1. Give an example of a complete metric space which is homeomorphic to an incomplete metric space.

2. Let \((E, d)\) be a metric space and \(S \subseteq E\) a subset. Show that \(A \subseteq S\) is open relative to \(S\) if and only if \(A = S \cap U\) for some \(U \subseteq E\) open.\(^1\)

3. Let \((X, d)\) be a metric space. Assume \(f : X \to \mathbb{R}\) and \(g : X \to \mathbb{R}\) are uniformly continuous on \((X, d)\) and \((\mathbb{R}, |\cdot|)\), with \(|\cdot|\) the absolute-value norm.
   
   (a) Show that \(f + g : X \to \mathbb{R}\) is uniformly continuous, where \((f + g)(x) = f(x) + g(x)\).
   
   (b) Show that \(\max\{f, g\} : X \to \mathbb{R}\) is uniformly continuous, where \(\max\{f, g\}(x) = \max\{f(x), g(x)\}\).
   
   (c) Give a counterexample to the following statement: \(f \cdot g : X \to \mathbb{R}\) is uniformly continuous on \((X, d)\) and \((\mathbb{R}, |\cdot|)\), where \(f \cdot g = f(x) \cdot g(x)\).

4. A function \(f : X \to Y\) is open if \(\forall A \subseteq X\) such that \(A\) is open, \(f(A)\) is open. Show that any continuous open function from \(\mathbb{R}\) into \(\mathbb{R}\) is strictly monotonic.

5. Let \(X = [0, 1)\) and define
   
   \[d(x, x') := \inf \{|x - x' + k| : k \in \mathbb{Z}\}, \quad \forall x, x' \in X\]
   
   Show that \(d(x, x')\) is a metric over \(X\). \(\text{Hint:}\) find the \(k\) that yields the infimum for each pair \((x, x') \in X^2\).

6. For some metric space \((X, d)\), take any two sets \(A, B \subseteq X\) such that \(\text{int} A = \text{int} B = \emptyset\), and \(A\) is closed. Prove that \(\text{int}(A \cup B) = \emptyset\).

---

\(^1\) \(A \subseteq S\) is open relative to \(S\) if \(\forall x \in A \exists r_x > 0\) such that \(B_{r_x}(x) \cap S \subseteq A\).