Economics 204
Fall 2010
Problem Set 6
Due Mon, Aug 16 by 9am in Oleksa's GSI mailbox (Evans, 5th floor)

1. Suppose $\Gamma : X \to 2^Y$ is a correspondence defined by $\Gamma(x) = \{f_1(x), ..., f_N(x)\}$, where $f_i : X \to Y$ is a continuous function for all $i \in \{1, 2, ..., N\}$. Prove that Γ is both lhc and uhc.

2. Let $f : \mathbb{R}^n \to \mathbb{R}^n$ be a C^1 function. Define $F : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ by
$$F(x, \omega) = f(x) + \omega$$
Show that there is a set $\Omega_0 \subset \mathbb{R}^n$ of Lebesgue measure zero such that, if $\omega \notin \Omega_0$, then for each x_0 satisfying $F(x_0, \omega_0) = 0$ there is an open set U containing x_0, an open set V containing ω_0, and a C^1 function $h : V \to U$ such that for all $\omega \in V$, $x = h(\omega)$ is the unique element of U satisfying $F(x, \omega) = 0$.

3. Let $f : X \to X$ be continuous. Give an example of a set $X \subseteq \mathbb{R}^n$ and a continuous function f, such that f does not have a fixed point and X is:
   (a) closed, bounded, but not convex;
   (b) convex, closed, but not bounded;
   (c) convex, bounded, but not closed.

4. Let A be a nonempty, compact and convex subset of $\mathbb{R}^2$ such that if $(x, y) \in A$ for some $x, y \in \mathbb{R}$, then there exists some $z \in \mathbb{R}$ such that $(y, z) \in A$. Prove that $(x^*, x^*) \in A$ for some $x^* \in \mathbb{R}$. (Hint: Use Kakutani's Fixed Point Theorem)

5. Let A and B be nonempty, convex subsets of $\mathbb{R}^n$ with $\text{int}A \neq \varnothing$. Using the Separating Hyperplane Theorem, prove that there exists $p \in \mathbb{R}^n$ with $p \neq 0$ such that $\sup p \cdot A \leq \inf p \cdot B$ if and only if $\text{int}A \cap B = \varnothing$. (Hint: You might want to use the result of Theorem 1.11 in de la Fuente, p.234.)

6. Consider the second order linear differential equation given by $y'' = 4y' - 8y$.
   (a) Show how this equation can be rewritten as the following *first* order linear differential equation of two variables:
$$\bar{y}'(t) = A\bar{y}(t)$$
   (b) Verbally describe the solutions of the first order system by analyzing the matrix A.
   (c) Solve the system when $y(0) = 3$ and $y'(0) = 7$.