Economics 204 Summer/Fall 2018
Final Exam

Answer all of the questions below. Be as complete, correct, and concise as possible. There are 6 questions for a total of 165 points possible; point values for each problem are in parentheses. For questions with subparts, each subpart is worth the same number of points. You have 180 minutes to complete the exam. Use the points as a guide to allocating your time. You may use any result from class with appropriate references unless you are specifically being asked to prove it.

Do not turn the page until the exam begins.

1. (15) Define or state each of the following.

   (a) convergence of a sequence $\{x_n\}$ to a point x in a metric space (X, d)

   (b) linear transformation between vector spaces X and Y over the same field F

   (c) Separating Hyperplane Theorem

2. (30) Let $r \in (0, 1)$. Show that for every $n \in \mathbb{N} = \{1, 2, 3, \ldots\}$,

$$\sum_{k=0}^{n} r^k = \frac{1 - r^{n+1}}{1 - r}$$

3. (30) Let X be a finite-dimensional vector space, and V and W be vector subspaces of X such that $V \cap W = \{0\}$.

   (a.) Show that $V + W = \{x \in X : x = v + w \text{ for some } v \in V, w \in W\}$ is a vector subspace of X.

   (b.) Show that $\dim (V + W) = \dim V + \dim W$.

4. (30) Let (X, d) be a metric space and $C \subseteq X$ be compact. Let $A \subseteq C$ be a nonempty set. Suppose that for each $x \in C$ there exists $\varepsilon_x > 0$ such that $A \cap B_{\varepsilon_x}(x)$ is either empty or is a finite set. Show that A is a finite set.

5. (30) Let $f : [a, b] \to [a, b]$ where $a, b \in \mathbb{R}$ and $a < b$. Suppose f is continuous on $[a, b]$ and differentiable on (a, b). Suppose f has a fixed point x^* such that $x^* \in (a, b)$ and $f'(x^*) > 1$. Show that f has at least three fixed points.

   (**Hint:** Draw a picture.)

6. (30) Let $X \subseteq \mathbb{R}^n$ and $\Psi : X \to 2^{\mathbb{R}}$ be an upper hemicontinuous correspondence with nonempty, compact values, so $\Psi(x) \subseteq \mathbb{R}$ is a nonempty compact set for each $x \in X$. Let $C \subseteq X$ be compact. Show that Ψ attains its maximum and minimum on C. That is, if

   $$M = \sup\{y \in \mathbb{R} : y \in \Psi(x) \text{ for some } x \in C\} \quad m = \inf\{y \in \mathbb{R} : y \in \Psi(x) \text{ for some } x \in C\}$$

   then show that there exist $x_M, x_m \in C$ such that $M \in \Psi(x_M)$ and $m \in \Psi(x_m)$.