1. Suppose $\Gamma : X \to 2^Y$ is a correspondence defined by $\Gamma(x) = \{f_1(x), ..., f_N(x)\}$, where $f_i : X \to Y$ is a continuous function for all $i \in \{1, 2, ..., N\}$. Prove that $\Gamma$ is both lhc and uhc.

2. Let $f : \mathbb{R}^n \to \mathbb{R}^n$ be a $C^1$ function. Define $F : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ by

$$F(x, \omega) = f(x) + \omega$$

Show that there is a set $\Omega_0 \subset \mathbb{R}^n$ of Lebesgue measure zero such that, if $\omega \notin \Omega_0$, then for each $x_0$ satisfying $F(x_0, \omega_0) = 0$ there is an open set $U$ containing $x_0$, an open set $V$ containing $\omega_0$, and a $C^1$ function $h : V \to U$ such that for all $\omega \in V$, $x = h(\omega)$ is the unique element of $U$ satisfying $F(x, \omega) = 0$.

3. Let $f : X \to X$ be continuous. Give an example of a set $X \subseteq \mathbb{R}^n$ and a continuous function $f$, such that $f$ does not have a fixed point and $X$ is:

(a) closed, bounded, but not convex;
(b) convex, closed, but not bounded;
(c) convex, bounded, but not closed.

4. Let $A$ be a nonempty, compact and convex subset of $\mathbb{R}^2$ such that if $(x, y) \in A$ for some $x, y \in \mathbb{R}$, then there exists some $z \in \mathbb{R}$ such that $(y, z) \in A$. Prove that $(x^*, x^*) \in A$ for some $x^* \in \mathbb{R}$. (Hint: Use Kakutani’s Fixed Point Theorem)

5. Let $A$ and $B$ be nonempty, convex subsets of $\mathbb{R}^n$ with $\text{int} A \neq \emptyset$. Using the Separating Hyperplane Theorem, prove that there exists $p \in \mathbb{R}^n$ with $p \neq 0$ such that $\sup p \cdot A \leq \inf p \cdot B$ if and only if $\text{int} A \cap B = \emptyset$. (Hint: You might want to use the result of Theorem 1.11 in de la Fuente, p.234.)

6. Consider the second order linear differential equation given by $y'' = 4y' - 8y$.

(a) Show how this equation can be rewritten as the following first order linear differential equation of two variables:

$$\tilde{y}'(t) = A\tilde{y}(t)$$

(b) Verbally describe the solutions of the first order system by analyzing the matrix $A$.

(c) Solve the system when $y(0) = 3$ and $y'(0) = 7$. 

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