\partial A$;
   (e) $\partial(A \cup B) \subseteq \partial A \cup \partial B$.