1. Let $A$ be an $n \times n$ matrix.
   
   (a) Show that if $\lambda$ is an eigenvalue of $A$, then $\lambda^k$ is an eigenvalue of $A^k$ for $k \in \mathbb{N}$.
   
   (b) Show that if $\lambda$ is an eigenvalue of the matrix $A$ and $A$ is invertible, then $1/\lambda$ is an eigenvalue of $A^{-1}$.
   
   (c) Find an expression for $\det(A)$ in terms of the eigenvalues of $A$.
   
   (d) The eigenspace of an eigenvalue $\lambda_i$ is the kernel of $A - \lambda_i I$. Show that the eigenspace of any matrix $A$ belonging to an eigenvalue $\lambda_i$ is a vector space.

2. Let $V$ be an $n$-dimensional vector space. Call a linear operator $T : V \to V$ idempotent if $T \circ T = T$. Prove that all such operators are diagonalizable (that is, any matrix representation $A = Mtx_U(T)$ is diagonalizable). What are the eigenvalues?

3. Let $V$ be a finite-dimensional vector space and $W \subset V$ be a vector subspace. Prove that $W$ has a complement in $V$, i.e., there exists a vector subspace $W' \subset V$ such that $W \cap W' = \{0\}$ and $W + W' = V$.

4. Let $U$ and $V$ be vector spaces. Suppose $T : U \to V$ is a linear transformation and $v \in V$. Prove that, if the preimage $T^{-1}(v)$ is non-empty, and $u \in T^{-1}(v)$, then $T^{-1}(v) = \{u + z | z \in \ker T\} = u + \ker T$.

5. Let $V$ be a finite dimensional vector space and $T, S \in L(V, V)$. Prove that $TS$ is invertible if and only if $T$ and $S$ are invertible.

6. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be given by $T(x, y) = (4x - 2y, x + y)$. Let $V$ be the standard basis and $W = \{(5, 3), (1, 1)\}$ be another basis of $\mathbb{R}^2$.

   (a) Find $Mtx_V(T)$.

   (b) Find $Mtx_W(T)$.

   (c) Compute $T(4, 3)$ using the matrix representation of $W$. 